The nature of gamma-ray emission of Tycho’s supernova remnant


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Abstract: A nonlinear kinetic theory of cosmic ray (CR) acceleration in supernova remnants (SNRs) is employed to investigate the properties of Tycho’s SNR and their correspondence to the existing experimental data, taking into account that the ambient interstellar medium (ISM) is expected to be clumpy. It is demonstrated that the overall steep gamma-ray spectrum observed can be interpreted as the superposition of two spectra produced by the CR proton component in two different ISM phases: The first component, extending up to about 100 TeV, originates in the diluted warm ISM, whereas the second component, extending up to 100 GeV, comes from numerous dense, small-scale clouds embedded in this warm ISM. Given the consistency between acceleration theory and the observed properties of the nonthermal emission of Tycho’s SNR, a very efficient production of nuclear CRs in Tycho’s SNR is established. The excess of the GeV-emission due to the clouds’ contribution above the level expected in the case of a purely homogeneous ISM, is inevitably expected in the case of type Ia SNe.

Keywords: (ISM:)cosmic rays, acceleration of particles, shock waves, supernovae individual(Tycho’s SNR), radiation mechanisms:non-thermal, gamma-rays:theory

1 Introduction

The detection of high-energy (HE; 100 MeV $\leq E \leq 100$ GeV) and very high energy (VHE; $E \geq 100$ GeV) $\gamma$-ray emission from supernova remnants (SNRs) is extremely important, because it provides direct evidence for the acceleration of charged particles (atomic nuclei and/or electrons) inside SNRs to energies that are comparable to those of the gamma rays. Based on such detections one can hope to eventually confirm the idea that SNRs are indeed the main source of nuclear cosmic rays (CRs) up to energies of about $10^{17}$ eV in the Galaxy, as widely expected [1,2,3,4,5], see however [6].

With significant detections at GeV as well as at TeV energies of otherwise simple objects like remnants of type Ia supernovae in the Galactic gas disk, there is still the question whether the circumstellar environment is uniform in gas density or not, and what role this non-uniformity plays for the overall observed $\gamma$-ray spectrum and morphology. This question is discussed here for Tycho’s SNR. The remnant is spatially unresolved in $\gamma$-rays but otherwise very well studied. The spatially-integrated TeV-spectrum, detected by the VERITAS array [7], is quite compatible with a theoretical model for a type Ia - explosion in a strictly uniform Interstellar Medium (ISM), studied in detail in a previous paper [4]. However, the recent Fermi Large Area Telescope (Fermi-LAT) detection of high energy $\gamma$-rays above 400 MeV [8] disagrees with this simple nonlinear model, showing a significant GeV excess. It has been attempted to understand this result assuming a spectrum of CR protons $N \propto e^{-\gamma}$ with a spectral index $\gamma = 2.2$. This spectrum is considerably steeper than the spectrum predicted in [4], that implies $\gamma \approx 2$. To obtain this result, Bohm diffusion for all the accelerated particles was assumed [9]. However such an interpretation contains an internal contradiction: Bohm diffusion at the highest particle energies involved is inconsistent with such a steep proton spectrum.

Considerable increase of the maximal CR energy due to magnetic field amplification is expected only in the case of a hard CR spectrum with $\gamma < 2$, where the CRs with the highest energies provide the main contribution to the overall CR energy content. Fortunately, such a spectrum is expected to be produced by SN shocks.

In this paper a new interpretation of the detected $\gamma$-ray spectrum of Tycho’s SNR will be given. It is based on the expectation that the actual interstellar medium (ISM) is clumpy instead of being purely homogeneous [10,11].

2 Method and Results

The present form of the solutions of the nonlinear acceleration equations, considered here [12], assumes spherical symmetry. In this approximation it is possible to predict the temporal and radial evolution of gas density, pressure, and mass velocity, together with that of the energy spectrum, as well as the spatial distribution of CR nuclei and electrons, including the properties of their non-thermal radiation.

This theoretical model has been used in detail to investigate Tycho’s SNR as the remnant of a type Ia SN [13] in a homogeneous ISM, in order to compare the results with the existing data [4]. It was argued that consistency of the standard value of stellar ejecta mass $M_{ej} = 1.4 M_{\odot}$ and a total hydrodynamical explosion energy $E_{ej} = 1.2 \times 10^{51}$ erg [14] with the gas dynamics, acceleration theory and the existing $\gamma$-ray measurements required the source distance $d$ to exceed 3.3 kpc in order to be consistent with the existing HEGRA upper limit for TeV $\gamma$-ray emission. The corresponding ambient gas number density $N_{g} = \rho / m$ (where $\rho$ is the gas density and $m$ is the proton mass) had then to be lower than 0.4 cm$^{-3}$. On the other hand, the rather low distance estimates from independent measurements together with internal consistency arguments of the
As can be seen from Fig. 1 a new IT AS [7] corresponds very well to the above expectation. The new spectral index was measured within the kinetic nonlinear theory (shown by the dashed line) is well consistent with the VERITAS measurement. This new calculation was performed following the usual procedure as described in [4]. For the proton injection rate \( \eta \approx 3 \times 10^{-4} \) this is still compatible with the above-mentioned shock modification and softening of the observed radio synchrotron emission spectrum. The new distance \( d = 3.8 \text{ kpc} \) and the corresponding new ambient ISM number density \( N_1 = 0.25 \text{ cm}^{-3} \) were taken in order to fit the observed TeV \( \gamma \)-ray emission [7].

However, as mentioned in the Introduction, the \( \gamma \)-ray spectrum measured by the Fermi LAT at energies 400 MeV to 100 GeV [8] is considerably (by a factor 2 to 5) above the value predicted by the kinetic theory (see Fig. 1). This excess of GeV \( \gamma \)-ray emission, when compared with the theoretical predictions, requires a more detailed consideration of this object and its environment, taking into account new physical factors which had been hitherto neglected.

The physics aspect which is not included in the present kinetic and (“renormalized”) spherically symmetric model is an essential inhomogeneity of the ambient ISM on spatial scales that are smaller than the SNR radius. This inhomogeneity is not the result of the progenitor star’s evolution towards the final supernova explosion, for example in the form of a wind and a corresponding modification of the circumstellar environment. It is rather an inherent nonuniformity of the average ISM on account of the interplay between its radiative heating by the diffuse galactic UV field and the radiative cooling of the gas [10] and (ii) the stochastic agitation of the ISM by the mechanical energy input and gas heating from supernova explosions [17][18]. The first effect is a thermal instability and thus a mechanism for small-scale cloud formation in the ISM driven by runaway radiative cooling [19]. Specifically the balance between line-emission cooling and gas heating due to the ultraviolet background radiation leads to two thermally stable equilibrium ISM phases [11]. One of them is the so-called warm interstellar medium with a typical gas number density \( N_{g1} \approx 0.1 \text{ cm}^{-3} \) and temperature \( T_1 \approx 8000 \text{ K} \), the other one a cold neutral medium with \( N_{g2} \approx 10 \text{ cm}^{-3} \) and \( T_2 \approx 100 \text{ K} \). According to simulations the scale of dense clouds is typically \( l_c = 0.1 \text{ pc} \) [20][21][22][23].

The second effect is a general compressible turbulence of the ISM, at least on scales in excess of \( \approx 1 \text{ pc} \), driven by the Galactic supernova explosions. According to MHD-simulations it involves high- and low-temperature gas components out of ionization equilibrium on all larger scales [23], for a review see [24]. While the overall picture of the ISM is clearly not simple, the evolution of a young SNR like that of T ycho’s SN will encounter a single realization of the stochastic ensemble of density fluctuations. For the energy spectrum of energetic particles, accelerated at the blast wave, small-scale high-density fluctuations of the ISM play the most conspicuous role because they produce mainly particles with energies far below the cutoff energies for a uniform circumstellar medium. To estimate the spectral changes due to upstream density variations in an analytical model, the typical ISM is therefore treated here as a generalized two-phase medium, composed of a pervasive warm/bot ISM (phase I) – called here for brevity “warm” ISM – and small-scale dense clouds (phase II), embedded in this warm ISM.

In order to determine the specific properties of the CRs and their nonthermal emission in the case of such a generalized two-phase ISM the latter is approximated here in a simple form, as a uniform warm phase with gas number density \( N_{g1} \) plus an ensemble of small-scale dense clouds with gas number density \( N_{g2} \). The warm diluted ISM phase is assumed to have a volume filling factor \( F_1 \approx 1 \), whereas the clouds occupy a small fraction of space with filling factor \( F_2 \ll 1 \). It is in addition assumed that most of the gas mass is contained in the warm phase, which means that \( F_1 N_{g1} \gg F_2 N_{g2} \). Then the SN shock propagates in the two-phase ISM without essential changes compared with the case of a purely homogeneous ISM with number density \( N_{g1} \). Therefore it produces inside the phase I of the ISM roughly the same amount of CRs and nonthermal emission as in the case of a homogeneous ISM. Then one has to estimate the additional contribution of the clouds in order to determine the overall spectrum of CRs.

The large-scale SN blast wave, interacting with each single cloud, produces a pair of secondary transmitted and reflected shocks. The reflected shock propagates in the warm isobaric ISM already heated by the blast wave. Due to this fact its Mach number is quite low and therefore its contribution to the overall CR production can be neglected.

The size \( R_2 = l_c/2 \) and the speed \( V_2 \approx (N_{g1}/N_{g2})^{1/2} V_1 \approx 10^{-2} V_1 \) of the transmitted shock are both considerably

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1. Gravitationally bound molecular clouds are not considered here, because they are large-scale massive elements of the ISM. If present, they would require an individual treatment.

Figure 1: Spectral energy distributions of the \( \gamma \)-ray emission from T ycho’s SNR as functions of \( \gamma \)-ray energy \( E_\gamma \), calculated for a source distance \( d = 3.8 \text{ pc} \), together with the experimental data obtained by Fermi and VERITAS. Dashed and solid lines represent the contribution of the warm-phase ISM and the total \( \gamma \)-ray energy spectrum that includes the contribution of the clouds (see main text), respectively.
smaller than the corresponding values of the SN blast wave. Therefore, according to \[25\], and using \( A = 10 \) for both shocks, the maximum momentum of CRs produced inside the cloud

\[
p_{\text{max2}} = \frac{R_{\text{w}}V_{\text{s2}}}{(R_{\text{w}},V_{\text{s1}})}p_{\text{max1}}
\]

(1)
is much smaller than the maximal momentum \( p_{\text{max1}} \) of the CRs produced by the SN shock in the warm ISM: \( p_{\text{max2}} \ll p_{\text{max1}} \). Since the ram pressure \( pV_{\text{s}}^2 \) is expected to be the same in both cases, and since the amplified magnetic field pressure reaches roughly the same fraction of the ram pressure, the magnetic field values are roughly the same in these two cases: \( B_{\text{2}} \sim B_{\text{1}} \).

An estimate of the CR spectrum produced by the transmitted shock starts from the expression for the CR distribution function

\[
f = \frac{q\eta N_{\text{g}}}{4\pi p_{\text{max1}}^3} \left( \frac{p}{p_{\text{max1}}} \right)^{-q},
\]

(2)

which is valid at all momenta \( p \geq p_{\text{max1}} \) up to the cutoff momentum \( p_{\text{max}} \) in the case of an unmodified shock, and roughly valid within the momentum range \( p_{\text{min}} \leq p < 10p_{\text{max1}} \) of the subshock in the case of a modified shock. Using this expression for the case of the SN blast wave and for the transmitted shock, the ratio of the two corresponding distribution functions can be found as

\[
\frac{f_2}{f_1} = \frac{q_2}{q_1} \left( \frac{N_{\text{g2}}}{N_{\text{g1}}} \right)^{1-(q_2-3)/2} \left( \frac{p}{p_{\text{max1}}} \right)^{q_1-q_2}.
\]

(3)
in \( p_{\text{min}} < p \leq 10p_{\text{max1}} \), where also the expression for the injection momentum \[26\] \( p_{\text{min}} \approx m_p V_{\text{s}} \) was used.

According to the calculation from section 2, the power law spectrum with the index \( q_1 = 4.3 \), determined by the subshock compression ratio, extends up to the CR momenta \( p \sim 10p_{\text{max1}} \).

Since the cutoff momentum \( p_{\text{max2}} \ll p_{\text{max1}} \) of the CR spectrum, produced by the transmitted shock, is much lower than the corresponding value for the blast wave, \( p_{\text{max1}} \approx 10^3m_p c \), one can neglect the modification of the transmitted shock. This leads to \( q_2 \approx 4 \).

Taking into account that the \( \gamma \)-ray production is proportional to the gas density, for \( \gamma \) \( F_{\gamma} \geq 1 \) GeV one can write the relation between the fluxes of \( \gamma \)-rays produced due to the two shocks considered:

\[
F_{\gamma2}(\gamma) = aF_{\gamma1}(\gamma)\exp(-\epsilon_1/\epsilon_2_{\text{max2}}).
\]

(4)

Here the factor \( a \) is determined by the expression

\[
a = \frac{q_2}{q_1} \left( \frac{N_{\text{g2}}}{N_{\text{g1}}} \right)^{1.5} \left( \frac{10c}{V_{\text{s}}} \right)^{0.3}
\]

(5)

and the \( \gamma \)-ray cutoff energy is \( \epsilon_{\text{max2}} \sim 0.1p_{\text{max2}}c \) since on average the energy of the \( \gamma \)-rays resulting from inelastic proton-proton collisions is about one tenth of the proton energy: \( \epsilon_\gamma \sim 0.1 \) pc.

Substituting the values of the SN shock speed \( V_{\text{s}} = 5000 \) km/s \[27\] \[31\], the number density for the warm phase of the ISM \( N_{\text{g1}} = 0.25 \) cm\(^{-3} \), as well as suitable fit parameter values for the cold phase of the ISM in the form of \( N_{\text{g2}} \approx 23N_{\text{g1}} \) and \( F_2 = 0.005 \), results in \( p_{\text{max2}} \approx 10^{-3}p_{\text{max1}} = 10^3\) \( m_p \) and \( a \approx 800 \). It is noted here that such parameter values for the cold ISM phase correspond rather well to the results of numerical modelling of the two-phase ISM \[28\].

Then the flux of \( \gamma \)-rays produced inside the clouds can be written in the form

\[
F_{\gamma2}(\gamma) = 4F_{\gamma1}(\gamma)\exp(-\epsilon_1/100\text{GeV}).
\]

(6)

The total \( \gamma \)-ray flux \( F_\gamma = F_{\gamma1} + F_{\gamma2} \) expected from Tycho’s SNR for a two-phase ISM is shown in Fig. 1. One can see that it fits the existing data in a satisfactory way. Note that the considerable increase (by a factor of 5) of the \( \gamma \)-ray emission at energies \( \epsilon_\gamma < 100 \) GeV over and above the case of a purely homogeneous ISM is due to the contribution of clumps which contain only 10% of the ISM mass.

### 3 Discussion

Small-scale dense clumps of sizes \( l_c \ll 0.1 R_c \) can be also produced by the accelerating CRs themselves within the precursor as the result of the so-called acoustic instability \[28, 29, 30\].

Small-scale bright structures of angular size \( 10^\circ \) were recently detected in nonthermal X-rays \[31\]. For a source distance \( d = 3.8 \) kpc the corresponding spatial size is \( l \approx 0.2 \) pc, which would be roughly consistent with the sizes of the expected clumps. However, the acceleration in such clouds is not expected to reach electron energies in the TeV range which could lead to synchrotron X-ray emission. Therefore these small-scale X-ray structures cannot be considered as an indication that dense gas clumps of size \( l_c \sim 0.1 \) pc indeed exist inside Tycho’s SNR. Their existence rather derives from the general properties of the ISM, as discussed above.

The contribution of dense gas clumps in different kinds of emission can be roughly estimated as follows. First, consider the thermal X-ray emission. Since, besides other factors, the flux of thermal X-ray emission \( F_X \propto N_{\text{H}}N_{\text{g}} \) is proportional to the gas density \( N_{\text{g}} \) and the total gas mass \( M_\text{g} \) of the source, we have \( F_{X2}/F_{X1} = (N_{\text{g2}}/N_{\text{g1}})^2 \approx 2.6 \). Since the temperature difference in the two gas phases is not an essential factor for the soft X-rays with energies below \( 2 \) keV \[32\] we conclude that the soft thermal X-ray emission should be dominated by the contribution of dense gas clumps. In the hard X-ray range above \( 2 \) keV, on the other hand, the luminosity is sensitive to the gas temperature, roughly as \( F_X \propto T^{2.1} \propto V^{4.2} \) \[32\] which make the contribution of dense gas clumps relatively small due to their lower temperature. The expected luminosity of individual clumps is considerably higher (by a factor of about 500) compared with the surrounding diluted gas of the same volume. However each instrument sees the remnant in projection. Therefore the expected ratio of X-ray fluxes from the projection volume \( V = \pi l^2L \) containing the clump to the nearby one of the same size which does not contain the clump is \( r = (V + 500V_c)/V_c \), where \( V_c = \pi l^2L/6 \) is the clump volume, \( p \) is the cross-section of the volume, and \( L \) is the line of sight length. A maximal value of this ratio \( r \approx 1 + 330l_c/L \) is achieved for \( \rho = l_c/2 \). It follows from this expression that the contrast of X-ray emissions varies from \( r \approx 4 \) for the central part of the remnant where \( L \approx 10 \) pc to about \( r \approx 25 \) at the limb region with \( L \approx 1.4 \) pc. Therefore we conclude that these clumps could be detected in soft X-rays from the limb regions. In their study with Chandra \[33\] observe some small contribution of thermal X-rays from the regions occupied by the
swept-up ambient gas. They concluded that it is not clear whether the faint lines or other residual emission comes from the ejecta or whether they arise from shocked ambient medium. In line with the idea of the present paper it is suggested here that this emission comes from small-scale clouds in the surrounding ISM. The difficulty is of course that the X-ray emission of Tycho’s SNR is dominated by the nonthermal X-ray emission. In the context of X-ray emission also the observed irregularities in the blast wave position around the remnant [34] could at least be partly due to the interaction with ambient clouds.

Secondly, the contribution of dense gas clumps to the synchrotron emission of SNRs is estimated here. From their measurements of the variations in the expansion parameter in the radio range and comparison with X-ray features [35] suggested the presence of ambient clouds, shocked by the blast wave. Since the radio synchrotron emission is produced by electrons with energies less than 1 GeV, Eq.\(^{(3)}\) with \(p = m_{e}c\) is used in order to estimate the ratio of the total synchrotron fluxes originating within the two gas phases: \(F_{\gamma} / F_{\nu} \propto (f_{\gamma} / f_{\nu}) (f_{\gamma} / f_{\nu}) \approx 0.1\). This shows that the contribution of dense gas clumps to the radio emission is expected to be small. The luminosity of individual clumps compared with the neighbouring region of the same volume is about 15. Therefore one should be able to detect the clumps with an instrument of corresponding angular resolution from the limb region where the contrast is expected to be \(r \approx 2.9\). The synchrotron X-ray energy flux scales as \(F_{\gamma} \propto F_{\nu} K_{\gamma} N_{\gamma} V_{\gamma} \propto F\). Therefore the expected X-ray flux from all the clumps within the SNR, \(F_{\gamma} \approx (f_{\gamma} / f_{\nu}) (f_{\gamma} / f_{\nu}) \approx 5 \times 10^{-3} F_{\gamma}\), is small compared with the total flux \(F_{\gamma}\).

The dense clumps of size \(l_{c} \approx 0.1\) pc have an angular size of about 6\(^{\circ}\). Structures of such sizes (or even smaller) near the outer shock of Tycho’s SNR have been studied in optical H\(_{\alpha}\) emission [36, 37]. It is difficult to conclude whether the dense clumps can be detected in optical emission, even though the lack of smoothness of the optical filaments points in this direction.

According to the above considerations, the excess of \(\gamma\)-ray GeV emission above the level expected in the case of a purely homogeneous ISM due to the contribution of small-scale interstellar clouds is inevitably expected in the case of type Ia SNe situated in a relatively dense ISM inside the Galactic disk. It is not clear whether such an effect is expected in the case of a SNR situated in a much more rarefied ISM, like SN 1006, in the uppermost part of the Galactic gas disk. On a speculative basis, similar effects may also take place in Cassiopeia A, where the observed \(\gamma\)-ray spectrum [38] looks very similar to that of Tycho’s SNR. The difference would be that in the case of Cassiopeia A numerous observed knots from the supernova ejecta might take over the role of pre-existing interstellar clumps.

4 Summary

The \(\gamma\)-ray spectrum of Tycho’s SNR, consistent with the measurements by Fermi and VERITAS, is proposed to be the superposition of two spectra: the first part, extended up to about 100 TeV, is produced by the SN blast wave within the dilute “warm” phase of the ambient ISM, whereas the second part, with a cutoff at about 100 GeV, originates in dense clouds embedded in this warm ISM. The remarkable connection between CR production and the physical nature of the Galactic ISM becomes evident through the characteristics of the spatially integrated \(\gamma\)-ray emission of the SNR sources.

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