Cosmic Ray Fronts ahead of SNR Shocks

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Abstract: The length of a CR-shock precursor L_p is believed to be set by the ratio of the CR mean free path λ to the shock speed, i.e., $L_p \sim c\lambda/V_{sh} \sim cr_g/V_{sh}$, which is independent of the CR pressure P_c. We argue that while the CR pressure builds up ahead of the shock, an acoustic instability, driven by the CR pressure gradient, results in precursor steepening into a strongly nonlinear front. Its size scales with *the CR pressure* as $L_f \sim L_p \cdot (L_s/L_p)^2 (P_c/P_g)^2$, where L_s is the scale of the developed acoustic turbulence, and P_c/P_g is the ratio of CR to gas pressure. Since $L_s \ll L_p$, the precursor scale reduction must be significant even for a moderate gas heating (reduction of the $(P_c/P_g)^2 \gg 1$ factor).

Keywords: acceleration of particles – cosmic rays – shock waves —supernova remnants — turbulence – nonlinear phenomena

1 Introduction

The supernova remnant (SNR) shocks are often observed as surprisingly thin filaments, particularly in the X-ray band. The emission is thought to be due to the super-TeV shock accelerated, synchrotron radiating electrons, occasionally also seen in gamma-rays (as inverse Compton up-scattered background photons). The gamma emission may in some cases be contaminated or even dominated by the accelerated protons via π^0 decay [1]. A convincing demonstration of namely the latter scenario would, of course, be a *prima facie* evidence for the acceleration of also the main, i.e. hadronic component of galactic CRs in SNRs. To improve our understanding of the SNR morphology would be an important step towards this goal.

The key element of the broadly accepted CR acceleration mechanism, the diffusive shock acceleration (DSA), is multiple crossing of the shock front with a $\sim V_{sh}/c$ energy gain after each crossing. In doing so particles diffusively escape from the shock, on average to a distance $L_p \sim \kappa(p)/V_{sh}$. Here κ is the momentum dependent diffusion coefficient and V_{sh} is the shock velocity. An obvious morphological consequence of this process should be and extended $\sim L_p$ shock precursor filled with synchrotron radiating electrons. However, X-ray observations provide no evidence for such extended precursors [2]. Note that L_p grows with particle momentum as almost certainly does $\kappa(p)$, so particles of higher energy should make thicker emission filaments than do the lower energy particles. This trend does not seem to be supported by the observations either [6].

Besides, the particle diffusion coefficient κ is believed to be close to its Bohm value, $\kappa \sim cr_g(p)/3$, which requires strong magnetic fluctuations $\delta B_k \sim B_0$ at the resonant scale $k \sim 1/r_g(p)$. The high level of fluctuations is achieved through one of the three instabilities driven by accelerated particles. The first one is the well known [19] ion cyclotron resonant instability of a slightly anisotropic (in pitch angle) CR distribution ahead of the shock [3]. The turbulence saturation level for this instability is moderate, $\delta B \sim B_0$.

The second instability, is a nonresonant instability driven by the CR current. As opposed to the previous instability, this one cannot be stabilized by the quasilinear deformation of the CR distribution function since in the upstream plasma frame the driving CR current persists, once the CR cloud is at rest in the shock frame. It also generates a broad spectrum of waves, and the longest ones were claimed to be stabilized only at the level $\delta B \gg B_0$, due to the lack of more efficient than the magnetic tension stabilization mechanisms at such scales [4]. Bell also pointed out that in the most interesting regime, the instability is driven by a fixed CR return current through the Ampere force $\mathbf{J}_c \times \mathbf{B}$.

The third instability is an acoustic instability driven by the pressure gradient of accelerated CRs upstream [11]. The pressure gradient is clearly a viable source of free energy for the instability. Curiously enough, the acoustic instability has received much less attention than the first two. Moreover, in many numerical studies of the CR shock acceleration, special care is taken to suppress it. The suppression is achieved by using the fact that a change of stability occurs at that point in the flow where $\partial \ln \kappa / \partial \ln \rho \simeq -1$ (for both stable and unstable wave propagation directions).

Here κ is the CR diffusion coefficient, and ρ is the gas density. Namely, one requires this condition to hold identically all across the shock precursor, i.e., where the CR pressure gradient $\nabla P_c \neq 0$. Not only is this requirement difficult to justify physically, but, more importantly, an *artificial* suppression of the instability eliminates its *genuine* macroscopic and microscopic consequences, as briefly discussed below.

Now the question is which instability dominates the CR dynamics? Given the finite precursor crossing time, it is reasonable to choose the fastest growing mode and consider the development of a slower one on the background created by the fast mode after its saturation. The resonant cyclotron instability is likely to dominate at the outskirt of the shock precursor where both the CR current and pressure gradient (driving the other two instabilities) are weak, whereas the pitch angle anisotropy is strong enough to drive the resonant instability. Recall that the anisotropy is typically inversely proportional to the local turbulence level which must decrease with the distance from the shock. In this paper, however, we focus on the main part of the shock precursor where both the CR-pressure gradient and CR current are strong, and consider the Bell and Drury instabilities as the strongest candidates to govern the shock structure. In fact, these instabilities are coupled, not only by the common energy source but also dynamically. The coupling of the magnetic and the density perturbations, akin to the modulational instability, will be the subject of a separate publication. This paper will be limited to a simpler situation in which one of the instabilities dominates. In particular we will identify conditions under which the acoustic instability grows more rapidly. Then, we determine the shock structure resulting from the nonlinear development of the acoustic instability.

2 Comparison of the growth rates

The derivations of the linear dispersion relations of the Bell and Drury modes can be found in refs.[11, 4]. For the magnetic field and density perturbations of the form, $\tilde{B} \propto \exp(-ikx \pm i\omega t)$ and $\tilde{\rho} \propto \exp(ikx \pm i\omega t)$, in an approximate symmetric representation, in which the diffusive CR damping of acoustic mode and the resonant CR contribution to the nonresonant magnetic instability are neglected, these relations can be written as follows

$$\omega^2 = k^2 C_A^2 - 2\gamma_B k C_A \tag{1}$$

$$\omega^2 = k^2 C_s^2 - 2i\gamma_D k C_S \tag{2}$$

Here C_A and C_S are the Alfven and the sound speeds, respectively while

$$\gamma_B = \sqrt{\frac{\pi}{\rho_0}} J_c/c \ \text{ and } \gamma_D = -\frac{1}{2\rho_0 C_S}\frac{\partial \overline{P}_c}{\partial x} \tag{3}$$

The last expressions approximately represent the maximum growth rates of the nonresonant B-field (Bell) perturbations and acoustic density (Drury) perturbations, driven by the CR return current and by the CR pressure gradient, respectively. Note that the instabilities are different in that the Bell mode is unstable in the limited k-band where $0 < k < 2\gamma_B/C_A$, and γ_B is only the maximum growth rate that is obviously achieved at $k = \gamma_B/C_A$, whereas the acoustic mode saturates with k at $k \gtrsim \gamma_D/C_S$ at the level γ_D.

Next, we compare the growth rates of the two instabilities. The comparison should be done for the same equilibrium distribution of CRs upstream of the subshock, i.e., in the CR precursor. Apparently, there is a difficulty here. The acoustic instability analysis presumes a certain level of magnetic fluctuations (e.g. $\delta B \sim B$) to pondermotively couple CRs to the gas flow, and thus ensure the equilibrium. In the Bell's stability analysis, it is assumed that the undisturbed B-field is along the shock normal and $\mathbf{J_{CR}}$ points into the same direction. Then, the CR pressure gradient cannot be statically compensated since $\mathbf{J_{CR}} \times \mathbf{B} = 0$. Implicitly though, one may assume that the resonant instability provides Alfven waves which balance the CR pressure gradient along the main field through the wave pondermotive pressure at larger scales. Then, the Bell instability develops at the scales much shorter than the gyro-radii of the current-carrying particles. Note that such treatment cannot be extended to the larger scales without further considerations [9, 5]. In a strongly modified shock, the equilibrium distribution upstream in a steady state is given by [14]:

$$f = f_0(p) \exp\left[\frac{q(p)}{3\kappa(p)}\phi(x)\right], \ x \geq 0 \tag{4}$$

where ϕ is the flow potential, $u = \partial\phi/\partial x < 0$, $f_0(p)$ is the CR distribution and $q(p) = -\partial \ln f_0/\partial \ln p$ the spectral index at the subshock. On the other hand, if the shock modification is negligible, the equilibrium is simply

$$f = f_0(p) \exp\left[\frac{1}{\kappa(p)}\phi(x)\right], \ x \geq 0 \tag{5}$$

One sees that the only difference between the last two representations of the particle distribution upstream is the $q/3$ factor in the exponent of eq.(4). This quantity is well constrained in the nonlinear solution given by eq.(4): $3.5 < q(p) < q_{sub}$, where $q_{sub} = 3r_{sub}/(r_{sub}-1)$ with r_{sub} being the subshock compression ratio. For the ratio of acoustic to magnetic growth rates, eq.(3), we obtain

$$\frac{\gamma_D}{\gamma_B} = \frac{C_A}{C_S}\frac{c^2}{3\omega_{ci}}\left\langle \frac{p^2}{\sqrt{1+p^2}}\frac{q}{3\kappa}\right\rangle \tag{6}$$

where p is in units of mc. We have introduced the spectrum averaged quantity as follows

$$\langle\cdot\rangle \equiv \frac{\int(\cdot) f_0(p) p^2 \exp(q\phi/3\kappa)\, dp}{\int f_0(p) p^2 \exp(q\phi/3\kappa)\, dp} \tag{7}$$

For the test particle solution, given by eq.(5), one should replace $q/3 \to 1$. For the Bohm diffusion coefficient $\kappa = r_g(p)c/3$, we obtain

$$\frac{\gamma_D}{\gamma_B} = \left\langle \frac{q}{3} \right\rangle \frac{C_A}{C_S} \quad (8)$$

From the last formula we may conclude that the acoustic instability dominates the magnetic one in the case of low $\beta \equiv C_S^2/C_A^2 \ll 1$ upstream, regardless of the degree of nonlinearity of acceleration. The reason for such a counter-intuitive relation between the growth rates of these two instabilities is that a stronger magnetic field (low-beta plasma) supports a stronger CR pressure gradient which drives the acoustic instability. Note, that plasma heating upstream would increase the role of magnetic instability unless the large scale magnetic field is also strengthened, e.g. through an inverse cascade of the turbulent magnetic energy [9]. On the other hand, the development of the acoustic instability makes (Sec.4) the precursor shorter. This boosts the gradient driven acoustic instability and leaves less room for the current driven magnetic instability. Next, we consider the CR transport in a developed acoustic turbulence with an admixture of Alfven waves, presumably generated by cyclotron instability at a distant part of the shock precursor and convected into its core.

3 CR transport in Shock Precursor

We assume that magnetic perturbations in the precursor are of the following two types. First, there are conventional shear Alfven waves, stemming from the CR cyclotron instability. Second, there are compressible magnetosonic perturbations generated by the Drury instability. The pitch angle scattering of CRs in the both wave fields has been calculated in many publications. We can use the expressions given by eqs.(6,7) in ref.[7]. After summing the Bessel function series and retaining only the magnetic parts of the scattering wave fields, for the pitch-angle diffusion coefficients we obtain

$$\begin{aligned} D_\mu^A &= -\left(1-\mu^2\right)\sum_{\mathbf{k}}\frac{1}{\xi^2}\int_0^\infty I^A\left(k_\parallel,k_\perp,\tau\right) \\ &\times \cos\left(k_\parallel c\mu\tau\right)d\tau\frac{\partial^2}{\partial\tau^2}J_0\left(2\xi\sin\frac{\Omega\tau}{2}\right) \end{aligned} \quad (9)$$

$$\begin{aligned} D_\mu^S &= \frac{1}{3}\left(1-\mu^2\right)\sum_{\mathbf{k}}\frac{1}{\xi^2}\int_0^\infty I^S\left(k_\parallel,k_\perp,\tau\right)\frac{k_\parallel^2}{k_\perp^2}d\tau \\ &\times \left[\Omega^2\cos\left(k_\parallel c\mu\tau\right)-\xi^{-2}\frac{\partial^2}{\partial\tau^2}\right]J_0\left(2\xi\sin\frac{\Omega\tau}{2}\right) \end{aligned} \quad (10)$$

Here I^A and I^S are the normalized (to $B_0^2/8\pi$) spectral densities of magnetic fluctuations of Alfven and slow magnetosonic wave components of the upstream turbulence, μ is the cosine of the particle pitch angle, $\xi = k_\perp c\sqrt{1-\mu^2}/\Omega$, J_0 is the Bessel function, and Ω is the relativistic gyrofrequency. The spatial diffusion coefficient can be evaluated as follows (e.g.[13])

$$\kappa = \frac{c^2}{8}\left\langle \frac{1-\mu^2}{\mathscr{D}_\mu} \right\rangle \quad (11)$$

where $\mathscr{D}_\mu = \left(D_\mu^A + D_\mu^S\right)/\left(1-\mu^2\right)$ and $\langle\cdot\rangle$ denotes here the pitch angle averaging. For the turbulence spectra extended in $k_\perp$ (such as the Goldreich-Shridhar cascade [12]), the scattering frequency $\mathscr{D}_\mu$ peaks at $|\mu| = 0,1$ [7]. Indeed, the scattering is known to be strongly suppressed for $\xi \gg 1$ (high frequency perturbation of particle orbits), so that particles with $|\mu| \approx 1$ are subjected to a more coherent (not so rapidly oscillating) wave field. Furthermore, particles with $\mu \approx 0$ are effectively mirrored by the compressible component of magnetic turbulence. Once there are peaks at $|\mu| = 0,1$, $\mathscr{D}_\mu$ should also have minima in between, i.e. at some $|\mu| = \mu_0$, where $0 < \mu_0 < 1$. Upon writing

$$\left\langle \frac{1-\mu^2}{\mathscr{D}_\mu} \right\rangle = \int_0^1 \left(1-\mu^2\right)d\mu\int_0^\infty \exp\left(-\mathscr{D}_\mu t\right)dt$$

and evaluating the integral in μ by using the steepest descent method, we obtain

$$\kappa = \frac{2\pi c^2\left(1-\mu_0^2\right)}{\sqrt{\mathscr{D}_\mu\left(\mu_0\right)\mathscr{D}_\mu''\left(\mu_0\right)}}$$

where the double prime denotes the second derivative. Assuming that the compressible part of the turbulence, which originates from the acoustic instability, dominates ($D^A \ll D^S$), we can represent the momentum averaged $\bar{\kappa}$ as

$$\bar{\kappa} = \frac{\int \kappa(p) f(p) p^2 dp}{\int f(p) p^2 dp} = \frac{\bar{\kappa}_B}{F_S + \alpha} \quad (12)$$

Here $\bar{\kappa}_B$ is the momentum averaged Bohm diffusion coefficient $\kappa_B = cr_g(p)/3$, α is the normalized level of Alfvenic turbulence $\alpha \sim (\delta B_A)^2/B_0^2$, (originating from D^A), and F_S is the level of compressible turbulence (originating from D^S). The latter can be calculated in the simplest 1D model in which the acoustic waves unstably grow at a rate γ_D and then steepen into an ensemble of shocks (shocktrain) [15, 17]

$$F_S = \sigma\left(\frac{\partial P_c}{\partial x}\right)^2 \quad \text{with} \quad \sigma = \left(\frac{L_s}{\rho C_S^2}\right)^2 \mathscr{F}\left(\vartheta_{nB}\right) \quad (13)$$

Here L_s is the average distance between shocks in the ensemble, and $\mathscr{F} \sim 1$ depends predominantly on the shock obliquity and on the specific model for the shocktrain formation [18].

4 Turbulent CR Front Solution

In this section we derive the equations for the shock transition from the kinetic convection-diffusion equation. Using the expressions (12) and (13) for the momentum averaged diffusion coefficient, we can write the convection-diffusion equation in a steady state as follows

$$\frac{\partial}{\partial x}\left(ug+\frac{\bar{\kappa}_B}{\sigma\left(\partial P_c/\partial x\right)^2+\alpha}\frac{\partial g}{\partial x}\right)=\frac{1}{3}\frac{\partial u}{\partial x}p\frac{\partial g}{\partial p} \qquad (14)$$

where $g=fp^3$ and u is the (positive) magnitude of the flow speed. We assume that the CR pressure integral is dominated by particles with $p\gg mc$, but there is a spectral break at some $p=p_*$ which makes the integral finite without introducing a cut-off momentum [15]. This makes the well known two-fluid model [10] suitable for our purposes. Integrating the appropriately weighted convection-diffusion equation (14) in momentum, and supplementing the result with the conservation of the mass and momentum fluxes across the shock transition, we arrive at the following closed system of equations that governs the shock transition

$$\frac{\partial}{\partial x}\left(uP_c+\frac{\bar{\kappa}_B}{\sigma\left(\partial P_c/\partial x\right)^2+\alpha}\frac{\partial P_c}{\partial x}\right)=-\frac{1}{3}\frac{\partial u}{\partial x}P_c$$

$$\rho u=\rho_1 u_1=const$$

$$\rho u^2+P_c=\rho_1 u_1^2=const \qquad (15)$$

Here P_c is the CR pressure, ρ_1 and $u_1=V_{sh}$ are the far upstream density and flow speed. From these equation we obtain the following equation for the shock front structure

$$\left(\frac{\partial P_c}{\partial x}\right)^2+\frac{L_B}{\sigma P_c\left(1-P_c/P_{c2}\right)}\frac{\partial P_c}{\partial x}+\frac{\alpha}{\sigma}=0$$

where $P_{c2}=(6/7)\,\rho_1 u_1^2$, $L_B=\bar{\kappa}_B/u_1$ and we assumed that thermal gas is not heated appreciably, so its pressure can be neglected in eq.(15) [14]. The shock solution can be obtained from the last equation in a closed form as $x=x\left(P_c\right)$. Introducing a dimensionless coordinate $z=xL_B/\sigma P_{c2}^2$ and $\Phi=2P_c/P_{c2}-1$ we can rewrite the last equation describing the front transition as

$$\left(\frac{\partial \Phi}{\partial z}\right)^2+\frac{8}{1-\Phi^2}\frac{\partial \Phi}{\partial z}+4a=0$$

where the transition is governed by a single parameter $a=\alpha\sigma\left(P_{c2}/L_B\right)^2$. Obviously, a smooth transition exists only for $0<a<4$. If $a\ll 1$ $\Phi=-\tanh\left(az/2\right)$, so that the scale of the front is the familiar $L_B=\bar{\kappa}_B/u_1$. If, however, $a\sim 1$, the scale of the shock (front) transition reduces to

$$L_f=\frac{\sigma}{4L_B}P_{c2}^2\simeq\frac{1}{4}\frac{L_S^2}{L_B}\left(\frac{P_{c2}}{\rho C_s^2}\right)^2.$$

5 Conclusions

We have obtained a one parameter family of smooth, strongly nonlinear shock front transitions which are typically significantly shorter than the conventional ($\sim\kappa/V_{sh}$) shock precursors. The phenomenon in general is somewhat similar to the transport bifurcation phenomenon, which is a subject of active research in magnetic fusion (L-H bifurcations, [8, 16]).

The critical parameter that governs such transitions (parameter a, Sec.4) depends (through the growth rate of the front supporting acoustic turbulence, γ_D) on the thermal gas pressure inside the front, de facto on the turbulent heating efficiency. The smooth transition exists for $a<4$, so that in the case $a>4$, a gaseous subshock must form. This however, should not result in a longer precursor. The determination of the heating efficiency, and thus a more complete study of the shock fronts obtained in this paper will be the subject of a separate publication.

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