Drift motions of galactic cosmic rays in the regular galactic magnetic field

SHOKO MIYAKE¹, SHOHEI YANAGITA²

¹ College of Community Development, Tokiwa University, Mito, Ibaraki 310-8585, Japan
² Faculty of Sciences, Ibaraki University, Mito, Ibaraki 310-8512, Japan

miyakesk@tokiwa.ac.jp

Abstract: The effects of the drift motion of the galactic cosmic rays (GCRs) in the regular galactic magnetic field (GMF) on its propagation in the interstellar space are numerically investigated. The calculations are made for two models of the magnetic field, axisymmetric spiral structure and bisymmetric spiral structure. We assume the GCRs are originated in supernova remnants (SNRs) and adopt the stochastic method in numerical calculations to take into account the discreteness of the sources both in space and time. The spatial distribution of SNRs where the GCRs arriving at the solar system were originated depends strongly on the GMF model adopted due to the effects of the drift motions and the dependence of the diffusion coefficient on the magnetic field strength. The characteristic source distribution may have some relation to the anisotropy of the GCRs which arrive at solar system.

Keywords: galactic cosmic ray, propagation, galactic magnetic field, supernovae

1 Introduction

The propagation of galactic cosmic ray (GCR) in the interstellar space has normally been investigated in the frame work of the diffusion convection equation assuming that the distribution of the sources of GCRs is continuous both in space and time. Various physical processes involving the propagation of GCR such as diffusion in space, nuclear reactions, convection by the galactic wind, and various energy changes are taken into account explicitly in the partial differential equations, however, studies which addressed the effect of drift motions of GCR in the galactic magnetic field (GMF) are few. As is well known, the drift motions in the heliospheric magnetic field are indispensable ingredients to understand the 22 year cycle of the solar modulation phenomena of GCR (e.g. [1]; [2]; [3]). Similarly it is expected the drift motions of GCR may play an important role in the propagation of GCR in the galaxy.

We have investigated numerically the effects of the drift motions of GCR on its propagation for two models of global regular GMF, for axisymmetric spiral structure (ASS) and for bisymmetric spiral structure (BSS). Here we assume the GCRs are originated in supernova remnants as indicated by various evidences and adopt the stochastic method in numerical calculations to take into account the discreteness of both in space and time the distribution of the sources. The method is based on the equivalence of a coupled set of stochastic differential equations (SDE) to the Parker’s convection-diffusion equation describing the propagation of GCR. This SDE method has been applied successfully to the investigation of the solar modulation of GCR in the heliosphere (e.g. [4]; [5]), the galactic modulation of extragalactic hypothetical CRs [6] and the structure of the CR electron halo in starburst galaxies [7].

In this paper, we present the details of the results for the effects of the drift motions on the expected distribution of the sources of GCR protons and electrons for the two models of the GMF, ASS and BSS.

2 Numerical model

The SDE equivalent to the Parker’s diffusion-convection equation describing the propagation of GCRs is written as

\[
\frac{dX}{dt} = (\nabla \cdot \kappa + \mathbf{V} + \nabla \cdot \mathbf{d}) dt + \sum \sigma_s dW_s(t),
\]

\[
\frac{dP}{dt} = -\frac{1}{3} P(\nabla \cdot \mathbf{V}) dt - dP_{\text{sync}} - dP_{\text{IC}},
\]

where \(X\) and \(P\) indicates the position of the guiding center and the momentum of the particle respectively, \(t\) is the time, \(\mathbf{V}\) is the velocity of the galactic wind, \(\mathbf{d}\) is the drift velocity describing the drift motions of GCR in the galactic magnetic field (GMF), \(\nabla \cdot \kappa\) is the divergence of the diffusion coefficient tensor, \(\sum \sigma_s dW_s\) is a Wiener process given by the Gaussian distribution, \(P_{\text{sync}}\) and \(P_{\text{IC}}\) is the synchrotron and the inverse Compton momentum loss in the case of electrons. In this study, we have neglected the existence of the galactic wind for simplicity, so the terms \(\frac{1}{3} P(\nabla \cdot \mathbf{V}) dt\) and \(-\frac{1}{3} P(\nabla \cdot \mathbf{V}) dt\) in (1) become zero. We adopted

\[
\kappa = \frac{1}{3} l_{m,f.p.} v,
\]

\[
l_{m,f.p.} = 3 \times 10^{17} \left( \frac{P}{1 \text{GeV}} \frac{1}{\text{C}} \right)^{2-\delta} \left( \frac{B}{1 \mu \text{G}} \right)^{-1} \left[ \text{cm}^2/\text{sec} \right],
\]

\[
\mathbf{V}_d = \frac{\nu v}{3q} \times \left( \frac{B}{B^2} \right),
\]

\[
\frac{dP_{\text{sync}}}{dt} = 4 \frac{\sigma_T}{\Gamma} \frac{c \beta}{8 \pi} B^2,
\]

\[
\frac{dP_{\text{IC}}}{dt} = 4 \frac{\sigma_T}{\Gamma} \frac{c \beta}{8 \pi} U_{ph},
\]

where \(l_{m,f.p.}\) is the mean free path of the particle, \(\nu\) is the velocity of the particle, \(B\) is the GMF strength, \(\delta\) is the power law index describing the Alfvén wave spectrum (e.g. [8]), \(\sigma_T\) is the Thomson cross section, \(\Gamma\) is the Lorentz factor, and \(U_{ph}\) is the energy density of the interstellar
photons. We assume $\delta = \frac{1}{2}$ for a Kraichnan spectrum. We also assume $U_{ph}$ are 0.26 eV cm$^{-3}$ for CMB, 0.20 eV cm$^{-3}$ for reemitted radiation from dust grains, and 0.45 eV cm$^{-3}$ for stellar radiation, respectively.

We adopt the value of the parameters for the models of ASS and BSS from the reference [11] which describes better the polarized synchrotron emission as seen by the WMAP satellite.

The components for the ASS model of GMF are given by

$$B_r = B_0(r) \sin(p) \cos(\chi(z)),$$

$$B_\phi = B_0(r) \cos(p) \cos(\chi(z)),$$

$$B_z = B_0(r) \sin(\chi(z)),$$

where $p$ is the pitch angle, $B_0(r)$ is the field strength, and $\chi(z)$ is the tilt angle. We adopted

$$\chi(z) = \chi_0 \tanh \left( \frac{z}{1 \text{ kpc}} \right),$$

$$B_0(r) = \frac{3r_1 + 24}{r_1 + r},$$

where $z$ and $r$ are given in kpc and $B_0(r)$ in $\mu$G. We assume $r_1 = 30$ kpc, $p = 14.6^\circ$, and $\chi_0 = 18.5^\circ$ for the ASS model. These values are consistent with the best fit parameters of the mask 6 shown in Table 4 of [11].

The components for the BSS model of GMF are given by

$$B_r = B_0(r) \cos(\phi \pm \eta \ln \left( \frac{r}{8 \text{ kpc}} \right)) \sin(p) \cos(\chi(z)),$$

$$B_\phi = B_0(r) \cos(\phi \pm \eta \ln \left( \frac{r}{8 \text{ kpc}} \right)) \cos(p) \cos(\chi(z)),$$

$$B_z = B_0(r) \sin(\chi(z)),$$

where $\eta = \frac{1}{2 \tan(\phi)}$. Here we assume “Positive” BSS model (BSS$_+$) and “Negative” BSS model (BSS$_-$) which correspond to the positive and negative sign inside the big parenthesis for $B_r$ and $B_\phi$. We assume $r_1 = 40$ kpc, $p = 10.5^\circ$, and $\chi_0 = 17.6^\circ$ for the BSS$_+$ model, and also assume $r_1 = 35$ kpc, $p = 7.2^\circ$, and $\chi_0 = 19.2^\circ$ for the BSS$_-$ model. These values are consistent with the best fit parameters of the mask 6 shown in Table 4 of [11].

Figure 1 shows plane views of the strength of GMF and stream line of the drift velocity at $z = 1$ pc. We can see the obvious differences between ASS model and BSS$_\pm$ model. The directions of drift motions in the southern part of the galaxy are reversed from those in the northern part because the directions of the magnetic field lines are reversed.

In this study, we are interested only in the GCRs arriving at the solar system. When the final state is fixed like this study, we can solve the SDEs backward in time. In our simulation, particles start at the solar system and run backward in time until they arrive at some SNR active in accelerating GCRs as described below. The momentum spectrum $f_{X}(p)$ at arbitrary position $X$ is written as a convolution of the spectrum $f_{X_0}(p_0)$ at the GCR source with the normalized transition probability $F(p_0, X_0 | p, X)$ obtained by our SDE method as

$$f_{X}(p) = \int f_{X_0}(p_0) F(p_0, X_0 | p, X) d p_0.$$

Here we assume the spectrum at all of the GCR sources is $f_{X_0}(p_0) \propto (n_p^2 c^4 + p_0^2 c^2)^{-1.6} / p_0$. In our simulation, we don’t set the escape boundary of the GCRs in the Galaxy. Some particles can’t reach any active SNR, even if we follow their trajectory for a long time. We set the maximum time of simulation in each particle to $5 \times 10^9$ yr. We neglected those particles which cannot reach any active SNR from the result.

For a galactocentric Cartesian coordinate system $(x, y, z)$, we assume the solar system is locates at $(8.5$ kpc, 0 kpc, 0 kpc). We also assume that the SN occurs at random in time and space within the radius of 20 kpc from the center of the Galaxy. Our stochastic simulation needs to set life and size of the SNR. The life of the SNR corresponds to the duration of the acceleration of GCRs in the SNR. From the recent studies of some SNRs, it is suggested that the escape time of GCRs from their accelerating region depends on their energy. In our simulation we assume for simplicity the acceleration of GCRs continues at a uniform rate in SNRs for $T_{SNR} = 10^5$ yr after the SN explosion irrespective of the types of the SN. We assume also the accelerated GCRs escape their birth place uniformly. The radius of SNRs is assumed to be fixed as 30 pc following the Sedov model by adopting the values of $10^8$ yr, $10^{51}$ ergs and 1 proton cm$^{-3}$ for the age, the expansion energy and the ambient matter density, respectively. In this simulation, the active SNR is defined as the region of $|X_{SNR} - X| < 30$ pc during $T_{SNR} \leq t \leq T_{SNR}$. Where $X_{SNR}$ and $T_{SNR}$ are the position and time when SN occurred. We assume the surface density of SNR rate has a galactocentric radial dependence scaled to the molecular cloud given by [9], and has a Gaussian height distribution such as
\[ P_{SN} = \left(2\pi\alpha_{SN}^2\right)^{-1/2} \exp\left(-\frac{z^2}{2\alpha_{SN}^2}\right) \left[\text{kpc}^{-1}\right]. \] 

where \( \alpha_{SN} = 0.070 \text{ kpc} \). The rate of SN is assumed to be 3 SNs per 100 yr in the Galaxy.

### 3 Results and Conclusions

We collected the lists of SNRs which were picked up as a source of GCRs by our simulation for the three types of GMF model. Figure 2 (a), (b), and (c) shows the spatial distribution of the source SNRs for the electrons observed with a fixed energy of 100 GeV at the solar system overlaid with the contours of the strength of the magnetic field for GMF, ASS, BSS, and BSS+, respectively. The position of the SNRs are indicated as blue points. Red point indicates the solar system. In these simulations we have not take account of the drift motions of GCR electrons in GMF. As seen in Figure 2 (a), the sources of the GCR electrons for ASS model is distributed almost isotropically in the region of several kpc centered at the solar system. On the contrary for BSS, models, the sources of GCR electrons have a tendency to be distributed along regions with relatively weak GMF strength. This tendency for the distribution of the source SNRs may come from the dependence of the mean free path on the strength of the ambient magnetic field. Diffusion velocities of electrons are much higher in regions with weaker magnetic field, because we assumed the mean free path is inversely proportional to the magnetic field strength. We may expect GCRs can arrive at the solar system from more distant SNRs if they propagate through regions with relatively weak magnetic field.

Figure 3 is the same as Figure 2, but here we take account of the drift motions in GMF. Thin lines indicate directions of the drift motion. Contrary to our expectation the role of drift motions in the propagation of GCRs seems to be not so important, because the characteristics in the resultant distribution of the source SNRs are qualitatively the same as in the result for the case without drift motions although a little differences can be seen. These results may be understandable if we recognize that the expected drift velocity is less than the typical diffusion velocity of GCRs in the galaxy. We also found small effects of the drift motion on the age and path length distribution of the GCRs which arrive at the solar system, though we do not show the details here.

The effects of the drift motion may be seen in the energy spectrum of TeV electrons. As is well known, very high energy electrons can reach the solar system only from local sources, because electrons with energy, say above TeV, suffer from severe energy loss by synchrotron emission and the inverse Compton scattering during their journey from distant sources. Accordingly only a few local SNRs may contribute to the electron spectrum at the earth in these high energy regions. The electron spectrum from each local SNR may be affected by how well the source is connected to the solar system magnetically. These studies are now in progress.

Electrons and protons drift away in the opposite direction because the drift velocity depends on the sign of the charge of particles. Figure 5 shows the distribution of source SNRs of protons observed at the earth with a fixed energy of 100 GeV for the same GMF configurations shown in Figure 2 for electrons. We cannot recognize any clear evidences of differences in the distribution of the source SNRs between protons and electrons except that the SNRs for protons distribute in much larger regions which may be expected from the difference in energy.

Finally we point out an interesting finding by our simulations of anisotropic distribution of the source SNRs both for electrons and protons in case of BSS as shown in Figures 2,[3][4] This anisotropic distribution of the sources may be reflected to and observed as anisotropies in the intensity of GCRs.

Acknowledgment: This work has been supported in part by KAKENHI.

### References

Figure 2: (a) Source distribution of the GCR electrons (100 GeV at solar system) for ASS model. The drift motion is not considered in the calculation. (b) Same as (a) for BSS+ model. (c) Same as (a) for BSS− model.

Figure 3: (a) Source distribution of the GCR electrons (100 GeV at solar system) for ASS model. The drift motion is considered in the calculation. (b) Same as (a) for BSS+ model. (c) Same as (a) for BSS− model.

Figure 4: (a) Source distribution of the GCR protons (100 GeV at solar system) for ASS model. The drift motion is considered in the calculation. (b) Same as (a) for BSS+ model. (c) Same as (a) for BSS− model.