The Distribution of the Zodiacal Light Brightness Varying with Time in the Geocentric Ecliptic Coordinate

YUANLEI ZOU1, ZHENSEN WU1, SHENMIAO HAN1
1School of Science, Xidian University, Xi’an, 710071, China
wuzhs@mail.xidian.edu.cn

Abstract: Based on T. Kelsall’s model of the brightness of the zodiacal light, into which the earth’s orbit radius varying with time is introduced, it is derived that the relation of the zodiacal light brightness and time, and the distribution of the brightness of the zodiacal light on a given date in the geocentric ecliptic coordinate. After the analysis of the influence of the time on the zodiacal light brightness is made, it is indicated in this paper that the zodiacal light brightness varying with time is caused by the heliocentric distance, which also varies with time and is approximately seen as the distance between the sun and the detector.

Keywords: T.kesall’s model, zodiacal cloud, zodiacal light, geocentric ecliptic coordinate, Earth’s orbit

1. Introduction

The zodiacal light is an important astronomical phenomena commonly occurring in spring or autumn, considered as the interplanetary dust’s thermal radiation in infrared band and the scattering light in visual bands that the light from the sun is scattered by the interplanetary dust (IPD) cloud. The IPD cloud which centers on the sun is symmetry and its main axis lies in the ecliptic plane. Many scientists have done many researches on it and give some models about its structure, distribution of brightness and distribution of density. For instance, the Helios 1 and 2 spacecraft mapped the distribution of the zodiacal light[1]. To model the zodiacal light observed by the Clementine aircraft, J.M.Hann&H.A.Zook[2] described the brightness of the zodiacal light along the ecliptic in given direction in 2001. Based the observation of IRAS, COBE and ISO, William T.Reach(1997) pointed out that the zodiacal light was caused by the smooth cloud, dust-bands, circumsolar rings and dust trails[3]. Most of studies above can describe the main features of zodiacal light, but can’t explain well that the brightness of the zodiacal light varies with the time and location of observation. So it will bring the trouble when we need know the IPD cloud’s structure and the zodiacal light brightness viewed from the earth.

To solve the problem, based on T.Kessal’s model of the zodiacal light brightness[4], into which the earth’s orbit radius varying with time is introduced, it is derived that the relation between the zodiacal light brightness and time. The processing steps are as following: Firstly, by solving the Earth’s ellipse orbit equation describing how the earth runs around the sun, it can be determined the relation of the Earth orbit’s radius \( R \) and eccentric anomaly \( E \) and the relation between true anomaly \( f \) and \( E \). Secondly, from the Kepler Equation, the eccentric anomaly \( E \) which depends on time \( t \) can be obtained, then we get the final numerical values of \( R \) and \( \lambda \) (the longitude of the Earth in the heliocentric ecliptic coordinate). Finally, \( R \) and \( \lambda \) were submitted into T.Kessal’s model, so the brightness distribution of the zodiacal light in space will be obtained.

2. T. Kelsall model

2.1. The ecliptic coordinate

Figure 1. The particle P in the ecliptic coordinate
2.2. The zodiacal cloud

The zodiacal light observed brightness should include the information of multi-component particles such as Smooth Zodiacal Cloud, Dust Bands and circumsolar Ring. In this paper, we just discussed the Smooth Zodiacal Cloud for its contribution to the zodiacal light brightness is most and has a relative simple structure for modeling.

In the heliocentric coordinate, the center of the zodiacal cloud \( (X_0, Y_0, Z_0) \) is set as its origin, so the new coordinate system \( (X', Y', Z') \) can be expressed as:

\[
\begin{align*}
X' &= X - X_0 \\
Y' &= Y - Y_0 \\
Z' &= Z - Z_0
\end{align*}
\]

Where \( R_c \) is defined as the distance from \( P(X,Y,Z) \) to the center of the zodiacal cloud \( (X_0, Y_0, Z_0) \) and \( Z_c \) is defined as the height above the symmetry plane of the zodiacal cloud:

\[
Z_c = X' \sin \Omega \sin i - Y' \cos \Omega \cos i + Z' \cos i
\]

Where \( \Omega \) and \( i \) respectively are the ascending and inclination node of the symmetry plane of the zodiacal cloud tilted with the ecliptic plane.

In T. Kelsall’s model, the density of the zodiacal cloud was given as following formula, which is divided into radial and vertical terms:

\[
n(X,Y,Z) = n_0 R^{-\gamma} f(\xi)
\]

Where \( \xi = |Z_c|/R \) describes that the particle separates from the symmetry plane of the zodiacal cloud, meaning the angle between P-Sun and the symmetry plane of the zodiacal cloud. \( \beta, \gamma \) and \( \mu \) are free parameters obtained by simulating the DIRBE data.

2.3. The zodiacal light brightness

From above that, there is a clear introduction about the density of the zodiacal cloud. Based on the single scattering theory, the zodiacal light brightness could be considered as the sum of scattered light of the dust particle lying in the direction which the COBE/DIRBE observed.

If the scatter phase function \( \Phi_\lambda(\theta) \) and the albedo \( A \) are given, the brightness of the zodiacal light will be given as:

\[
Z = \int n(X,Y,Z) A_\lambda F_\lambda \Phi_\lambda(\theta) ds
\]

From the above equation, we still can not get the integral result directly in \( S \). But we will get the relations between \( S \) with \( \theta \) in fig.1 as following:

\[
\frac{S}{\sin(\theta - \epsilon)} = \frac{R}{\sin(\epsilon)} = \frac{R_0}{\sin(\pi - \theta)} \quad (7)
\]

Then:

\[
S = R_0 \frac{\sin(\theta - \epsilon)}{\sin(\theta)} \quad (8)
\]

Where \( \epsilon = \text{arccos}(\cos \phi \cos \beta) \), \( \phi = \lambda - \lambda_0 \), and \( \epsilon \) is the elongation angle. So Eq. (1.6) will be transformed as following equation:

\[
Z_\lambda(\beta,\phi,t) = \int_\lambda n(\beta,\phi,s) A_\lambda F_\lambda \Phi_\lambda(\theta) d\theta
\]

From Eq (1.9) and (1.10), if the direction of view \( (\beta,\phi) \) and the Earth’s orbit radius \( R_o \) are given, the brightness of the zodiacal light will be obtained. Finally, we can get the distribution of the zodiacal light brightness in the geocentric zodiacal coordinate in space which also varies with the observed time.

2.4. Determine the \( R_o \) and \( \lambda_0 \) depended on observed time

The Earth runs around the sun in ellipse orbit which has a slightly changing in different years because of the disturbance. In this paper, the orbit is presumed as a fixed one lying in the ecliptic plane. And the relationship between \( R_o \), \( f \) and \( E \) can be expressed as follows. [5-6]:

Vol. 10, 140
To obtain the value of $E$, the kepler’s equation is solved by a iterative method as follows.

$$E - e \sin E = n(t - \tau) = M$$ \hspace{1cm} (13)

Where $e$ is eccentricity rate of the Earth, $n$ is the daily motion (degrees/day), and $t$ is a day counting in Julian Number. $\tau$, the time at Perihelion, should be expressed as the same day count. $M$ is Mean anomaly.

Presumed the day number as $d$, it can be computed in Julian Day Number[7]:

$$d = \frac{367 \times Y - (7 \times (Y + ((M + 9)/12)))}{4} + (275 \times M)/9 + D - 730530$$ \hspace{1cm} (14)

And adapt “the J2000 Julian Day Number” which started from 2000.1.1, so:

$$d_0 = \frac{367 \times 2000 - (7 \times (2000 + ((1 + 9)/12)))}{4} + (275 \times 1)/9 + 1 - 730530$$ \hspace{1cm} (15)

$$\tau = \frac{367 \times y - (7 \times (y + ((1 + 9)/12)))}{4} + (275 \times 1)/9 + 3 - 730530 - d_0$$ \hspace{1cm} (16)

$\tau$ and $d_0$ are the time at Perihelion and the J2000 Julian Day Number, respectively.

So we can iteratively solve equation (1.4) as followed:

$$E_1 = M$$

$$E_2 = M + e \sin E_1$$

$$E_3 = M + e \sin E_2$$

$$\cdots$$

$$E_n = M + e \sin E_{n-1}$$

If $|E_{n+1} - E_n| \leq 10^{-8}$, we will get the $E = E_n$.

3 Results

3.1. The Earth’s situation in space

Figure 2. The longitude of the Earth in the heliocentric ecliptic coordinates $\lambda$ vs. time (the J2000 Julian Day)

In Fig.2 and Fig.3, in 2000, the ecliptic longitude of the Earth varies linearly with time for the Daily motion $n$ is set as a constant (360/365.257degrees/day). In fact, the value of $n$ is largest at perihelion and smallest at aphelion. The Earth orbit radius varies periodically with time, and its largest value can be expressed as $R_{\text{max}} = 1.016\text{AU}$ on $t=180\text{day}$. It’s just in September during which the Earth is near the aphelion; its smallest value also can be expressed as $R_{\text{min}} = 0.983\text{AU}$, $t=0(365)\text{ day}$, and it’s just in January during which the Earth is near the perihelion. So the Earth’s situation can be determined by $\lambda$ and $R_{\text{a}}$.

3.2. The density distribution in $(\beta, \lambda)$ at $S=1\text{AU}$

Figure 4. The distribution of particle density in Z axis and $d$ ($d = \sqrt{X^2 + Y^2}$)

Fig.4 illustrates that the zodiacal cloud particle centers on the Sun and distributes symmetrically in the both sides of ecliptic plane. The density ranges from $10^{-3} \text{ cm}^{-3}$ to $10^{-7} \text{ cm}^{-3}$ and decays fast away from the Sun.

3.3. The zodiacal light brightness vs. time

Figure 5. The zodiacal light brightness vs. time
Fig.5 describes the zodiacal light observed brightness varying with time, which is detected towards the ecliptic north pole by the COBE/DIRBE, so $\beta = 90^\circ$ and $\phi = 0^\circ$ ($\phi = \lambda - \lambda_e$). In fig.6, we consider the influence of $R_\odot$ varying with the observed time on T.kelsall’s zodiacal light brightness model, from which it is obtained that the zodiacal light brightness varying with time. Obviously, the brightness has the same profile as the one in Fig.6 and Fig.7. In Fig.7, when $(\beta = 0^\circ, \phi = 30^\circ)$ and $(\beta = 0^\circ, \phi = 15^\circ)$, all of them have the same brightness scale as the one in fig.6. In Fig.2 and Fig.6, the $R_\odot$ profile reverses the brightness’s, on the same day which $R_\odot$ has its max $R_{\odot \max}$, but the zodiacal brightness has its min value. So the zodiacal brightness varying with the observed time is caused by the Earth’s orbit changes, this may differ from the explanations that the zodiacal cloud is titled with the ecliptic plane [4].

In Fig.8, on the given day such as 2000.1.3, we have modeled the brightness distribution of zodiacal light at the wavelength of 1.25μm. And at this wavelength, the parameters referred in T.kesall’s model are shown in table 1.

<table>
<thead>
<tr>
<th>$n_0$ (cm$^{-3}$)</th>
<th>$a$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.13 \times 10^7$</td>
<td>1.34</td>
<td>4.14</td>
<td>0.942</td>
<td>0.189</td>
</tr>
<tr>
<td>$i$</td>
<td>$\Omega$</td>
<td>$X_0$</td>
<td>$Y_0$</td>
<td>$Z_0$</td>
</tr>
<tr>
<td>2.03$^\circ$</td>
<td>77.7$^\circ$</td>
<td>$1.9 \times 10^3$</td>
<td>$5.48 \times 10^3$</td>
<td>$-2.15 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 1 the parameters referred in T.kesall’s model

4 Conclusions

In this paper, based on T.kelsall’s model, two important conclusions can be obtained that are distribution of the brightness of the zodiacal light and the density of the zodiacal cloud on a given date in the geocentric ecliptic coordinate. We have considered the influence of $R_\odot$ varying with the observed time on T.kelsall’s zodiacal light brightness model, and have made a conclusion that the zodiacal brightness varying with the observed time is caused by the Earth’s orbit change.

5 References

[1] C.Leinert, et al., The Zodiacal Light from 1.0 to 0.3 AU as Observed by the Helios Space Probes. Astronomy And Astrophysics, 1981, vol. 103, 177-188.