3-d anisotropy during the Forbush decrease of the galactic cosmic ray

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Abstract: We study the temporal changes of the three dimensional (3D) anisotropy of galactic cosmic rays (GCR) during the Forbush decrease (Fd). A new model based on the Parker’s transport equation has been developed to describe the expected distributions of the radial, latitudinal and heliolongitudinal gradients and 3D anisotropy of GCR during the Fd. Proposed model incorporates consistent, divergence-free interplanetary magnetic field (IMF) with the sector structure, derived by solving Maxwell’s equations with a heliolongitudinally dependent solar wind velocity reproducing in situ observations during the Fd. The changes of the IMF turbulence in the range of the frequencies $10^{-6}$ – $10^{-5}$ Hz and the magnitudes of the IMF in the vicinity of the disturbances in the interplanetary space are considered as the general cause of the Fd. We found the reasonable compatibility between the observed and expected GCR intensity during Fd, but there is not satisfactorily correspondence in the behavior of the 3D anisotropy.

Keywords: Forbush decrease, galactic cosmic ray intensity, anisotropy.

1 Introduction

The substantial disturbances in the interplanetary space observed after the powerful coronal mass ejecta and solar flares are accompanying by the short period decreases (Forbush decrease) of the galactic cosmic ray (GCR) intensity [1]. Forbush decrease (Fd) of GCR intensity observed by super neutron monitors can provide extremely useful information about the structure of the interplanetary magnetic field (IMF) turbulence [2-5]. The sporadic Forbush decreases (Fd) characterize a rapid decrease phase during one-two days and recovery phase lasting for few days; they are caused by the transient disturbances associated with the powerful solar flares. Recurrent Fd have the approximately symmetric decrease and recovery phases and last 10-15 days; they are caused by the rotating disturbances in the interplanetary space associated with the long-lived active heliolongitudinal zones on the Sun. Normally, the amplitudes of the recurrent Fd are less than the amplitudes of the sporadic Fd. The behavior of anisotropies in the GCR stream during the Fd is very interesting for the investigation. In connection with this, the theoretical modeling plays an important role to explain the information obtained from the sporadic and recurrent Fd observed by neutron monitors. In present work we investigate three-dimensional (3D) anisotropy of GCR during the Fd based on theoretical modeling of the recurrent Fd using Parker transport equation [6]. We compare the modeling results with the neutron monitors experimental data. 

Generally short period change of the GCR intensity is non stationary process and for its describing the Parker’s time-dependent transport equation must be applied. An initial phase (rapid decreasing of the intensity) of the powerful sporadic Fd belongs to these types of phenomena. However, the recovery period for the great of majority of the sporadic Fd and the whole period of the recurrent Fd can be considered as a steady-state process.

Figure 1: Change of the GCR intensity (1 day) from the Moscow neutron monitor during the Fd in period of February 9- March 1, 2003
In this paper we model the recurrent Fd and consider that the change of the density of GCR versus the heliolongitude could be ascribed to the stationary time profile of density of cosmic rays caused by the relative motion of Earth in the disturbed vicinity of the interplanetary space [4].

As a case study for the modeling we consider the recurrent Fd taking place in the February 9- March 1, 2003. (figure 1).

2 Model of the Fd

We model Fd of the GCR intensity based on the stationary Parker’s transport equation [6]:

\[ \nabla \cdot \left( K_{ij} \cdot \nabla f \right) - \left( v_d + V \right) \cdot \nabla f + \frac{1}{3} \nabla \cdot \nabla f = 0. \quad (1) \]

Here \( f \) and \( R \) are omnidirectional distribution function and rigidity of cosmic ray particles, respectively; \( V \) – solar wind velocity, \( v_d \) is the drift velocity \( v_{dr} = \frac{\partial K_{ij}}{\partial x_j} \) [7], \( K_{ij} \) and \( K_{ij}^a \) are the symmetric and antisymmetric parts of anisotropic diffusion tensor \( K_{ij} = K_{ij}^s + K_{ij}^a \) of GCR for two dimensional (2D) IMF.

An applying of the stationary model (\( \partial V/\partial t = 0 \)) to describe the Fd is justified, when the amplitudes of the Fd of GCR intensity generally are \( \leq 5-6\% \) in the energy range of 10 GeV and duration is reasonably large (~15 days). The stationary model the Fd of the GCR intensity is realized by the changes of the expected GCR intensity versus the heliolongitudes (the value of the heliolongitudes 13.3° corresponds to the 1 day).

The equation (1) for the dimensionless variables \( f \) and \( r \) in the spherical coordinate system \(( r, \theta, \varphi)\) can be written:

\[ A_{i} \frac{\partial^2 f}{\partial \theta^2} + A_{i} \frac{\partial^2 f}{\partial \varphi^2} + A_{i} \frac{\partial^2 f}{\partial \theta \partial \varphi} + A_{i} \frac{\partial^2 f}{\partial \varphi \partial \theta} + A_{i} \frac{\partial^2 f}{\partial r \partial \theta} + A_{i} \frac{\partial^2 f}{\partial r \partial \varphi} + A_{i} \frac{\partial^2 f}{\partial \varphi \partial r} + A_{i} \frac{\partial^2 f}{\partial \theta \partial r} + A_{i} \frac{\partial^2 f}{\partial \varphi \partial r} = 0 \]

The coefficients \( A_{i} \), \( i = 1, 2, \ldots, 11 \) are functions of the spherical coordinates \( r, \theta, \varphi \) and rigidity \( R \) of GCR particles.

2.1 Solution of the Maxwell equations

To construct a relatively realistic model of the Fd there should be taken into account changes of the solar wind velocity during the Fd and corresponding IMF (\( B_x, B_y \) and \( B_z \)) components depending on spatial coordinates. However, in this case the validity of the Maxwell’s equation \( \nabla \cdot B = 0 \) should be kept for the time and spatially dependent solar wind velocity.

Maxwell’s equations for the IMF strength \( B \) have a form e.g.[8]:

\[
\begin{align*}
\frac{\partial B}{\partial t} &= \nabla \times (V \times B) \\
\text{div} B &= 0
\end{align*}
\] (3)

So, we have to solve the system of equation (3) to obtain the IMF components corresponding to the assumed solar wind velocity. To solve the system of equations (3) in general form is difficult. We consider a stationary case \( \frac{\partial B_y}{\partial t} = 0, \frac{\partial B_z}{\partial t} = 0 \) and assume that an average value of the heliolatitude component of the solar wind velocity \( V_\theta \) equals zero. Then the system of equations (3) in the heliocentric spherical \( (r, \theta, \varphi) \) coordinate system can be reduced, as

\[ \begin{align*}
\sin \theta \frac{\partial B_x}{\partial r} + \cos \theta \frac{\partial B_r}{\partial \theta} + r \sin \theta \frac{\partial B_r}{\partial \varphi} &= 0 \\
\frac{1}{r} \frac{\partial}{\partial \varphi} \left( r \frac{\partial B_r}{\partial \varphi} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( r \sin \theta \frac{\partial B_r}{\partial \theta} \right) &= 0
\end{align*} \] (4)

We consider 2D IMF so the latitudinal component \( B_\theta \) of the IMF equals zero \( (B_\theta = 0) \) and solve the system of equations (4) by the method described in detail in [9,10].

In the system (4) we implement the approximation of the solar wind velocity during 27 days i.e. \(-1\) solar rotation period (figure 2) during period of February 6- March 4, 2003 (period in the region of the considered Fd). The solar wind velocity was approximated by formula:

\[ V_r = V_0 \left( 1 - 0.22 \sin (\theta + 1.45) + 0.06 \sin (2\varphi + 1.57) \right) \]

\[ -0.09 \sin (3\varphi + 8.70) \] (5)

and corotational velocity \( V_\varphi = -\Omega r \sin \theta \).

Figure 2: Change of the solar wind speed (1 day from ACE) in period of February 6- March 4, 2003 (dashed line), and its approximation (solid line).

The experimental data of the \( B_x, B_y \) components of the IMF during the considered period show sector structure (figure 3ab), while components found as the solutions of Maxwell’s equation do not provide sector structure. However, we implement in the numerical model of the Fd of the GCR intensity the hybrid scheme which includes \( B_x, B_y \) components obtained as the solutions of the Maxwell’s equation and sector structure according to formula [7]:

\[ B = \left(1 - 2H\left(\theta - \theta_0\right)\right) \left( B_x e_x + B_y e_y \right) \]

(6)
where $H$ is the Heaviside step function changing the sign of the global magnetic field in each hemisphere, $e_\eta$ and $e_\phi$ are the unite vectors directed along the component $B_\eta$ and $B_\phi$ of the IMF and $\theta$ corresponds to the heliolatitudinal position of the HNS taken [11], as:

$$\theta = \frac{\pi}{2} + \delta \sin \left( \varphi + \frac{\pi}{2} \right)$$  \hspace{1cm} (7)

where $\delta = 50^\circ$ is the observed mean tilt [12] angle during the considered period and the value $60^\circ$ - shifting angle of the HNS with respect to the heli долгitudes. Figure 3ab presents the heliolongitudinal changes of the $B_\eta$ and $B_\phi$ components of the IMF obtained as the solution of Maxwell’s equations, as assumed in the model of the Fd, taking into account sector structure of the IMF in comparison to the ACE data [13] of the IMF.

![Figure 3ab](image)

Figure 3ab. Heliolongitudinal changes of the (a) $B_\eta$ and (b) $B_\phi$ components of the IMF (solid line) near Earth orbit obtained as the solution of Maxwell’s equations for the solar wind velocity given by Equation (5) taking into account sector structure of the IMF and the $B_\eta$ and $B_\phi$ components in the period of February 6- March 4, 2003 (dashed line).

Figure 3ab presents the coincidence of the experimental and theoretical components of the IMF included in the model of the Fd of the GCR intensity.

### 2.2 Solution of the transport equation

In the model we assume that the Fd of the GCR intensity is caused by the change of the diffusion coefficient owing to the varying of the IMF turbulence. [2-5] showed that the temporal changes of the rigidity spectrum of the Fd of the GCR intensity found by neutron monitors experimental data can be provided from the theoretical modeling only if the change of the IMF turbulence is assumed.

In the proposed model the change of diffusion coefficient $K_\eta$ takes place in the disturbed vicinity of the interplanetary space responsible for the Fd. The changes of the structure of the IMF turbulence are characterized by the changes of the exponent $\nu$ of the PSD of the IMF turbulence [14]. So, we assume that the exponent $\nu$ of the PSD of the IMF turbulence changes versus the heliolongitudes (the value of the heliolongitudes 13.30° corresponds to the one day -86400 s). The parallel diffusion coefficient $K_\eta$ of cosmic ray particles has a form: $K_\eta = K_0 K(r) K(R, \varphi)$, where

$$K_0 = 4.5 \times 10^{31} \text{ cm}^2 / \text{s} \quad K(r) = 1 + 0.5 (r / 1 U) \quad K(R, \varphi) = R^2 \nu(\varphi)$$

where we assume that $\nu(\varphi) = 0.8 - 0.15 \times (\cos(\varphi) - 0.7)$. The disturbed vicinity of the interplanetary space responsible for the Fd is restricted in distance, heliolongitudes and heliolatitudes as: $r < 8.4 U$, $\varphi \in (40^\circ;320^\circ)$, $\theta \in (60^\circ;120^\circ)$. The ratios of $\beta$ and $\beta_i$ of the perpendicular $\kappa_\perp$ and drift $\kappa_\parallel$ diffusion coefficients to the parallel diffusion coefficient $K_\parallel$ of the GCR particles are given in standard form, $\beta = \frac{K_\perp}{K_\parallel} = (1 + \omega^2 \tau)^{-1}$ and $\beta_i = \frac{K_\perp}{K_\parallel} = \omega^2 (1 + \omega^2 \tau^2)^{-1}$, where

$$\omega = 300 \lambda_\parallel R^{-1}, \quad B \text{ is the strength of the IMF. These expressions of } \beta \text{ and } \beta_i \text{ are acceptable for the scattering of the GCR particles to which neutron monitors and ground muon telescopes respond and it is in accord with the quasi linear theory [14, 15].}

The components $B_\eta$ and $B_\phi$ obtained from the solution of system of equations (4) appear in the spiral angle, $\psi = \arctan \left( \frac{B_\phi}{B_\eta} \right)$, i.e. in the principal directions of the anisotropic diffusion tensor; while the magnitude $B = \sqrt{B_\eta^2 + B_\phi^2}$ is introduced via the value of the parallel diffusion coefficient, $K_\parallel$.

The expected changes of the density of the GCR for the rigidity of 10 GV during the Fd are presented in figure 4. Figure 4 shows the reasonable compatibility between the observed and expected GCR intensity during Fd.

![Figure 4](image)

Figure 4. Changes of the expected (solid line) amplitudes of the Fd of the GCR intensity for the rigidity of 10 GV during the Fd corresponding to the streams ($S_\eta$, $S_\phi$, $S_\eta$) as follows:

$$A_\eta = -\frac{3 S_\eta}{\nu}, A_\phi = -\frac{3 S_\phi}{\nu}, A_\eta = -\frac{3 S_\phi}{\nu}$$  \hspace{1cm} (8)

Components of corresponding anisotropy of GCR are e.g., [16]:

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\[ A_v = -\frac{3}{V} CV + \frac{3}{V} K_H \left[ K_{\alpha \beta} \nabla_v f + K_{\alpha \gamma} \nabla_\varphi f + K_{\gamma \varphi} \nabla f \right] \]
\[ A_\theta = \frac{3}{V} K_H \left[ K_{\alpha \beta} \nabla_\varphi f + K_{\alpha \gamma} \nabla_v f + K_{\gamma \varphi} \nabla f \right] \]
\[ A_\varphi = \frac{3}{V} K_H \left[ K_{\alpha \beta} \nabla f + K_{\alpha \gamma} \nabla_\varphi f + K_{\gamma \varphi} \nabla_v f \right] \]

where \( V \) is the cosmic ray particles velocity, \( C = 1.5 \) is the Compton-Getting factor, \( \nabla_v f = \frac{\partial f}{\partial v} \), \( \nabla_\varphi f = \frac{\partial f}{\partial \varphi} \), \( \nabla f = \frac{\partial f}{\partial r} \) are radial, heliolatitude and heli-longitude gradients in percents per 1 AU, respectively.

The results of calculations are presented in figure 5.

![Figure 5](image_url)

Figure 5: Harmonic diagram of the expected GCR anisotropy obtained for the model of the Fd. On the vertical and horizontal axes are the radial and azimuthal components of the GCR anisotropy in %, respectively.

### 3 Experimental data

We have compared the theoretical results with the experimental data of 3D GCR anisotropy found by the IZMI-RAN group [17].

![Figure 6](image_url)

Figure 6: Harmonic diagram of the expected anisotropy obtained for the model of the Fd (solid line) and experimental one (dashed line). On the vertical and horizontal axes are the radial and azimuthal components of the GCR anisotropy in %, respectively.

The experimental data
We have compared the theoretical results with the experimental data of 3D GCR anisotropy found by the IZMI-RAN group [17].

Figure 6 presents comparison of the theoretical modeling results with the experimental data. For better visualization the expected azimuthal component of GCR anisotropy \( A_\varphi \) is multiplied by two, and radial component \( A_v \) is two times reduced. It is seen from figure 6 that unfortunately, there is not satisfactorily correspondence between the theoretical and experimental results of the 3D anisotropy. These results show that due to complexity of the Fd there is difficult to compose the realistic model fully describing observational data.

### 4 Conclusions

We have composed the 3-D model of the Forbush decrease based on the Parker’s transport equation. The model incorporates a consistent, divergence-free IMF derived from Maxwell’s equations with the heliolongitudinally dependent solar wind velocity reproducing in situ observations. In the model it is also taken into account sector structure of the IMF according to experimental data.

The proposed model of the Fd is compatible with the observational changes of the GCR intensity, but it does not satisfactorily describe the behavior of the 3D GCR anisotropy. This phenomenon we relate to the complexity of the realistic Fd modeling.

### 5 References