Abstract: Cosmic ray intensities are computed in the inner and outer heliosphere using a two-dimensional, time-dependent modulation model. Results are compared to IMP-8, Ulysses, Voyager 1 and Voyager 2 observations for compatibility. A time-dependence for the assumed transport parameters are constructed by incorporating recent theoretical advances. This approach gives results which compared well with the traditional compound approach of Ferreira and Potgieter (2004) and also to the observations along Voyager 1, Voyager 2 and at Earth on a global scale. However, for extreme solar maximum conditions the computed step-like modulation is not as pronounced as observed, indicating that some merging in the form of global interaction regions is needed. The new approach indicate that for this particular polarity cycle, time-dependent changes in the diffusion coefficients are more important compared to previous polarity cycles i.e. for this particular polarity cycle the drift effects are downplayed by changes in the diffusion coefficients.

Keywords: Cosmic rays, heliosphere, diffusion coefficients, drifts, compound approach.

1 Introduction
Cosmic ray transport is influenced by solar activity and this leads to ~11 year and ~22 year modulation cycles in the cosmic ray intensities. Diffusion, convection, energy changes and drifts are the different processes which modulates cosmic ray intensity in the heliosphere. The modulation of galactic cosmic rays in the inner and outer heliosphere over various solar cycles is simulated in this work using a 2D time-dependent numerical model (see [9], [15]).

To describe the long-term time-dependent modulation of cosmic rays in the heliosphere Ferreira and Potgieter [15] developed an empirical approach where model results were compared to observations to construct a time-dependence in the transport parameters, called the compound approach. This was done because of a lack of a clear theory on how the diffusion and drift coefficients should change over a solar cycle. In this study the compound approach of [15] is improved by incorporating the time-dependence in the diffusion and drift coefficients, recently derived by [1], [2], [3] and [7], into the model. Modeling results are compared to different spacecraft observations in the inner and outer heliosphere for compatibility.

2 Model
The model is based on the numerical solution of the Parker transport equation [5]. This equation is solved in terms of time \( t \) and rigidity \( P \) in two-dimensional space \( (r, \theta) \) with \( r \) radial distance and \( \theta \) polar angle.

Two diffusion coefficients of particular concern for this study are those in the radial direction \( (K_{rr}) \) and the polar direction \( (K_{\theta\theta}) \) respectively given by,

\[
K_{rr} = K_1 \cos^2 \psi + K_{\perp r} \sin^2 \psi, \quad (1)
\]

\[
K_{\theta\theta} = K_{\parallel \theta}, \quad (2)
\]

While the drift coefficient \( (K_A) \) is given by (see [14]),

\[
K_A = K_{\parallel \theta} \frac{\beta P}{3B} \frac{10P^2}{10P^2 + 1}. \quad (3)
\]

In Equations [1] \( K_{\parallel |} \) is the diffusion coefficient parallel to the heliospheric magnetic field (HMF), \( K_{\perp r} \) the perpendicular diffusion coefficient in the radial direction, \( K_{\parallel \theta} \) the perpendicular diffusion coefficient in the polar direction, \( B \) is the HMF magnitude, \( \psi \) is the spiral angle of \( B, \beta \) the ratio between the particle speed to the speed of light and \( K_{\parallel \theta} \) is a dimensionless constant which scale \( K_1 \) and has a value from 0 to 1 representing zero drift to full drift. For an illustration of the dependence of these coefficients on \( r, \theta \) and \( P \), see [11].

An expression by [2] for the parallel mean free path for protons (damping model) at Earth with rigidity \( 10^{-1} \) MV is given as \( \lambda_1 \propto P^{1/3} \). We assume,

\[
\lambda_1 = C_1 \left( \frac{P}{P_0} \right)^{1/3} \left( \frac{r}{r_0} \right)^2 f_2(t), \quad (4)
\]

where \( C_1 \) (in units of AU) is a constant determining the absolute value of mean free path, \( P_0 = 1 \) MV, \( r_0 = 1 \) AU, \( C_2 \) a constant determining the radial dependence and \( f_2(t) \) a time-dependent function as discussed below.

For the perpendicular diffusion coefficient we assume (see [6],[8]):

\[
K_{\perp r} = aK_1 f_3(t), \quad (5)
\]

\[
K_{\parallel \theta} = bK_1 F(\theta) f_3(t) f_2(t), \quad (6)
\]

where \( a \) and \( b \) are dimensionless constants, \( F(\theta) \) a function enhancing \( K_{\parallel \theta} \) toward the poles by a factor of 6 and \( f_3(t) \) a different time-varying function, discussed below.

3 Time-dependence in transport parameters
In the compound approach of [15] all diffusion and drift coefficients were multiplied by \( f(t) \), a time dependent function given by,

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**Time-dependent cosmic ray modulation in the heliosphere**

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**Abstract:** Cosmic ray intensities are computed in the inner and outer heliosphere using a two-dimensional, time-dependent modulation model. Results are compared to IMP-8, Ulysses, Voyager 1 and Voyager 2 observations for compatibility. A time-dependence for the assumed transport parameters are constructed by incorporating recent theoretical advances. This approach gives results which compared well with the traditional compound approach of Ferreira and Potgieter (2004) and also to the observations along Voyager 1, Voyager 2 and at Earth on a global scale. However, for extreme solar maximum conditions the computed step-like modulation is not as pronounced as observed, indicating that some merging in the form of global interaction regions is needed. The new approach indicate that for this particular polarity cycle, time-dependent changes in the diffusion coefficients are more important compared to previous polarity cycles i.e. for this particular polarity cycle the drift effects are downplayed by changes in the diffusion coefficients.

**Keywords:** Cosmic rays, heliosphere, diffusion coefficients, drifts, compound approach.
Fig. 1: Proton observations (symbols) are shown as a function of time for Voyager 1, Voyager 2, IMP 8 and Ulysses. Also shown are the computed 2.5 GV proton intensities at Earth and along the Voyager 1 and 2 trajectories. From [11]

\[ f(t) = \left( \frac{B_0}{B(t)} \right)^{\alpha(t)/\alpha_0}, \quad (7) \]

with \( \alpha \) the tilt angle in degrees, \( \alpha_0 = 40^\circ \) and \( B_0 = 5 \) nT.

However for this study the theoretical advances in transport parameters by [1], [2], [3] and [7] are incorporated into our time-dependent transport model to simulate the time-dependence for the transport parameters. The observed magnetic field magnitude \( B \), magnetic field variance \( \delta B^2 \) and tilt angle are transported from Earth into the outer heliosphere resulting in a time-dependence for the diffusion parameters. The magnetic field variance \( \delta B^2 \), was calculated using the OMNI magnetic field observations (from http://cohohweb.gsfc.nasa.gov). The observed hourly averages of the total field magnitude were binned in one year intervals, and then the statistical variance in each interval was calculated.

The time-dependence for \( K_1 \) is attained from an expression for parallel free mean path \( \lambda_\parallel \) for protons given by [3] and since we consider only the influence of time varying quantities \( B \) and \( \delta B^2 \) on \( \lambda_\parallel \), we approximate the complicated equation by assuming that (see also [12, 13])

\[ \lambda_\parallel \propto \left( \frac{\delta B}{B} \right)^2 \left( \frac{1}{\delta B} \right)^{\frac{3}{4}}. \quad (8) \]

The time dependence, \( f_2(t) \), is then given by

\[ f_2(t) = C_4 \left( \frac{1}{\delta B(t)} \right)^{\frac{3}{4}}, \quad (9) \]

where \( C_4 \) is a constant in units of \((\text{nT})^2\).

While for the time-dependence of the perpendicular diffusion coefficients, \( f_3(t) \), we approximate the expression for \( \lambda_\perp \) as given by [11] by assuming

\[ \lambda_\perp \propto \left( \frac{\delta B}{B} \right)^{\frac{4}{3}} \left( \frac{1}{\delta B} \right)^{\frac{1}{3}}. \quad (10) \]

Resulting in,

\[ f_3(t) = C_5 \left( \frac{\delta B(t)}{B} \right)^{\frac{4}{3}} \left( \frac{1}{\delta B(t)} \right)^{\frac{1}{3}}, \quad (11) \]

where \( C_5 \) is a constant in units of \((\text{nT})^{2/3}\).

A time-dependence for the drift coefficient is constructed from the theoretical work done by [7], where \( \dot{K}_A \) is scaled with respect to \( \delta B \). In this work we assume a similar dependence for the drift coefficient on solar activity but instead of \( \delta B \) the tilt angle \( \alpha \) is utilised to scale \( \dot{K}_A \) for increasing solar activity. This is done to compute charge-sign dependent modulation over a solar cycle. A time-dependent function \( f_1(t) \) is assumed such that the drift coefficient, \( K_A \propto f_1(t) \). The function is given by,

\[ f_1(t) = \frac{75^\circ - \alpha(t)}{\alpha_k}, \quad (12) \]

where \( \alpha_k = 1^\circ \).
Time-dependent cosmic ray modulation in the heliosphere

4 Results and discussions

Time-dependent cosmic ray modulation is calculated over multiple solar cycles and results are shown in Figure 1. The figure shows the computed result of 2.5 GV protons at Earth and along Voyager trajectories compared to IMP 8 (from http://astro.nmsu.edu), Ulysses (from [4]), Voyager 1 and Voyager 2 (from http://voyager.gsfc.nasa.gov) observations for compatibility. Note that Ulysses did move to higher latitudes and larger distances and therefore cannot be compared in detail with IMP 8 data without correcting for latitudinal and radial gradients (see [4]). However, for this study these spacecraft data is utilised and compared with computed result at Earth since they largely agree on the modulation amplitude from solar minimum to solar maximum.

Figure 1 shows that after incorporating the recent theoretical advances, the model computed results which are compatible to the observations at Earth and along the Voyager 1 and 2 trajectories. For the period ∼1985-2009, the model successfully computed intensities at Earth when compared to IMP 8 and Ulysses spacecraft observations but it failed to reproduce the measured step increase/decrease during the solar maximum periods.

The model also successfully reproduced the cosmic ray modulation in the outer heliosphere. During the period ∼1986-1989, the measured intensities of Voyager 2 were higher compared to Voyager 1. For this period protons drift in mostly along the heliospheric current sheet towards Earth. While Voyager 1 travelled to higher latitudes, during this period, Voyager 2 stayed close to the equatorial plane and therefore higher intensities are measured. This feature is simulated by the model. However, for the period ∼1992 onwards the intensities along the Voyager 1 is higher compared to Voyager 2 with the model also reproducing these features. The computed intensities along both Voyager trajectories also show that it needs some improvements for extreme solar activity conditions where merging of interactions regions seems necessary, as seen in the inner heliosphere.

Computed intensities based on the previous compound approach are shown as the dashed line in Figure 2 while the approach incorporating the improved theory is shown as the solid line. From this figure we see that the two approaches give very similar results, with the previous compound approach yielding a better comparison for some and the improved approach for other periods. Differences between both models and the observations exist, especially for the ∼1985-1989 period along the Voyager 1 trajectory where the two approaches result in lower intensities than the observations. For the period ∼1994-2000 the previous compound approach gave intensities higher than what was observed. However, for this period the modified compound approach successfully reproduced the observations on a global scale. For the periods ∼2002, ∼2005 and ∼2007, the modified compound approach gave higher intensities when compared to observations while the previous compound approach calculated compatible results.

However, both the modeling results show that they failed to reproduce the step increases or decreases in cosmic ray intensities in the spacecraft measurements during solar maximum periods. This is an indication that both approaches, where values at Earth for B, dB2 and α are transported into the heliosphere, do require additional merging of these propagating barriers into more pronounced barriers along the way.
In Figure 3, the computed cosmic ray intensities for a no drift, e.g., $K_{A0} = 0$, and $K_{A0} = 0.8$ scenario are shown at Earth, from 1984 onwards and compared to IMP 8 and Ulysses observations. The figure shows that drifts are required to compute a compatible result at Earth over a solar cycle. From the figure it follows that the non-drift scenario results in a computed amplitude between solar minimum and maximum which is much larger for the period $\sim$2004-2010 (present polarity cycle) when compared to previous polarity cycles. This indicates that for this particular polarity cycle, time-dependent changes in the diffusion coefficients are more important compared to previous polarity cycles i.e. the cosmic ray modulation is no longer largely determined by changes in the drift coefficient but also by solar cycle related changes in diffusion coefficients. See also Vos et al. this proceedings and [10].

## 5 Summary and conclusions

Time-dependent cosmic ray modulation of cosmic rays in the heliosphere is computed over multiple solar cycles using a two-dimensional time-dependent modulation model. The results are compared to IMP-8, Ulysses, Voyager 1 and Voyager 2 observations for compatibility. Recent theoretical advances [1], [2], [3] and [7] in transport parameters are incorporated into the model to improve the previous compound approach of [15]. This approach gives compatible results at Earth and along both Voyager trajectories when compared to observations on a global scale. The results also compare well with the traditional compound approach. However, for extreme solar maximum conditions the computed step-like modulation is not as pronounced as observed, indicating that some merging in the form of global interaction regions is needed. It is also shown that the cosmic ray modulation during the recent solar minimum period was no longer largely determined by changes in the drift coefficient but also by solar cycle related changes in diffusion coefficients.

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### References