A new 3D transport and radiation code for galactic cosmic rays

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Abstract: We show the necessity for a new approach towards comprehensive and consistent simulations of the propagation of galactic cosmic rays. Our developments are optimised for addressing the spatially 3-dimensional inhomogeneous diffusion problem and utilise contemporary numerical methods. We aim to address the transport problem in a full 3-dimensional environment. For that, we test the transition from 2D to 3D simulation results within an existing propagation code. We present sub-kpc scale simulations that allow the investigation of small-scale structures regarding different model conditions such as variety regarding non-axisymmetric cosmic ray source distributions. These results are discussed critically and motivate our development of a new transport code for galactic cosmic rays. The capabilities of this code are outlined.

Keywords: cosmic rays, propagation,

1 Introduction

The challenge in modelling the propagation of cosmic rays (CRs) in our Galaxy is at least twofold: Implementing a realistic and accordingly complex physics model of CR propagation in our Galaxy and devising a numerical scheme that can solve the underlying transport equations accurately but efficiently. These two conflicting aspects are often consolidated by making several more or less justified simplifications in the underlying CR propagation physics and/or the input parameter space (i.e. galactic magnetic field, gas distribution, radiation fields, isotropic diffusion). Consequently, current available transport codes feature CR propagation models of varying complexity [1], from single species fluid models to codes that account for a nuclear reaction network of multiple CR species. DRAGON [2] and GALPROP [3] are considered the most capable of the currently publicly available codes. Both compute the CR distribution in our Galaxy by solving the CR transport equation for each CR species using a mostly second-order Crank-Nicolson discretisation. The most prominent CR interactions with the radiation fields and matter distributions in our Galaxy are taken into account, furthermore both allow the computation of secondary particles, e.g. γ-rays. Gamma-rays are of particular interest as they offer a testing ground for the validity of CR particle transport modelling results via comparison to measurements of the galactic diffuse γ-ray emission.

Despite their capabilities current propagation scenarios are still relying on simplifications that have been rendered obsolete by tighter constraint on the input parameters as provided by new experimental results, i.e. matter distributions, magnetic field models, radiation fields, CR source distributions, and perhaps more importantly by the increase in computational power that allows a more realistic 3-dimensional non-isotropic treatment of CR propagation. Only recently steps have been made to overcome the 2D-paradigm that clearly demonstrate that new insights on current topics in astroparticle physics can be gained [4].

We show if and how 3-dimensional sub-kpc scale propagations scenarios can be treated using GALPROP and test the validity by comparing these results to 2-dimensional simulations. We demonstrate the capabilities of 3-dimensional simulations by introducing a non-axisymmetric CR source distribution. In this context we dis-
cuss current limitations of GALPROP thereby motivating
the necessity of new code developments.

2 Beyond the 2D-paradigm

2.1 2D to 3D comparison

We test consistency of the solution for a 3D-propagation
scenario obtained by GALPROP with that obtained in the
2D case by formulating a 2D (z,r)-coordinates) scenario us-
ing an axisymmetric source distribution (source_model=1)
and an equivalent 3D propagation scenario that should in
principle result in the same CR distribution. We use a spatial
resolution of 0.1 kpc for the z-direction and 0.15 kpc for the
r,x,y-directions. In these scenarios our Galaxy is confined
within a box ranging from x,y = ±15 kpc (r = 15kpc in the
2D scenario) and a height ranging from z = ±4 kpc. Here
we discuss protons and electrons only, although tests with
nuclie up to Z = 8 have been performed. The energy grid
uses 23 logarithmically equidistant energy points ranging
from 100 MeV to about 1 TeV. The size of the time steps
used by the solver ranges from 10^8 yrs to 10^2 yrs. We
find that the 2D and 3D proton distributions match and that
any deviations are on the percent level. This is exemplary
shown for \( E_{\text{kin}} = 100 \) MeV protons in Figure 1. Up to a cer-
tain degree of accuracy, GALPROP provides a consistent
solution. We use the CR proton spectrum at Earth to quan-
tify the remaining discrepancies between the 2D and 3D
solutions and investigate their dependence on parameters
that govern the numerical solver. This is shown exemplary
for the parameter timestep_repeat (the number of itera-
tions in each time step) in Figure 2. In case of CR proton
spectra the 2D and 3D scenarios seem to converge toward a
common solution. We note however that in case of the 3D
scenario and for the number of iterations exceeding 200, the
required computing time becomes excessive. Figure 2 also
shows that the relative deviations of the solutions obtained
in the 2D scenario decrease with increasing number of iter-
ations, a further indication of convergence. The same holds
true for the 3D scenario. These test are necessary because
GALPROP does not properly control convergence towards
a numerical solution nor offers feedback on the error of the
particular solution.

We performed a similar analysis for electrons. As an ex-
ample we show the spatial electron distribution at an
energy of \( E_{\text{kin}} = 1.2 \) TeV in Figure 4, where deviations are
considerably larger. In Figure 4 we show that the solutions
of the 2D and 3D scenarios for energies above 200 GeV do
not match as well as for CR protons. We use the electron
spectra at Earth to quantify the deviations and find that the
deviations increase with increasing energy. We do not find
any dependence on the number of iterations in each time
step nor the size of the smallest time step. If this discrepancy
hists at an error in the numerics, e.g., that the high energy
boundary conditions are handled incorrectly, or that the
time scales associated with energy loss are simply too small,
is still under investigation.

2.2 3D modelling

3D-simulations allow us to model propagation scenarios
that do not feature \( \phi \)-symmetry. To demonstrate this we
implemented a source distribution following a logarithmic
spiral arm pattern derived from COBE observations of FIR
cooling lines [5]. Figure 5 shows the resulting distribution
of CR protons at an energy of \( E_{\text{kin}} = 444 \) MeV. This allows

1. The corresponding GALDEF files are available upon request.
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3 A new CR transport code

In the previous section we identified the lack of control over convergence in the GALPROP code as a fundamental problem for the numerical solver. While we showed that it is actually possible to find a meaningful quantification of the convergence of the code, the less experienced user of such a code, is hardly interested in repeatedly conducting convergence studies between any major change in the physical setup.

This problem is actually connected to the way the transport equation is solved within GALPROP: the transport equation is integrated in time until the user-specified total propagation time is reached. In this case the user has to make sure that the overall time integrated by the solver is sufficiently long for a steady state solution to be achieved. An obvious alternative to this scheme would be to compute a steady-state solution directly without any time-integration. This, however, forbids a time-splitting approach as utilised within GALPROP: a first order operator splitting method is applied in a way that all dimensions are alternately solved for, while keeping the remaining coordinate dimensions fixed.

Therefore, we are deploying an alternative solver which no longer uses the dimensional operator splitting. This of course has the drawback that for a Crank-Nicolson dis-
cretisation of the diffusion problem the resulting matrix-
equation no longer has tri-diagonal form. While the tri-
diagonal form has the advantage that a solution can be com-
puted directly and efficiently, modern numerical methods
handle sparse matrix problems very efficiently, too. We
are currently testing the implementation of such a method.
In particular the solution of the steady state problem is very
efficient as it does not invoke any time-integration steps.
Here we are using two different approaches in parallel. On
the one hand we implemented a method, which solves the
steady state problem only. This has the advantage that there
is no longer a decrease in accuracy due to the order of the
time discretisation. On the other hand we also implemented
a time-dependent solver, where we can follow the time-
dependent evolution of the Cosmic Ray distribution within
the Galaxy. This is, e.g., used in cases where temporally
variable sources are needed to be considered. For the latter
we use the result from the steady state solver as an initial
condition that is to be modified by temporal effects. The
new solver uses operator splitting for the time-dependent
problem. We use adapted solvers for each physically dis-
tinct term in the transport equation instead of simple dimen-
sional operator splitting as used within GALPROP. That is
we apply different solvers for spatial convection, spatial dif-
fusion and diffusion and energy losses in momentum space.
Within the solution of the transport problem we allow for
a full spatial variation of the diagonal components of the
diffusion tensor. Corresponding analytical tests show satis-
factory results.

A specific description of these new solvers will be given
in an upcoming publication where we will show the conver-
gence properties and the numerical error by comparison to
analytical tests. In particular, the final result will no longer
rely on convergence criteria to be selected by the user. This
allows the user to concentrate on the physical problem at
hand and facilitate consistent comparisons within the com-
unity. Our new CR transport code is capable of utilizing
modern parallel computing architecture by using the Mes-
sage Passing Interface (MPI) standard.

4 Summary and Discussion

New observational constraints on the input parameters of CR
transport models as well as the increase in available
computing power necessitate a transitions from 2D mod-
elling towards 3D simulations of CR propagation in our
Galaxy. We study this transition from 2D to 3D simul-
ations using an existing code (GALPROP), and find signifi-
cant deviations for electrons with energies higher than 200
GeV. For protons we find a general agreement except for
discrepancies on the percent level. Our comparison shows
the need for practical and universal convergence criteria.
Using sub-kpc scale 3D simulations we investigate a non-
axisymmetric CR source distributions following a spiral
arm pattern that allows us to address scientific questions
inaccessible to 2D simulations. We demonstrate the need
for an efficient CR transport code that makes use of parallel
computing architectures.

Motivated by our findings we present a brief overview
of our new CR transport code. This code uses contempo-
rary numerical methods in a way that the user no longer
unknowingly faces convergence issues. The new code will
rely on a number of solvers, each optimized for different
physical sub-processes in the CR propagation equation, and
is capable of treating anisotropic diffusion. Our CR trans-
port code utilises modern parallel computing architectures.

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