Electron and Positron Modulaton in the Heliosphere

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Abstract: This work presents a new stochastic (Monte-Carlo) method for modeling the solar modulation of electrons and positrons in the 3-dimensional heliosphere. The model incorporates effects of diffusion, convection, gradient and curvature drift, neutral sheet drift, and adiabatic cooling. All transport quantities are computed ab initio i.e., turbulence quantities are modeled based upon turbulence transport theory, and the cosmic ray diffusion tensor is computed based upon quasilinear theory (for parallel transport) and the nonlinear guiding center theory (for perpendicular transport). Novel features of the model include the use of spherical coordinates which makes the code much more efficient than prior models that employed Cartesian coordinates and a new rigorous method for computing neutral sheet drift. Modeling results are compared with observations of cosmic electrons and positrons.

Keywords: Stochastic approach, cosmic-ray modulation, numerical method.

1 Introduction

Solar modulation is the process by which the Sun impedes the entry of cosmic rays into the solar system, thereby altering the intensity and energy spectrum of the cosmic rays. This process is governed by Parker's equation which includes four different modulation mechanisms: diffusion, convection, adiabatic cooling, and drifts. Convection and adiabatic cooling have been successfully modeled. Determining the diffusion effect is a challenging astrophysical problem, because it requires an understanding of the properties of magnetic fields and turbulence throughout the heliosphere, while also demands accurate theories for determining particle transport properties — such as the diffusion tensor — from the properties of the turbulence [18]. Another challenge is to incorporate the drift effects in numerical simulations which is especially difficult in the vicinity of the heliospheric current sheet (HCS).

It is important to consider particle drifts — gradient and curvature drifts as well as drift along the HCS — in modeling cosmic ray transport inside the heliosphere because there is a well-known difference in cosmic ray modulation in successive polarity periods [11, 9]. Although gradient and curvature drifts are well understood and can be calculated based on the mean magnetic field (see, e.g., [14]), a generally accepted model for HCS drift is still elusive. Early approaches [14, 9, 13] take the HCS drift as a δ-function limit of the regular gradient and curvature drifts, and implement it as a jump condition. Other approaches include simply neglecting drifts (see, e.g., [11, 7, 1]), emulating drift in 2D models (see, e.g., [20]), using conservation of drift flux in 2D models [6], or calculating a 2D drift velocity field [2]. The exact technique used for HCS drift is not always specified (see, e.g., [15, 16]). See Burger and Potgieter 1989 [5] for a review of a number of models for HCS drift. In the present approach we assume that the 3D HCS is locally flat and then use the drift velocity field of [4], derived through particle tracing. The HCS is described by a transcendental function [13, 15, 21] which usually makes the simulation prohibitive in terms of computing time if the drift velocity field of [4] is used in three dimensions. By making suitable assumptions the computing time can however be reduced considerably without compromising accuracy unduly.

We investigate the different behaviors with particles of opposite charge signs or in different solar cycles using our newly developed stochastic method [19] and compare the results with measured data by spacecraft. Our new approaches and the HCS model is presented in Section 2. The results are discussed in Section 3. At last, Section 4 concludes this article.

2 Stochastic Model

The governing equation for solar modulation is Parker's well-known transport equation [17, 12],

$$\frac{\partial f}{\partial t} = \nabla \cdot (\boldsymbol{\kappa} \cdot \nabla f - \mathbf{V} f) + \frac{1}{3p^2}(\nabla \cdot \mathbf{V})\frac{\partial p^3 f}{\partial p} , \quad (1)$$

where $f(\mathbf{r}, p, t)$ is the omni-directional distribution function (i.e., the phase space density averaged over solid angle in momentum space), with p the particle momentum, $\mathbf{r}$ the spatial variable and $\mathbf{V}$ the solar wind velocity. The spatial diffusion tensor, $\boldsymbol{\kappa}$, can be decomposed into two parts, $\boldsymbol{\kappa}_s$, the symmetric part, and $\boldsymbol{\kappa}_A$, the anti-symmetric part. We use the analytic formula for $\boldsymbol{\kappa}_s$ based on quasi-linear theory and NLGC theory [18] in this paper.

2.1 Heliospherical current sheet

The tilt angle, α, of the heliospheric current sheet at any radial distance r is defined to be the maximal latitudinal excursion of the current sheet within a shell of radius r. We assume that the excursion is the same in the northern and southern hemispheres. Because we allow the tilt angle on the solar source surface to vary with time, the tilt angle at radius r is related to the tilt angle at the source surface r_s at some earlier time,

$$\begin{aligned}\alpha(t,r) &= \alpha\left(t-\frac{r-r_s}{V},r_s\right)\\ &= \alpha_0 \pm \frac{\pi}{11\text{years}}\left(t-t_0-\frac{r-r_s}{V}\right) . \end{aligned} \tag{2}$$

The expression after the first equal sign is more general and might be used, for instance, if the time variation of the current sheet is based on observations [8]. For concreteness, we adopt the expression after the second equal sign, which assumes a linear variation during an 11-year long solar cycle. Here, α_0 is the tilt angle at the source surface at some reference time t_0. The plus and minus signs apply when the tilt angle is respectively increasing or decreasing, and this sign needs to be reversed at the appropriate time in the solar cycle. Note that this version is a modification of [21]. The full shape of the current sheet in three dimensions is then defined as the surface that satisfies the equation,

$$\tan\left(\frac{\pi}{2}-\theta\right) = \tan\alpha(t,r)\sin\phi_0 , \tag{3}$$

where $\phi_0 = \phi + (r-r_s)\Omega_\odot/V - \Omega_\odot\,(t-t_0)$ and $\Omega_\odot$ is the sidereal solar rotation rate corresponding to a period of 25.4 days.

Please note that Equation 3 is valid for any tilt angle in theory. Jokipii and Thomas [13] presented an approximation to this equation which uses sine instead of tangent for a small tilt angle, i.e., $\alpha < 5$ degrees. However, even today many publications use this approximation for much larger α than its domain of validity, rather than the correct formula displayed in Equation 3 [15, 21]. According to Equation 3, The shape of the HCS is changing with time and position.

The parallel diffusion component, $\kappa_\parallel$, and the perpendicular component, $\kappa_\perp$, can be determined *ab initio* from turbulence models based on spacecraft measurements [18].

The general weak-scattering drift velocity of a particle with charge q, momentum p, and speed v is given by [14]. However, we replace the δ-function approximation with an improved forumla shown below in Equation 4.

2.2 HCS drift

For the current sheet drift speed/magnitude, we adopt a formula following [4, 3],

$$V_{dc} = \left(0.457 - 0.412\frac{d}{R_g} + 0.0915\frac{d^2}{R_g^2}\right) v , \tag{4}$$

where d is the distance to the current sheet and R_g is the Larmor radius. Note that this formula is valid when d is smaller than two times R_g and strictly speaking, it is only valid for a flat HCS. We assume locally the HCS is flat, in other words, we take the first order approximation. However, if there was a more accurate result for the magnitude of the HCS drift, our method is easy to be adapted.

We find that the HCS velocity for a 3D wavy HCS are given by

$$\begin{aligned}V_{dcr} &= \frac{r\sin\theta\Omega_\odot V_{dc}}{\sqrt{V^2+V_{\theta x}^2+r^2\sin^2\theta\Omega_\odot^2}} ,\\ V_{dc\theta} &= -\frac{\tan\alpha\sin\theta\cos\phi_0\left(V^2+r^2\sin^2\theta\Omega_\odot^2\right)}{\sqrt{V^2+V_{\theta x}^2+r^2\sin^2\theta\Omega_\odot^2}}\frac{V_{dc}}{V} ,\\ V_{dc\phi} &= \frac{VV_{dc}}{\sqrt{V^2+V_{\theta x}^2+r^2\sin^2\theta\Omega_\odot^2}} ,\end{aligned} \tag{5}$$

where $V_{\theta x}$ is given by

$$\begin{aligned}V_{\theta x} = &-\frac{\tan\alpha\sin\theta\cos\phi_0}{V}\left(V^2+r^2\sin^2\theta\Omega_\odot^2\right)\\ &\pm\frac{\pi}{11\text{years}}\frac{r^2\sin^3\theta\Omega_\odot}{V}\sin\phi_0\sec^2\alpha .\end{aligned} \tag{6}$$

Note that Equation 5 is derived analytically based on the shape of the HCS without further assumptions and is valid for large tilt angle. That is to say that if we have a better formula than Equation 4, we can still use Equation 5 with a different V_{dc}. If we choose a different heliospheric magnetic field, we could just change the value of the tangent of the field line and the normal vector of the HCS and find the corresponding directions of the HCS drift components.

2.3 Numerical integration

The stochastic method to solve Equation 1 can be divided into two steps. The first step is that we need to find the corresponding SDEs to Equation 1 based on Ito formula [10] and solve it. The second step is that we obtain the modulated distribution function from the solutions of the corresponding SDEs [19].

We choose backward-in-time method to integrate the SDEs. First we start from some initial position, chosen to be the Earth in this paper, and some initial time, integrate along the trajectory of the pseudo-particle, follow them back in time until they reach the boundary. Then we start the whole process again for another pseudo-particle. In this way, we have the steady-state solution. Because this

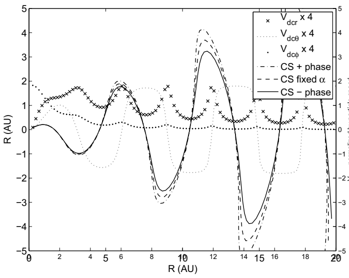

Figure 1: The trajectory of the HCS on the $r - \theta$ plane for a fixed ϕ and the wavy HCS drift on the same plane. "CS" denotes the current sheet. "+ phase" means we take the positive sign in Equation 2 after α_0 while "- phase" means the negative sign.

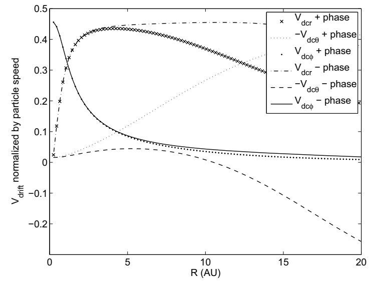

Figure 2: The wavy HCS drift along a field line with a tilt angle of 2 degrees.

is equivalent to set $t \rightarrow \infty$, we don't need and shouldn't specify the initial condition. In other words, the initial condition has no contribution to our solution.

All modulation parameters and boundary conditions are taken to be the same as given in [15]. More details of how to solve Equation 1 by using the stochastic method can be found in [19].

3 Results

The components of the HCS drift are shown in Figures 1 and 2. These figures apply to a particle right on the current sheet. Figure 1 shows the components of the HCS drift when ϕ is a constant and tilt angle is 18 degrees. The dashed line is the HCS with a fixed, i.e., time-independent, tilt angle. In this figure, we can see the time-dependent tilt angle clearly affects the shape of the current sheet. For example, at about 12 AU, the peak of the positive phase is about 1 AU larger than that of the negative phase. The difference becomes even larger with the increasing radial distance. However, the time-dependent tilt angle has a much smaller effect on the magnitude of the drift speeds. Therefore we only plot the drift speeds for the positive phase.

Figure 2 show the components of the HCS drift when θ is a constant, and where ϕ and r are warying in the same way as a Parker Field line. Note that in these two figures, $V_{d\theta}$ is negated. The time-dependent tilt angle shows very weak effect on $V_{dc\phi}$, but a dramatic effect on $V_{dc\theta}$.

The features shown in Figures 1 and 2 are different from those in [2]. First of all, the normalized V_{dcr} is independent of energy. Second, the normalized V_{dcr} is a smooth function of r whose maximum is reached at about 1.8 AU and then decreases all the way to 100 AU.

We made simulations for electrons and positrons in both polarities of a typical solar minimum, which are shown in Figure 3 and Figure 4. Figure 3 showed the comparison between our simulation of the spectra and the observed data by AMS for A > 0 and a tilt angle of 30 degrees. In this figure we can see the present model with our new treatment of the HCS drift qualitatively agrees with observations. Figure 4 showed the comparisons between our simulated the positron fractions and observations. Again, our new model agrees with observations.

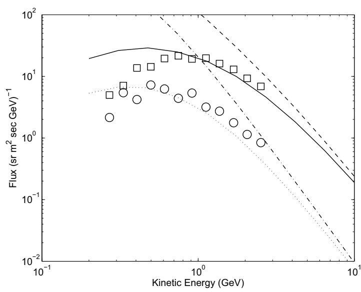

Figure 3: The spectra of protons and electrons. The dashed line and the dash-dotted line are the LIS for electrons and positrons respectively. The discrete symbols are AMS measurements during 1998. The solid line and the dotted line are spectra for electrons and positrons respectively.

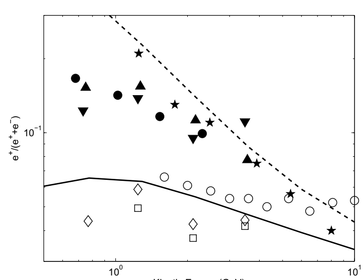

Figure 4: The positron fraction. The solid line and the dashed line are the simulation results for A $<$ 0 (empty symbols) and A $>$ 0 (filled symbols) respectively. Empty circles are PAMELA data. Empty diamonds, empty squares, and filled triangles are AESOP data. Filled circles and filled pentagrams are AMS data.

4 Conclusions

We demonstrated the method on how to analytically determine the 3D HCS drift based on the Parker field for the first time so far as we know. We also presented a new way to determine the magnitude of the HCS drift. By combining these two methods, we show that the 3D HCS drift is important for low energy particles and the modulation is stronger by comparing our results to observations.

We also note that our methods are extensible to other magnetic field configurations and to other approaches to determine the magnitude of the HCS drift.

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