Diffusion model of the solar energetic particle injection into interplanetary medium

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Abstract: The temporal and angular dependencies of the solar energetic particle flux injected into interplanetary space have been obtained from the solution of the particle anisotropic diffusion equation for pointed in space and impulsive in time source. The model, being the extension of the Ng and Gleeson’s model, describes the particle diffusion in the spherical shell with given thickness surrounding the Sun. It explains possible cause the long (up to 10 hours) solar energetic particle injection.

With account of the particle transport in the interplanetary medium (Krimigis’s model for high-energy particles) the model reproduces non-monotonic behavior in time the solar energetic particle intensity and exponential decrease in time of the particle flux observed in some real cases.

Keywords: Particle transport, injection, diffusion.

1 Introduction

Now it is established mainly from analysis of the ionization states and chemical abundances of solar energetic particles (SEP) fluences at the Earth orbit that two distinct classes of SEP events so–called impulsive and gradual exist [1]. The upper part of the Sun atmosphere is generation region of the SEPs in the gradual events. The shock driven out the Sun by the coronal mass ejection accelerates the particles [2]. SEP acceleration is ended off at exit of the shock in the interplanetary space where both SEPs and shock propagate. The closed magnetic regions placed in the lower part of the Sun atmosphere are generation regions of the SEPs in the impulsive events. The particles are accelerated by stochastic mechanism [4]. To exit in the interplanetary medium the SEPs have to intersect the Sun atmosphere. The investigations of the SEP injection in the impulsive events are interest because the angular and temporal dependences of the injected particle flux exert an substantial effect on the dynamics of the SEP flux in the interplanetary space. The different models of the SEP injection were considered. The Ng, Gleeson model explaining simultaneously four principal features of SEPs in impulsive events observed at $r = 1AU$ is most useful [5] and references there in).

However this model has sufficient disadvantage. It assumes impulsive release of the SEPs at a point and their subsequent propagation on the spherical surface presenting the Sun atmosphere with continuous leakage into the interplanetary space. As a result the SEP fluence at the lines of the magnetic field intersecting the point source at initial time moment is infinite. Furthermore, it’s not understand which properties of the real atmosphere are corresponded with the values of the model parameters obtained from comparison the calculation and observations. Here the improved variant of the Ng, Gleeson model of the SEP injection is presented.

2 The diffusive model of the SEP injection

It’s supposed the Sun atmosphere is the spherical shell bounded by two surface with radii $r_{\text{max}}$ and $r_{\text{min}}$. At initial time moment the SEPs are given as a point source in space and impulsive in time. The particles propagate by diffusive way. The desired solution has axial symmetry relative to spherical coordinate system whose polar axis intersects the point source. Correspondent equation has form

\[
\frac{\partial f}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( k_r r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k_\theta \sin \theta \frac{\partial f}{\partial \theta} \right),
\]

(1)

where $f$ is the particle distribution function; $k_r, k_\theta$ are components of the diffusion tensor taken as constants. Impulsive source in time may be taken into account as initial condition. Let us write the equation (1) in dimension less form using as a spatial scale $r_s$ and a time scale $t_s = r_s^2/k_r$, where $r_s$ is the Sun radius. The equation (1) may be solved by the method of the variable separation

\[
f = \frac{F_s}{4\pi r_s^3} \sum_{n=0}^{\infty} (2n + 1) P_n \cdot R_n(r,t),
\]

(2)
where \( F_g \) is injected particle distribution function, \( P_n \) are the Legendre polynomials of degree \( n \) and \( R_0 \) satisfies an equation

\[
\frac{\partial R_0}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial R_0}{\partial r} \right) - \frac{\varepsilon n(n+1)}{r^2} R_0 \tag{3}
\]

with the initial \( R_0(t \to 0) = e^{-\varepsilon(n+1)t} \delta(r - r_g) / r_g^2 \) and internal \( \frac{\partial R_0}{\partial r} \big|_{r_{\text{min}}} = 0 \), external \( R_0 \big|_{r_{\text{max}}} = 0 \) boundary conditions. Here \( \varepsilon = k_p/k_r \), \( r_g \) is radius of the source, \( \delta(r - r_g) \) is delta–function and it is used property of the Legendre polynomials

\[
\lim_{t \to 0} \sum_{n=0}^{\infty} (2n+1)e^{-\varepsilon(n+1)t} P_n = 2 \cdot (\cos \theta - 1) \tag{5}
\]

The equation (3) are solved numerically by the implicit difference method of 2-nd order of accuracy both in space and in time.

Different features of the SEP injection may be determined from the solution (2):

1) the particle flux through a unit element area

\[
j_{es} = -\frac{F_g}{4\pi r^2 t_s} \sum_{n=0}^{\infty} (2n+1)P_n \frac{\partial R_0}{\partial r} \big|_{r_{\text{max}}} \tag{4}
\]

2) the particle number escaped into outer space through a unit element area

\[
f_{es} = \int_0^t j_{es} dt = -\frac{F_g}{4\pi r^2} \sum_{n=0}^{\infty} (2n+1)P_n \int_0^t \frac{\partial R_0}{\partial r} \big|_{r_{\text{max}}} dt, \tag{5}
\]

3) whole particle flux

\[
J_{es} = \int_0^t \int_0^{r_{\text{max}}} j_{es} r^2 \sin \theta d\theta dr = -\frac{F_g r_{\text{max}}^2}{t_s} \frac{\partial R_0}{\partial r} \big|_{r_{\text{max}}}, \tag{6}
\]

4) whole particle number escaped from the atmosphere

\[
F_{es} = \int_0^t F_{es} dt = -F_g r_{\text{max}}^2 \int_0^t \frac{\partial R_0}{\partial r} r_{\text{max}} dt, \tag{7}
\]

5) particle number being in the shell at some time moment

\[
f_{res} = F_g \int_{r_{\text{min}}}^{r_{\text{max}}} r^2 R_0 dr, \tag{8}
\]

Accuracy of the numerical calculation can be obtained by using the law of particle conserving \( F_{es} + f_{res} = F_g \).

Figures 1, 2 present \( j_{es} \) (solid continuous curves) and \( f_{es} \) (dash curves) in dependence on time. In calculations presented in Fig. 1, 2 it is adopted respectively: 1) \( r_{\text{min}} = r_s \), \( r_{\text{max}} = 3r_s \), \( r_g = 1.5r_s \), \( \varepsilon = 1 \) and 2) \( r_g = 2.5r_s \), \( \varepsilon = 0.3 \).

As seen in Fig. 1, 2 the injection duration and its angular dependence depend on position of the source relative to the outer boundary of the shell, ratio of the components of the diffusion tensor and the radial diffusion coefficient.

The theory of the SEP acceleration in gradual events can be used for estimate of the diffusion coefficient in the Sun atmosphere. It is known that the acceleration time equals to travel time of the shock through the atmosphere [2]. The acceleration time and value of maximum momentum at accelerated particle spectrum relates by expression

\[
t_a = \frac{3\sigma}{(\sigma - 1)u_s^2} \int_{p_0}^{p_{\text{max}}} (k_1 + \sigma k_2) \frac{dp}{p}, \tag{9}
\]

where \( u_s \) is speed of plasma at region placed before the front; \( \sigma \) is the shock compression ratio; \( p_0 \) is momentum of the injected particle; \( k_1, k_2 \) are coefficients of space diffusion at regions placed before and behind front.

Figure 1: The particle flux (solid lines) and escaped particle number (dashed lines) through a unit element area plotted versus time. Upper curves correspond to values along the polar axis. The following curves correspond to values with step on 10° from polar axis.

Figure 2: Same as in Fig. 1 for other parameter values.
If take into account that \( u = V_S \) is speed of the shock front; \( \sigma = 4; k_1 = k_0(p/p_0)^{\alpha}, k_2 << k_1 \) then from (9) one can be obtained \( t_a \sim \delta r_e/V_S \) then taking into account indefinites one may be gotten \( k(p_{max}) \sim 0.1 r_e V_S [2] \), where \( \delta r_e \) is thickness of the atmosphere. In case \( V_S = 1000 \) km/s, \( p_{max} = 10 \) GeV/c the injection duration of particles with such energies is \( t_{inj} \sim t_s = r_e^2/k_r \sim r_e^2/k(p_{max}) \sim 2 \) h. So SEPs in impulsive events can to be in the solar atmosphere for some hours after their generation.

The SEP intensity observed at energy range of 84 to 200 MeV on board GOES–7 in events during 1989—1992 years was approximated within the diffusive model of the SEP propagation in the interplanetary space by means of fitting of the particle injection [6]. It was obtained that 17-th events can be explained by the particle injection for some hours. The single evidence of existence of the high energy SEPs near the Sun for some hours are the observation of the gamma–quants from \( \pi^0 \)-decay in events June 11 and 15, 1991 with cosmic observatories CGRO and GAMMA–1. The possible injection variant of the SEP events both classes has been suggested [6]: 1) particles are accelerated in the solar flares; 2) the SEPs fill the magnetic structures in which they are constructed for some hours; 3) SEPs are injected into interplanetary medium in consequence of destruction of these structures.

Within the presented model it’s obtained that the SEPs may to be near the Sun for some hours due to small—scale turbulence of the magnetic field of the solar atmosphere plasma.

The SEP features are obtained mainly from data of the detectors placed on the spacecrafts at the Earth orbit. The SEP propagation in the interplanetary space is determined by the diffusive transport equation

\[
\frac{\partial f}{\partial t} = \nabla (k \nabla f) - \nabla u f + \frac{\nabla u}{3} \frac{\partial f}{\partial p}, \tag{10}
\]

where \( f \) is the particle distribution function, \( k \) is a diffusion tensor, \( u \) is the solar wind velocity and \( p \) is a particle momentum. In the interplanetary medium \( k_\perp/k_{||} \ll 1 \), therefore the SEPs propagate independently in every magnetic flux tube. Here \( k_{||}, k_\perp \) are components of the diffusion tensor parallel and reverse to the regular interplanetary magnetic field. At the typical conditions mean free path of a proton with energy 100 MeV is about 0.1 \( r_e \), where \( r_e \) is astronomical unit. It is denoted that in interplanetary space the diffusion is fundamental process, in other words the first term in right–hand side of the equation (10) strongly exceeds second and third ones. Analytical solution of the diffusive equation in case the diffusion coefficient \( k = k_e(\varepsilon)(r/r_e)^{\beta}, 0 \leq \beta \leq 1 \) have been obtained by Krimigis [8]. Using it as a Green function one can be written the problem solution for the particles injected into the magnetic flux tube in form

\[
f = \frac{v_{max}^2}{r e c} \int_0^t j_{es}(\tau) \left( \frac{1}{(t - \tau)^{3/(2 - \beta)}} e^{-\frac{r_e^2 - \beta}{k_{es} r_{max}^2}} \right) dt, \tag{11}
\]

where \( I = 2(2 - \beta) k e \int_0^\infty x^{4/3} e^{-x} dx; r_{max} \) is radius of the outer boundary of the Sun atmosphere; \( j_{es} \) is density of the injected particle flux (4).

In principal, single event may to has two types of the injection. The fraction of the magnetic field lines go out from the source region into the interplanetary space. The SEPs being at these lines will quickly to go out into the interplanetary medium, slightly are scattered and form the particle fluence whose angular size on the whole coincides with the angular size of the source. Possibly, it is variant of the generation of the SEP fast component. Other fraction of the SEPs diffuses through the atmosphere, forms the injected particle flux whose angular size strongly exceeds the angular size of the source. It is possible variant of the formation of the SEP slow component. Such event can be presented by sum of two sources with different values of parameters.

Due to the Sun rotation the different magnetic flux tubes pass across a fixed point in space. The angular dependence of the injected particle flux and the Sun rotation can to influence on the SEP time–intensity at a fixed point. When initial longitude of the foot of the observer’s magnetic flux tube is more westernly relative to the flare site then it may to cause an exponential decrease of the SEP time–intensity, by contrast to power decrease of the one at the particle diffusion in every tube. At contrasted relative position of the ones one can be gotten non–monotonic behaviour of the SEP time–intensity.

As an example of an exponential decrease taken within developed the injection model Fig. 3 presents comparison...
the calculation and observation of the SEP time–intensity in February 23, 1956 event. In calculation following values of the parameters are used: \( r_{\text{min}} = r_s, r_{\text{max}} = 3r_s, \)
\( r_g = 2.8 \times r_s, \) \( k_g/k_r = 0.1, \) \( k_r = 3.4 \times 10^{17} \text{ cm}^2/\text{s}, \)
\( k_{\|} = k_{\|,e}(r/r_c)^\beta, \) where \( k_{\|,e} = 3.5 \times 10^{22} \text{ cm}^2/\text{s}, \beta = 0. \)
It is assumed that at initial time moment the foot of the observer’s magnetic flux tube intersects the point source. The observational data are registrations with the Chicago neutron monitor \([7]\). Dashed line is calculation of the SEP time–intensity propagating by diffusive way in a individual magnetic flux tube. As seen in Fig. 3 the model calculation presented by continious line reproduces observation well showing an exponential decrease.

Other example of the model use is presented in Fig. 4. The line presented by symbols are data of the SEP time–intensity propagating only because the small diffusion coefficient is needed for description of the increase stage and large one — for decrease stage. In Fig. 4 it is showed by thin solid line. In Fig. 4 the SEP time–intensity are obtained by means of sum of two sources. The following values of the parameters of the sources are used: 1) \( \varepsilon = 0.001, \) \( k_r = 1.4 \times 10^{17} \text{ cm}^2/\text{s}, \) \( r_g = 2r_s; \) 2) \( \varepsilon = 1, \) \( k_r = 1.4 \times 10^{17} \text{ cm}^2/\text{s}, \) \( r_g = 2r_s. \) For interplanetary propagation of the SEPs it is adopted \( k_{\|,e} = (r/r_c)^\beta, \) where \( k_{\|,e} = 10^{21} \text{ cm}^2/\text{s} \) is diffusion coefficient, \( \beta = 0. \) It is accepted that at initial time moment the foot of the observer’s magnetic flux tube is easterly relative the source site on \( 90^\circ. \) Too it’s assumed that amplitude of second source four times as many as one of the first source. In the Fig. 4 line marked by number 1 is time–intensity of the SEPs from the first source; by number 2 – from the second source; and by number 3 — sum of two sources.

### 3 Conclusion

The developed injection model of the SEPs in impulsive events allows to explain:

1) existence the SEPs near the Sun for some hours after their generation;
2) an exponential decrease of the SEP time–intensity at an fixed point in the interplanetary medium;
3) non–monotonic behaviour of the SEP time–intensity.

### References