Dear all,

Recently I read the difference scheme of galprop, i.e. file propel.cc. I notice that in v56, a r, z dependence of diffusion coefficient has been implemented. But I have a question about this.

The diffusion term is usually written as

$$-\nabla(D\nabla\psi) \ , \tag{1}$$

which in cylindrical coordinate is

$$\begin{aligned} & -\frac{1}{r}\frac{\partial}{\partial r}\left(D_{rr}(r,z)r\frac{\partial\psi}{\partial r}\right) - \frac{\partial}{\partial z}\left(D_{zz}(r,z)\frac{\partial\psi}{\partial z}\right) \\ &= -\frac{D_{rr}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) - \frac{\partial D_{rr}}{\partial r}\frac{\partial\psi}{\partial r} - D_{zz}\frac{\partial^2\psi}{\partial z^2} - \frac{\partial D_{zz}}{\partial z}\frac{\partial\psi}{\partial z} \ . \end{aligned}$$

The difference schemes for radial term in my mind are

$$\begin{aligned} -\frac{D_{rr}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) &= -\frac{D_{rr}}{r_i\Delta r}\left(r_{i+1/2}\frac{\psi_{i+1}-\psi_i}{\Delta r} - r_{i-1/2}\frac{\psi_i-\psi_{i-1}}{\Delta r}\right) \ , \\ &= -\frac{D_{rr}}{r_i\Delta r}\left(\frac{r_i+\Delta r/2}{\Delta r}\psi_{i+1} - \frac{2r_i}{\Delta r}\psi_i + \frac{r_i-\Delta r/2}{\Delta r}\psi_{i-1}\right) \ , \\ &= -D_{rr}\left(\frac{2r_i+\Delta r}{2r_i\Delta r^2}\psi_{i+1} - \frac{2}{\Delta r^2}\psi_i + \frac{2r_i-\Delta r}{2r_i\Delta r^2}\psi_{i-1}\right) \ . \end{aligned} \tag{2}$$

$$\begin{aligned} -\frac{\partial D_{rr}}{\partial r}\frac{\partial\psi}{\partial r} &= -\frac{(D_{rr,i+1}-D_{rr,i-1})}{2\Delta r}\frac{(\psi_{i+1}-\psi_{i-1})}{2\Delta r} \\ &= -\frac{(D_{rr,i+1}-D_{rr,i-1})}{(2\Delta r)^2}\psi_{i+1} + \frac{(D_{rr,i+1}-D_{rr,i-1})}{(2\Delta r)^2}\psi_{i-1} \end{aligned} \tag{3}$$

But in propel.cc, the difference scheme for the second term seems

$$\frac{D_{rr,i+1}-D_{rr,i-1}}{(2\Delta r)^2}\left(1+\frac{\Delta r}{r_i}\right)\psi_{i+1} - \frac{D_{rr,i+1}-D_{rr,i-1}}{(2\Delta r)^2}\left(1-\frac{\Delta r}{r_i}\right)\psi_{i-1} \ . \tag{4}$$

```
    // modification in propagation equation if diffusion coefficient depends on z,r
#pragma omp parallel for schedule(dynamic) default(shared) private(ir,iz,ip)
    for (ir = 1; ir < particle.n_rgrid-1; ++ir) {
        for (iz = 1; iz < particle.n_zgrid-1; ++iz) {
            for (ip = 0; ip < particle.n_pgrid; ++ip) {
                const double dDxxdr=(particle.Dxx.d2[ir+1][iz].s[ip]-particle.Dxx.d2[ir-1][iz].s[ip]) *pow(2.*particle.dr,-2.
);
                const double dDxxdz=(particle.Dxx.d2[ir][iz+1].s[ip]-particle.Dxx.d2[ir][iz-1].s[ip]) *pow(2.*particle.dz,-2.
);
//                std::cout<<"ddxxdr="<<ddxxdr<<" ddxxdz="<<ddxxdz<<" alpha_r1="<<(alpha1_r.d2[ir][iz].s[ip])<<" alpha_z1="<<
alpha1_z.d2[ir][iz].s[ip] <<std::endl;
                alpha1_r.d2[ir][iz].s[ip] -= dDxxdr*(1-particle.dr/particle.r[ir]);
                alpha3_r.d2[ir][iz].s[ip] += dDxxdr*(1+particle.dr/particle.r[ir]);
                alpha1_z.d2[ir][iz].s[ip] -= dDxxdz;
                alpha3_z.d2[ir][iz].s[ip] += dDxxdz;
            }//ip
        }//iz
    }//ir
```

Figure 1:

Compared with my scheme, there are additional two multipliers $\left(1-\dfrac{\Delta r}{r_i}\right)$ and $\left(1+\dfrac{\Delta r}{r_i}\right)$, as shown in Figure 1.