Stochastic acceleration of relativistic particles in a turbulent magnetic field

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Abstract: We numerically study the solutions of the momentum diffusion equation for the evolution of the energetic particle distribution in a turbulent magnetic field. We assume a given turbulence power spectrum $W(k) \propto k^{-q}$ for the magnetic turbulence within some finite range of turbulent wave-vector k, and consider variety of turbulence power spectrum indices in the Bohm diffusion ($q = 1$), Kolmogorov diffusion ($q = \frac{5}{3}$), Kreichnan diffusion ($q = \frac{3}{2}$), and hard-sphere approximation ($q = 2$), respectively. In the case of quasi-linear approximation for particle-wave interactions, the turbulent spectrum gives the energy diffusion rates $D(\gamma) \propto \gamma^q$, the acceleration timescale $t_{\rm acc} \propto \gamma^{2-q}$, and the escape timescale $t_{\rm esc} \propto \gamma^{q-2}$. We find that when the particles are confined to the turbulent acceleration region, i.e. without injection and escape, both cooling and acceleration result to the plasm system reach to a quasi-Maxwellian distribution. The injection and escape process may dominate the plasm system evolution of the spectrum.

Keywords: acceleration of particles – radiation mechanism: non-thermal

1 Introduction

It is assumed that the ultra-relativistic electrons are scattered by a magnetized cloud or by the magnetic inhomogeneities or plasma waves in stochastic acceleration mechanism. For a given Alfven velocity v_A of a turbulent magnetic field, since the characteristic acceleration scale is $t_{\rm acc} \propto (\frac{v_A}{c})^2$, the stochastic acceleration process is referred as a second-order Fermi acceleration process. Generally, for a non-relativistic turbulence, $v_A \ll c$, the efficiency of the turbulent acceleration mechanism is frequently questioned when compared to acceleration by shock[1]. However, in the relativistic regime, shock acceleration encounters several difficulties in accelerating particles to high energies (e.g. [2, 3, 4]). The numerical simulations indicated that, in the high velocities of the turbulent modes, $v_A \sim c$, the efficiency of the turbulent acceleration may be comparable to the efficiency of the shock acceleration [5]. Following their numerical results, the stochastic acceleration process were discussed in different astrophysical sources of high energy radiation, such as blazars (e.g. [6, 7]), extragalactic large-scale jet (e.g., [8]), clusters of galaxies (e.g., [9]), gamma-ray bursts (e.g., [10]), and ultra-high energy cosmic rays [11].

Earlier numerical studies of the stochastic acceleration were mostly concerned with methods for solving the particles transport equation (e.g., [12]) and considered the short time dependencies of diffusion characteristics on turbulence in a narrow dynamic range [13, 14], or considered rapid acceleration in fully turbulent strong fields [15]. In fact, the particles may be accelerated at the shock front, then escape into the downstream region of the shock, during this process, the particles is still accelerated by turbulent plasma waves. Therefore, the turbulent process may significantly affect the evolution of the particle spectrum. Motivated by these results, in this paper we attempt to investigate further influence of the stochastic acceleration of ultra-relativistic particles by a turbulent magnetic field for the evolution of the particles spectrum.

2 Stochastic acceleration

The description of Fermi type acceleration is in term of isotropic diffusion in momentum space. The evolution of the energetic particle distribution can be described by the momentum diffusion equation [16]:

$$\frac{\partial f(p,t)}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}[p^2 D(p,t)\frac{\partial f(p,t)}{\partial p}] \, , \qquad (1)$$

where $f(p,t)$ is the isotropic, homogeneous phase space density, p the dimensionless particle momentum, $p = \beta\gamma$, $D(p,t)$ the momentum-diffusion coefficient due to interactions with hydro-magnetic waves [17], γ the particle Lorentz factor of the particle, and β the particle velocity in units of light velocity c. The particle number density $N(p,t)$ is directly related to the phase space density:$N(p,t) = 4\pi p^2 f(p,t)$. For the ultra-relativistic particles, $\beta \approx 1$, the momentum becomes equivalent to the Lorentz factor of particle ($p = \gamma$). After including injection, radiation, and escape of the particles, Eq. (1) can be rewritten as (e.g. [6, 18]):

$$\frac{\partial N(\gamma,t)}{\partial t} = \frac{\partial}{\partial \gamma}\{[C(\gamma,t) - A(\gamma,t)]N(\gamma,t) + D(\gamma,t)\frac{\partial N(\gamma,t)}{\partial \gamma}\} - \frac{N(\gamma,t)}{t_{\rm esc}} + Q(\gamma,t), \qquad (2)$$

where $C(\gamma,t)$ is the radiative cooling parameter that describes the synchrotron and inverse-Compton (IC) cooling of the particles, $A(\gamma,t)$ is the acceleration term that describes the particle energy gain per unit time, $D(\gamma,t)$ is the momentum-diffusion coefficient due to interactions with magnetohydrodynamic waves, $E(\gamma,t)$ represents escape term, and $Q(\gamma,t)$ is the source term. Here we consider continuous injection case, i.e. the particles are continuously injected at the lower energy ($1 \le \gamma \le 2$) and systematically accelerated up to the equilibrium energy ($\gamma_{\rm e}$), where the acceleration is fully compensated for by the cooling. We take the decrease of the scattering efficiency in the Klein-Nishina regime into account using the approximation [19].

For an isotropic Alfvenic turbulence with one dimensional power spectrum $W(k) \propto k^{-q}$ in a finite wave-vector range $k_{\rm min} < k < k_{\rm max}$, the turbulence energy density is $\int_{k_{\rm min}}^{k_{\rm max}} W(k)dk = \delta B^2/(8\pi)$, and the turbulence is level $\zeta = \delta B^2/B^2$, where $k = 2\pi/\lambda$. The index q is the power spectrum index, and wave-vector $k_{\rm max}$ and $k_{\rm min}$ correspond to the shortest and longest waves in the system, respectively. Using above described wave spectrum, the momentum diffusion coefficient can be evaluated [20]: $D(\gamma) \approx \beta_A^2 \zeta \gamma^2 c r_g^{(q-2)} \lambda_{\rm max}^{(1-q)}$, where $\beta_A = v_A/c$ is the Alfven velocity normalized to the light speed, and $r_g = \gamma m_e c^2/(eB_0)$ is the gyroradius of a ultra-relativistic particle. We note that this results is valid for particles with gyroradii smaller than the correlation length of the turbulence field. The associated parallel spatial mean free path L is given by $L \approx (1/3)\zeta^{-1} r_g^{(2-q)} \lambda_{\rm max}^{q-1}$. This allows us to find the systematic acceleration timescale contained in Eq.(1) due to stochastic particle-wave interactions, $t_{\rm acc} = \gamma^2/D(\gamma) \approx \beta_A^{-2} L/c$. The escape timescale due to particle diffusion form the system of parallel spatial scale Λ can be given $t_{\rm esc} = \Lambda^2/k_{||}$, where $k_{||} = cL/3$ is the parallel spatial diffusion coefficient.

3 Particle spectra in the turbulent field

Based on the quasi-linear approximation for the particle-wave interactions, we now investigate influence of the stochastic acceleration of ultra-relativistic electrons by a turbulent magnetic field for the evolution of the electron spectrum. Four types of diffusions are considered: Bohm type ($q = 1$), Kolmogorov type ($q = \frac{5}{3}$), Kreichnan type ($q = \frac{3}{2}$). In our calculations, the parameters are used as follows: minimum and maximum Lorentz factors of par-

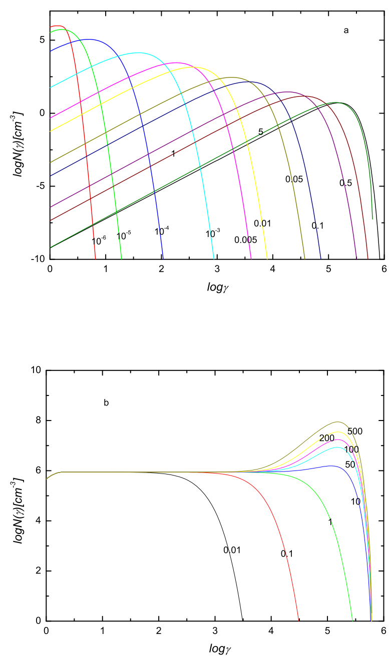

Figure 1: The evolution of the particles spectrum for the Bohm diffusion in the absence (left panel) and presence (right panel) of injection and escape of particles. The initial/injection spectrum with the particle energy $1 \le \gamma \le 2$. The stationary Maxwellian spectrum in left panel indicated by black line.

ticle are $\gamma_{\rm min} = 1$, $\gamma_{\rm max} = 10^6$, emission region size is $R = 10^{15}$ cm and light travel time is $t_{\rm cr} = R/c$. We considered that the magnetic is strongly turbulent with turbulence level $\zeta = \frac{\delta B^2}{B^2} = 1$. Since the acceleration rate $d\gamma/dt$ falls off as γ^{-1} for gyroradii larger than correlation length of the turbulence [21], we considered the longest wave being on the order of the size of the system, i.e. $\lambda_{\rm max} = 0.1R$. For simplification, we let the parallel spatial scale $\Lambda = R$ in our calculations. Below we consider three cases: steady state solution, time-dependent solution without escape and injection, and time-dependent solution without escape and injection.

In the first case, $\partial N(\gamma,t)/\partial t = 0$ in Eq. (2) and a general solution [22] is $N(\gamma) = x \exp[-\int_{\gamma_{\rm min}}^{\gamma_{\rm max}} \frac{C(\gamma)-A(\gamma)}{D(\gamma)} d\gamma]$, where x is a integration constant, which is estimated as $x = N_{\rm tot}/\exp[-\int_{\gamma_{\rm min}}^{\gamma_{\rm max}} \frac{C(\gamma)-A(\gamma)}{D(\gamma)} d\gamma]$, where $N_{\rm tot} = \int_{\gamma_{0\rm min}}^{\gamma_{0\rm max}} N_{\rm ini}(\gamma)d\gamma$ and $N_{\rm ini}(\gamma)$ is the initial electron distribution between in $\gamma_{0\rm min}$ and $\gamma_{0\rm max}$.

In our calculations, electrons are assumed to lose energy by synchrotron and IC scattering, where the synchrotron emission in a constant magnetic field with $U_{\rm B} \gg U_{\rm rad}$, the synchrotron self-absorption is not considered and IC cooling is assumed to occur in the Thomson regime. In

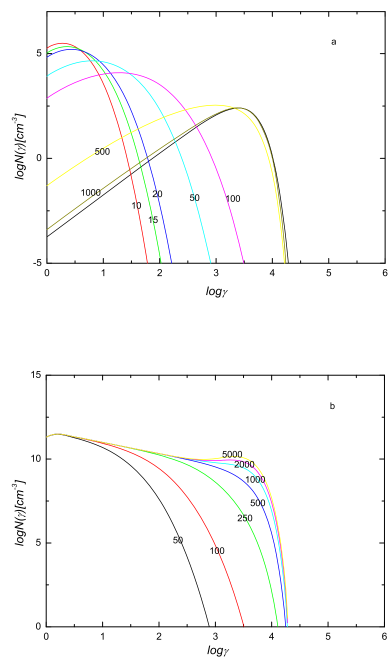

Figure 2: Same as Fig. 1 but for the Kolmogorov diffusion($q = 5/3$).

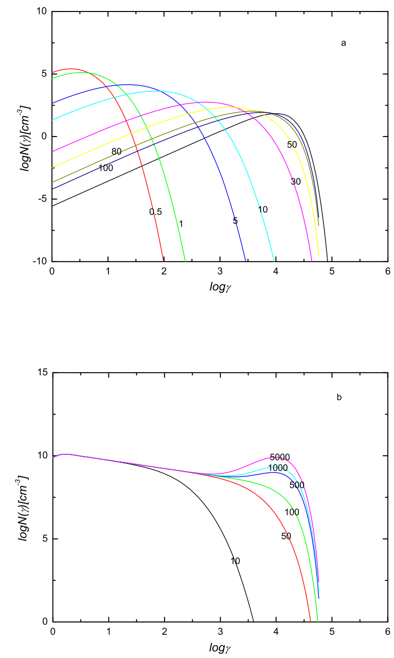

Figure 3: Same as Fig. 1 but for the Kreichnan diffusion ($q = 3/2$) .

this case, the stationary solution is given by $N(\gamma) = x\gamma^2 \exp[\frac{C_0(\gamma^{3-q}-1)}{D_0(q-3)}]$, where $C_0 = 4\sigma_T U_B/(3m_e c^2)$ and $D_0 = (1/3)\beta_A^2\zeta c[m_e c^2/(eB)]^{(q-2)}\lambda_{\max}^{(1-q)}$. It can be seen that the power law part of the function $N(\gamma)$ has the same slope and the shape of the exponential part depends on the cooling process and turbulent field properties. We expect this result to a quasi-Maxwellian distribution.

It should be noted that the scenario of the quasi-Maxwellian distribution of the particles was proposed for the arbitrary spectrum of the injected particles by Sauge & Henri (2004) [23], and was applied to model the activity of Mrk 501 in April 1997 [24]. In order to explain the multi-frequencies spectra of Mrk 421 and Mrk 501, a log-parabolic energy distribution, which is similar to a quasi-Maxwellian distribution, has been proposed [25][26], However, this log-parabolic distribution was obtained by assuming some specific acceleration process.

In the second case, we now numerically solve Eq. (2) by neglecting the particles escape and injection and analyze the initial spectrum evolution with a very narrow initial distribution of $1 \leq \gamma \leq 2$.

Finally, we consider the third case in which both particles escape and injection are included in Eq. (2). In this case, the escape in describes a diffusion of the particles into the region of the turbulent field where the magnetic field strength is significantly small and then the efficiency of the particle emission is also significantly less, the particles are assumed to be continuously injected at the lower energy($1 \leq \gamma \leq 2$), and systematically accelerated up to the equilibrium energy.

The calculated results in three cases are shown in Figs. 1-4.

For the particle spectra without escape and injection, the calculated results in the Bohm ($q = 1$), Kolmogorov ($q = \frac{5}{3}$), Kreichnan diffusion($q = \frac{3}{2}$), and hard-sphere approximation ($q = 2$) are shown in Figs.1(a), 2(a), 3(a), and 4(a), respectively. From these figure, there is a systematic decrease in the plasma density for the initial energy range and a simultaneous increase in the density around the equilibrium energy. In the Bohm diffusion, the plasma system fast evolves, and the acceleration will significantly decrease the density of the plasmas with the initial energy, accumulating a dominant part of the particles around the equilibrium energy and reaching the stationary distribution after the evolution time of $t_{\rm evo} \geq 5t_{\rm cr}$. In other cases of the Kolmogorov and the Kreichnan diffusions as well as the hard-sphere approximation, the plasma system reaches to the stationary distribution after $t_{\rm evo} \geq 1000t_{\rm cr}$, $t_{\rm evo} \geq 100t_{\rm cr}$, and $t_{\rm evo} \geq 80000t_{\rm cr}$, respectively. In the steady state, the spectrum is almost independent on the initial distribution, but dependent on the turbulence character.

For the particle spectra with escape and injection, the calculated results for four types of diffusions are shown in Fig. 1(b), 2(b), 3(b), 4(b), respectively. In this case, the

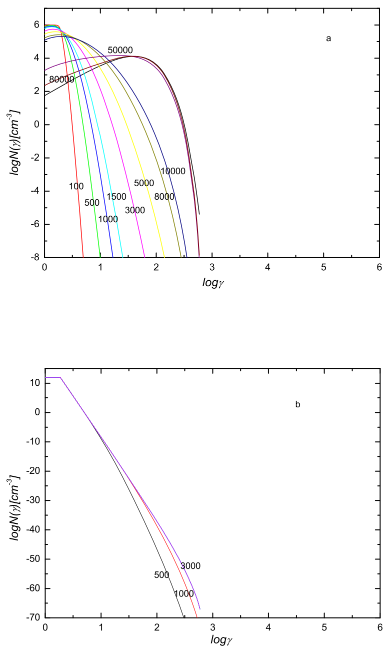

Figure 4: Same as Fig. 1 but for the hard-sphere approximation ($q = 2$).

competition between the acceleration and the escape produces a power law distribution that extends from the maximal energy of the injected particles up to equilibrium energy. Above the equilibrium energy, the parallel spatial diffusion process produces an exponential cutoff. The slope of the spectrum produced by the stochastic acceleration $n \simeq 1 + \frac{t_{\rm acc}}{2t_{\rm esc}}$. In our calculations, the characteristic acceleration time-scale and escape time-scale are free parameters, resulting in that the index depends on the initial assumption, and may give much more complex spectra in different turbulent energy spectrum.

4 Conclusion

Trough numerically solving the momentum diffusion equation, we studied the properties of stochastic acceleration of relativistic electrons due to particle-wave interaction in a turbulent magnetic field. In our calculations, it was assumed the turbulence power spectrum $W(k) \propto k^{-q}$ with $q = 1$, $q = 5/2$, $q = 3/2$, and $q = 2$ for the magnetic turbulence within some finite range of turbulent wave-vector k and the quasi-linear approximation for particle-wave interactions. From our calculations, We found that (1) the cooling and acceleration result to the plasm system reach to a quasi-Maxwellian distribution when the particles are confined to the turbulent acceleration region, i.e. without injection and escape; (2) the stationary state is almost independent of the initial distribution in the case of without injection and escape but dependent of the turbulence character, and (3) the injection and escape processes dominate the plasm system evolution of the spectrum.

Acknowledgments

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