We illustrate the theoretical description of this effect by Markus.Roth2@kit.edu where the fit of a smooth function to Auger data of cosmic rays following a log-normal distribution with average value \( \lnE \) and variance \( \sigma^2 = \ln \left( 1 + (\DeltaE)^2 \right) \) and thereby the primary spectrum is folded with an experimental resolution.

The energy resolution

\[
\DeltaE \approx \sigma \ln \left( \frac{E}{\lnE} \right) \approx \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left( -\frac{(\lnE - \lnE_0)^2}{2\sigma^2} \right).
\]

To perform the integration in Eq. 2, one should bear in mind that the energy resolution of a detector is energy dependent. Generally, it slightly improves towards higher energies like it is the case for the Pierre Auger Observatory. However, using the characteristic resolution of a detector and setting it constant over the whole energy range simplifies the calculation and still marks a sufficient approximation.

Folding a steeply falling flux leads to a shift upwards in energy, see Fig. 1. A detector will observe more events above a given threshold than it actually should, as a certain fraction of events happen to be overestimated in the reconstruction process due to the finite energy resolution. The set of events fulfilling this criterion are denoted as spillover.

Spillover has considerable impact on anisotropy studies. The usual approach is to use events above the Greisen-Zatsepin-Kuz’min cut-off energy (\( \approx 55\,\text{EeV} \)) and discard the rest as they are expected to be isotropic in their arrival directions and therefore would represent a hindering factor (e.g. [7]). Since the definition from Eq. 1 constitutes the basis of our considerations, we choose \( E_{1/2} = 10^{19.63} \text{eV} \approx 43\,\text{EeV} \) as energy threshold. However, the following mathematics also apply to any other choice.

Due to the folding a complete separation of spillover from signal events on shower-by-shower basis is not possible. Yet, Eq. 3 enables estimating their ratio. Limiting the upper bound of integration to the given threshold \( E_{1/2} \) truncates the spectrum such that the log-normal distribution solely smears events from below.
Figure 1: Spectra for a JEM-EUSO-like energy resolution of $\Delta E = 30\%$ (top) in comparison to an Auger-like resolution of $\Delta E = 12\%$ (bottom). Fit to real data refers to Eq. (1). The measured and truncated spectra are given by Eq. (2) and Eq. (4). The chosen threshold $E_{1/2}$ is represented by the grey vertical line.

Figure 2: Top: Spectra expected to be obtained by JEM-EUSO considering full FoV accounting for the respective energy resolution and effects of trigger efficiency which is defined in Eq. (5). Bottom: Trigger efficiency of JEM-EUSO considering the full FoV (solid line) and the aperture cut described in the text (dashed line).

$J_{\text{truncated}}(E) = \int_{E_{1/2}}^{E_{1/2}} J(E_0) \cdot f(E; \ln E_0, \sigma) \, dE_0.$ (4)

The ratio of spillover above the threshold is then given by

$R_{\text{spillover}}(E \geq E_{1/2}) = \frac{\int_{E_{1/2}}^{\infty} J_{\text{measured}}(E) \, dE}{\int_{E_{1/2}}^{\infty} J_{\text{measured}}(E) \, dE}.$ (5)

The obtained spectra are shown together in Fig. 1 for two different energy resolutions as they are featured by the Pierre Auger Observatory and JEM-EUSO. Isotropic spillover and the remaining (possible) anisotropic signal events are visualized by colored areas in red and blue.

An Auger-like energy resolution of $\Delta E = 12\%$ [?] yields a spillover ratio of about 20%. This ratio rapidly increases towards poorer resolutions to be almost 60% for a JEM-EUSO-like $\Delta E = 30\%$ [?]. The actual degree of correlation that is observable in UHECR arrival directions therefore gets obfuscated and the probability of indentifying specific sources decreases regardless of the overall sample size.

In case of JEM-EUSO the trigger efficiency of the detector also needs to be accounted for to obtain a more precise estimate. While the Pierre Auger Observatory is fully efficient well below the chosen threshold, JEM-EUSO just starts to become efficient. Considering its full field of view (FoV), JEM-EUSO is expected to feature 50% efficiency at $E = 3 \times 10^{19}$ eV and 90% efficiency at $E = 10^{20}$ eV [?].

Since not all events below $E_{1/2}$ trigger the detector, there are less events to be overestimated in energy and able to diminish the contribution of signal events beyond. An error function constitutes the common way to approximate the trigger efficiency $T(E)$ as a function of energy which is then given by

$T(E) = \frac{1}{2} \left( 1 - \text{erf}\left( \frac{\log_{10} E - \log_{10} E_{50}}{\log_{10} W} \right) \right).$ (6)

where $E_{50} = 3 \times 10^{19}$ eV is the energy of 50% efficiency and $\log_{10} W = 0.55$ characterizes the width of the transition region.

Applying aperture cuts in data improves the trigger efficiency for corresponding subsets of events. More stringent cuts are able to shift the transition region further downwards. Yet, the overall exposure decreases accordingly.

Restricting data to events that are located within a distance of $R < 125$ km from the center of FoV and incoming from zenith angles $\theta > 60^\circ$ results in 90% efficiency already for energies of $3 \times 10^{19}$ eV. The parametrization in Eq. (6) is then determined by $E_{50} = 10^{19.25}$ eV and $\log_{10} W = 0.25$. Both the efficiency accounting for the full FoV and the aperture cut are shown in the bottom panel of Fig. 2.

To incorporate its effects into the folding of the energy resolution, the trigger efficiency first has to be applied to the underlying true energy distribution

$J_{\text{triggered}}(E) = T(E) \cdot J(E).$ (7)

The spectrum of triggered events $J_{\text{triggered}}(E)$ then replaces $J(E)$ in Eq. (2) and Eq. (4) in order to obtain the new measured and truncated spectra, which are shown in the top panel of Fig. 2 together with the resulting ratio of spillover.

Accounting for effects of the trigger efficiency of JEM-EUSO (full FoV) improves the expected ratio of spillover by 9%. Yet, 49% of the events in the data sample still have to be considered as background.

3 Purifying the sample

Large ratios of spillover present a strong handicap to any anisotropy study in order to identify specific sources of UHECRs. The rather intuitive possibility to purify the sample again is to apply an additional energy cut in data with $E_{\text{cut}} > E_{1/2}$

$R_{\text{spillover}}(E \geq E_{1/2}) \rightarrow R_{\text{spillover}}(E \geq E_{\text{cut}}).$

The chosen energy threshold $E_{1/2} = 10^{19.63}$ eV to calculate the measured and truncated spectra remains fixed in the process.

Applying higher cuts progressively removes background events from the sample and consequently enhances the ratio, which is therefore an important approach for detectors that otherwise suffer from substantial spillover. JEM-EUSO, for instance, essentially aims at energies far beyond $E_{1/2}$. Using $E_{\text{cut}} = 10^{19.80}$ eV in the corresponding data yields a ratio of approximately 18% that is comparable to the one of the Pierre Auger Observatory. The evolution of the spillover ratio as a function of cut energy for different energy resolutions is illustrated in Fig. 2.

However, more stringent cuts rapidly reduce the overall sample size, which limits the maximum cut energy since a reasonable study can only be based on an adequate number of events. Due to the extremely large exposure using $E_{\text{cut}} = 10^{19.80}$ eV still leaves a considerable sample of 144 events per year for JEM-EUSO, while only about 8 events survive this cut for the Pierre Auger Observatory (see Tab. 1).

4 Conclusions

A reasonable energy resolution constitutes a crucial property of cosmic ray detectors to separate background from
Importance of energy resolution

Table 1: Expected number of events per year for different energy cuts. Trigger efficiency is accounted for in the calculation for JEM-EUSO. Its annual exposure considering the full FoV is expected to amount 60,000 km\(^2\) sr\(^{-1}\). The geometric aperture of Auger South is about 7,000 km\(^2\) sr. The effective site coverage is currently limited to 80%, which yields an annual exposure of approximately 5,500 km\(^2\) sr.

<table>
<thead>
<tr>
<th></th>
<th>(\mathcal{E} / (\text{km}^2 \text{sr yr}))</th>
<th>(\log_{10}(E_{\text{cut}} / \text{eV}))</th>
<th>(N_{\text{total}} / \text{yr})</th>
<th>(N_{\text{spillover}} / \text{yr})</th>
<th>(R_{\text{spillover}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pierre Auger Observatory</td>
<td>5,500</td>
<td>19.63</td>
<td>36</td>
<td>8</td>
<td>22%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19.80</td>
<td>8</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>JEM-EUSO (full FoV)</td>
<td>60,000</td>
<td>19.63</td>
<td>433</td>
<td>212</td>
<td>49%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19.80</td>
<td>144</td>
<td>25</td>
<td>18%</td>
</tr>
</tbody>
</table>

Possibly source-correlated signal events in order to identify sources of UHECRs. Poor resolutions cause a large ratio of background events and thereby obfuscate the observable degree of correlation.

Additional energy cuts in data may improve the ratio though only at the expense of a rapidly reduced number of events due to the steeply falling flux. Data samples of air shower arrays with an exposure comparable to the Pierre Auger Observatory do not allow applying high purifying cuts so that the approach only presents a limited opportunity for improvement.

However, the situation changes for JEM-EUSO which exceeds the annual exposure of Auger by approximately one order of magnitude. Energy resolution then becomes a secondary factor.

It should be noted that the transition from isotropy to anisotropy is not expected to be completely sharp as it was assumed in this case for illustrative purposes. Particles will not instantly lose all directional information if their energy falls below the chosen threshold. Especially not, if they originate from nearby sources (on extragalactic scales). To what extent particles still point back to their origin naturally depends on the strength of the intergalactic and galactic magnetic fields and the distance of the respective source (e.g. \(^{\text{[?]}}\)). Stronger fields extend isotropy to higher energies.

Extensive numerical simulations are required to determine the actual impact of spillover on the observable degree of correlation. Due to the limited information available about possible UHECR sources and intergalactic magnetic fields, there are many unknown parameters on which the outcome depends. These topics should be addressed in future studies.