Theoretical study of the 27-day variation of the galactic cosmic rays intensity in connection with solar wind parameters in different epochs of solar activity

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Abstract: Theoretical study of the 27-day variation of the galactic cosmic ray intensity requires an inclusion of data that can be treated as a general cause of considered recurrence. We propose a model of the 27-day variation of the galactic cosmic rays intensity based on the 3-D Parker’s transport equation with implemented in situ measurements of the changes in heliographic longitude of the interplanetary magnetic field and solar wind speed in different epochs of solar activity. Furthermore, the experimentally found exponent γ of the rigidity spectrum of the 27-day variation of the galactic cosmic rays intensity is, among other parameters, partially used for calculation of the diffusion coefficient incorporated in the modelling.

Keywords: 27-day variation of the GCR, rigidity spectrum, solar wind, interplanetary magnetic field

1 Motivation
The 27-day variation of the galactic cosmic rays (GCR) intensity was discovered in the middle of last century, but still is in the interest and there are lots of publications devoted to this problem (e.g. [21, 14, 20, 2, 6, 17, 12, 18, 22]). However, changes of the amplitudes of the 27-day variation of the GCR intensity versus the rigidity R were studied only by [8, 7]. There was demonstrated that the rigidity spectrum of the 27-day variation of the GCR intensity is hard in the maximum epochs, and soft in the minimum epochs of solar activity (SA).

Our purpose in this paper is twofold: (1) to model the 27-day variation of the galactic cosmic rays intensity based on the 3-D Parker’s transport equation with implemented in situ measurements of the changes in heliographic longitude of the interplanetary magnetic field and solar wind speed in different epochs of solar activity. Furthermore, the experimentally found exponent γ of the rigidity spectrum of the 27-day variation of the GCR intensity is, among other parameters, partially used for calculation of the diffusion coefficient incorporated in the modelling.

2 Methodology
The theoretically expected amplitudes and rigidity spectrum exponent γ of the GCR intensity 27-day variation we calculate using the Parker’s transport equation [19] containing all elementary processes taking place in the heliosphere: convection, diffusion, energy changes of the GCR particles owing to the interaction with solar wind’s inhomogeneities, drift due to the gradient and curvature of the regular interplanetary magnetic field (IMF) and on the heliospheric current sheet (HCS):

\[ V_i (\kappa_j \nabla f_j) - V_i (U_i f) + \frac{1}{3 R^2} \frac{\partial (f R^3)}{\partial R} (\nabla_i U_i) = \frac{\partial f}{\partial t} \]  

(1)

where \( f \) is the omnidirectional distribution function, \( R \) is the rigidity of the GCR particles, \( U \) is the solar wind velocity and \( t \)- time. The anisotropic diffusion tensor \( \kappa_i \) for the 3-D interplanetary magnetic field (IMF) is taken in to account as in [4, 3].

The parallel diffusion coefficient is considered as: \( \kappa_i = \kappa_0 \kappa_i (r) \kappa_2 (R) \), where \( \kappa_0 = 2 \times 10^{22} \text{cm}^2/\text{s} \) and \( \kappa_i (r) = 1 + 0.5 r/r_0 \); \( r \) is a radial distance and \( r_0 = 100 \text{AU} \) - the size of the modulation region, \( \kappa_2 (R) = a (R/100 \text{GV})^2 \). It matches to the dependence of the 27-day variation of the GCR intensity on the rigidity \( R \), rates of coefficients \( a \) and \( \gamma \) are taken into account based on the experimental results [8] and our postulations for different epochs of SA.

The GCR intensity in the interstellar space is taken according to [24, 5] as:

\[ I = 1.1 \times 10^{-2} \frac{1 + 5.85 f}{T + 1.187^{-4}} \]  

where \( T \) is the particles’ kinetic energy.

Eq. (1) was reduced to the linear algebraic system of equations by finite difference scheme and then was numerically solved using the Gauss - Seidel iteration method (e.g. [11]) for one rotation of the Sun, i.e. for instant state of the heliosphere, when the distribution of the GCR density is determined by the time independent parameters included in Eq. (1). In the model there is assumed the flat HCS according to our finding (e.g. [23]) that the amplitudes of the 27-day variation of the GCR intensity noticeably do not depend on the tilt angle of the HCS. The neutral sheet drift was taken into account according to the boundary condition method [10].

3 Results of mathematical modeling of the 27-day variation of the GCR intensity
To compose a consistent mathematical model we implement in situ measurements of solar wind parameters, namely solar wind speed and IMF strength. We consider two individual periods of solar rotation during different epochs of SA: October 7 – November 2 1997 (minimum epoch of solar activity, Fig. 1) and March 2-March 28 2013 (maximum epoch of solar activity, Fig. 2). Figures 1-2 presents daily changes of the IMF strength (B; Fig. 1a, 2a) and SW speed (U; Fig. 1b, 2b) in comparison with the GCR intensity measured by Rome neutron monitor (I; Fig. 1, 2).

We consider the correlations coefficients between the changes of the GCR intensity I and solar wind speed U \((r_1, r_2)\), also between I and IMF strength B \((r_3, r_4)\) as the weight values in approximations of U and B daily changes.
we insert a function $g$ of wind speed $U$, IMF strength $B$ according to in situ measurements, and $\kappa_2(R)$ have the expressions, for minimum epoch of solar activity (Eq. (3)):

$$U = r_1 U_0(\theta)(0.92 + [0.15\sin(\phi + 2.77) + 0.13\sin(2\phi + 5.71)]g(\rho));$$

$$B = r_2 B_0(1.04 + [0.07\sin(\phi + 2.05) + 0.31\sin(2\phi - 6.24)]g(\rho));$$

$$\kappa_2(R) = 5.5(R/10 GV)^{1.36}$$

for maximum epoch of solar activity (Eq. (4)):

$$U = r_3 U_0(\theta)(1.01 + [0.15\sin(\phi + 3.31) + 0.15\sin(2\phi + 1.06)]g(\rho)):$$

$$B = r_4 B_0(0.91 + [0.33\sin(\phi + 3.95) + 0.25\sin(2\phi - 6.06)]g(\rho));$$

$$\kappa_2(R) = 16(R/10 GV)^{0.6}$$

To avoid the intersections of the magnetic field lines we insert a function $g(\rho) = \exp(\rho(0.01 - \rho))/0.001$. The magnetic field lines of the slower solar wind behave as an asymptote with respect to the magnetic field lines of the faster solar wind and, due to the function $g(\rho)$, lines are asymptotically close to each other in a radial distance about 7.5-8 AU.

The level of correlation depicted by parameters $r_1$, $r_2$, $r_3$ and $r_4$ shows that the both- heliolontitudinal changes of the SW speed and the IMF strength, should be treated as the sources of the 27-day variation of the GCR intensity, with different contributions vs. solar activity stage (Fig. 1-2).

In situ measurements of the solar wind speed and the IMF strength as sources of the 27-day variation of the GCR intensity we implemented in our code; for this purpose we use the approximation to get analytic formula for those experimental data. Fig. 3 shows a comparison of the normalized experimental data (squares) with approximations (solid curves) described by the formulas (3-4). The correlation coefficients between experimental data and approximations are very high and have the following values: between the normalized SW speed and its approximation is 0.955 and 0.943 in the minimum and maximum epochs of SA, respectively; between the normalized IMF strength and its approximation is 0.934 and 0.931 in the minimum and maximum epochs of SA, respectively.

The values of the rigidity spectrum ($\delta D(R)/D(R)$) exponent $\gamma$ of the 27-day variation of the GCR intensity ($\delta D(R)/D(R) \propto R^{-\gamma}$) calculated using experimental data for considered solar rotations are: $\gamma \approx 1.4$ in 1997 and $\gamma \approx 0.6$ in 2013. Description of the calculation method of the power-law rigidity spectrum exponent $\gamma$ can be found in [1].

The solutions of the transport equation for the Earth orbit (at 1 AU) in the equatorial region ($\theta = 90^\circ$) present Figures 4 and Figures 5. Figures 4 illustrates that the expected rigidity spectrum of the 27-day variation of the GCR intensity in the maximum epoch of SA is harder than in the minimum epoch. In the minimum epoch expected exponent of the power-law rigidity spectrum $\gamma$ is equal to 1.3 (Fig. 4a) and in the maximum epoch $\gamma$ equals 0.8 (Fig. 4b). Figure 5 shows that experimental data-27-day wave of the GCR intensity (squares) satisfactorily coincides with the expected results (solid curve) in the minimum (Fig. 5a) and maximum (Fig. 5b) epochs of solar activity.
Fig. 3: The comparison of the normalized experimental data (squares) with approximation (solid curve) of the IMF strength (3a, c) and the solar wind speed (3b, d) in the minimum (3a, b) and maximum (3c, d) epochs of solar activity.

4 Temporal changes of the rigidity spectrum exponent $\gamma$ of the 27- day variation of the GCR intensity

Studying long- period changes of the rigidity spectrum exponent $\gamma$ of the 27- day variation of the GCR intensity ([8]) one can observe the polarity dependence in its behaviour- characteristic plateau pattern during positive polarity periods and pick in the negative polarity period (Fig. 6).

To clearly show the existence of the different periodicities in the temporal changes of the rigidity spectrum exponent $\gamma$ of the 27- day variation of the GCR intensity we use method of power spectrum density analyses (e.g. [9]) for the period of 1970-2004. Fig. 7 presents the relative power spectrum density (PSD) of the exponent $\gamma$ changes. There is clearly seen the existence of the periodicity of 142.9 $\pm$ 1.6 Carrington rotations, $\sim$ 11 years, in the PSD in the temporal changes of the rigidity spectrum exponent $\gamma$ of the 27- day variation of the GCR intensity.

In many papers are described different intermediate periodicities in the GCR intensity changes in the range from 0.5 to 3 years (e.g. [16, 13]). In this paper we study mid-term periodicities in the temporal changes of the rigidity spectrum exponent $\gamma$ of the 27- day variation of the GCR intensity. Figures 8a,b present results of the spectral analysis of the power-law rigidity spectrum exponent $\gamma$ of the 27- day variation of the GCR intensity. We found that the main periodicity in the temporal changes of the rigidity spectrum ex-

ponent $\gamma$ in the long period 1970-2004 is equal 35.7 $\pm$ 1.7 Carrington rotations, it is 2.6 $\pm$ 0.1 years. When we take into account shorter period (1976-1982), we found that there exist two main periodicities, which are: 26.6 $\pm$ 1.2 CR and 14.5 $\pm$ 0.7 CR. Thus, there should exist some mechanism being the reason of creation of those periodicities, which needs further studies.

5 Conclusions

1. We show the three-dimensional theoretical model of the 27- day variation of the GCR intensity for maximum and minimum epochs of SA with implemented in situ measurements of the heliolongitudinal changes of the SW speed and IMF strength being suitable to explain the experimental results. An expected rigidity spectrum of the 27- day variation of the GCR intensity is hard ($\gamma = 0.8$) in the maximum epoch, and soft ($\gamma = 1.3$) in the minimum epoch of solar activity.

2. We found that the main periodicity in the temporal changes of the rigidity spectrum exponent $\gamma$ of the 27- day variation of the GCR intensity in the long period are: 142.9 $\pm$ 1.6 and 35.7 $\pm$ 1.7 Carrington rotations, that are 10.6 $\pm$ 0.1 and 2.6 $\pm$ 0.1 years, respectively. In the shorter period there exist two main periodicities: 26.6 $\pm$ 1.2 CR and 14.5 $\pm$ 0.7 CR, that are 2.0 $\pm$ 0.1 and 1.1 $\pm$ 0.1 years, respectively.


References

Fig. 5: Comparison of the experimental data (the GCR intensity by Rome neutron monitor, squares, values on the right axes) with modeling results (relative density of the GCR particles \( n \) depending on \( \varphi \), solid curve, values on the left axes) in the minimum (5a, data of Rome neutron monitor from http://webusers.fis.uniroma3.it/svirco/pag2.html), and maximum (5b, data of Rome neutron monitor from http://www.nmdb.eu/) epochs of solar activity, on the helioequator (\( \theta = 90^\circ \)) at the Earth orbit (\( r = 1 \text{AU} \)); one day is equivalent to 13.3(3)\(^2\).

Fig. 6: Temporal changes of the one year averages of the rigidity spectrum exponent \( \gamma \) of the 27-day variation of the galactic cosmic rays intensity in 1965-2002 (smoothed over 3 years) with marked standard errors.


Fig. 7: Power spectrum density of the rigidity spectrum exponent \( \gamma \) of the GCR 27- day variation in 1970-2004 with marked 95% confidence level (dashed line).

Fig. 8: Power spectrum density of the rigidity spectrum exponent \( \gamma \) of the 27- day variation of the galactic cosmic rays intensity in 1970-2004 (8a) and 1976-1982 (8b) with marked 95% confidence level (dashed line).