UHECR Acceleration around Filaments of Cosmological Structure Formation

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Abstract: A mechanism for proton acceleration to \( \sim 10^{21} \) eV is suggested. It may operate in accretion flows onto thin dark matter filaments of cosmic structure formation. The flow compresses the ambient magnetic field to strongly increase and align it with the filament. Particles begin the acceleration by the \( \mathbf{E} \times \mathbf{B} \) drift with the accretion flow. The energy gain in the drift regime is limited by the conservation of the adiabatic invariant \( p^2_z / B(r) \). Upon approaching the filament, the drift turns into the gyro-motion around the filament so that the particle moves parallel to the azimuthal electric field. In this ‘betatron’ regime the acceleration speeds up to rapidly reach the electrodynamic limit \( c p_{\text{max}} = e B R \) for an accelerator with magnetic field \( B \) and the orbit radius \( R \) (Larmor radius). The periodic orbit becomes unstable and the particle slings out of the filament to the region of a weak (uncompressed) magnetic field, which terminates the acceleration.

To escape the filament, accelerated particles must have gyro-radii comparable with the filament radius. Therefore, the mechanism requires pre-acceleration that is likely to occur in structure formation shocks upstream or nearby the filament accretion flow. Previous studies identify such shocks as efficient proton accelerators to a firm upper limit \( \sim 10^{19.5} \) eV placed by the catastrophic photo-pion losses. The present mechanism combines explosive energy gain in its final (betatron) phase with prompt particle release from the region of strong magnetic field. It is this combination that allows protons to overcome both the photo-pion and the synchrotron-Compton losses and therefore attain energy \( \sim 10^{21} \) eV.

Keywords: ultra high energy cosmic rays, particle acceleration, accretion, astrophysical fluid dynamics

Introduction

To identify the UHECR sources it is necessary to test the putative extragalactic accelerators for their capability to accelerate protons beyond \( 10^{20} \) eV. Possible sites of the UHECR acceleration are the cosmic structure formation shocks. Remarkably, the diffusive shock acceleration mechanism falls short by one order of magnitude to produce particles in such shocks with the highest energy observed, i.e. a few \( 10^{20} \) eV [4, 1].

Below we demonstrate that protons accelerated in the structure formation shocks to \( \sim 10^{19.5} \) eV can be boosted to \( 10^{21} \) eV inside the same accretion flow. The suggested mechanism accelerates particles much faster than the DSA, thus sustaining against losses. It operates in plasmas accreting on to the gravitating dark matter (DM) filaments. Filaments, along with pancakes and knots are important elements of the cosmic structure formation which was established in a number of simulations (e.g., [5]).

Accretion flow

According to the the ΛCDM simulations, the gravitationally interacting dark matter (DM) particles aggregate to form a structure which then gravitationally drives conducting gas with the frozen in magnetic field. The matter accrets onto sheets, filaments and nodes and is thus organized in a “cosmic web” of massive nodes connected by filaments along which the matter flows towards nearby nodes. The rest of the space can be considered as low density, low magnetic field “voids”, e.g. [5].

Turning to the particle acceleration in such structures, we focus on a single filament with two nodes at its ends (a ‘dumbbell’), Fig. 1. Strong flow compression near the knots creates magnetic mirrors that confine energetic particles in the field-filament direction. A rarefied plasma accrets onto the filament from the surrounding void and stream then towards the nodes, while partially the plasma accrets onto the nodes directly from the void. It is not unreasonable to assume that, at least in some cases, the field is well aligned with the filament [3].

Particle acceleration in filaments

Consider a DM filament of radius \( R_f \) that accrets intergalactic gas in radial direction. We specify the magnetic field \( \mathbf{B} = (0, 0, -B) \) with \( B(r) \) depending only on \( r = \sqrt{x^2 + y^2} \), the distance to the filament axis (z-axis), while particle motion in z-direction is constrained by magnetic mirrors near the filament end nodes, so that the dynamics of accelerated particles is nearly perpendicular to \( \mathbf{B} \), i.e. \( p_z \ll p_r \approx p \). The equations of motion in the polar coordinates \((r, \theta)\) on the \((x, y)\) plane read...
where \( p_r \) and \( p_\theta \) are the radial and azimuthal components of the particle momentum, \( p = \sqrt{p_r^2 + p_\theta^2} \), \( r \) is the particle radial coordinate, \( \theta \) is the azimuthal angle. We scale \( B \) to its value at infinity, \( B_\infty = \text{const} \), both the radial coordinate \( r \) and the particle gyro-radius \( r_g(p) = pc/eB_\infty \) to \( R_B \) (which is the Bondi radius, \( R_B = \left(\gamma - 1\right)GM/c^2 \) with \( C_\infty \) being the speed of sound at the infinity), and time, to \( R_B/c \). Thus, the particle momentum \( p \) is measured in the units of \( eB_\infty R_B/c \). Since the azimuthal electric field \( E_\theta = -u_r(r)B(r)/c \), where \( u_r < 0 \) is the radial flow velocity, and since \( ru_rB = \text{const} \), the motion electric field \( E_\theta \propto 1/r \). Therefore, we introduced the following parameter \( v = -ru_rB/R_BcB_\infty < 0 \), that controls both the drift of energetic particles towards the filament and their acceleration. Note that due to the azimuthal symmetry, the angular variable \( \theta \) is ignorable. It is, however, useful in that it traces the particle energy. Indeed, by virtue of eqs. (1-2) \( p - v\theta = \text{const} \). Furthermore, the azimuthal component of the particle canonical momentum \( \mathcal{P}(t) = \Psi - rp_\theta \) decreases linearly with time; \( \mathcal{P} + vt = \text{const} \), where \( \Psi(r) \) is defined as follows \( \Psi = \int_0^r \rho B r dr \).

It is convenient to reduce the dynamical system given by eqs. (1-3) to the following 1D Hamiltonian system

\[
\begin{align*}
\dot{p}_r & = -p_\theta/p(B-p_\theta/r) \quad (1) \\
\dot{p}_\theta & = p_r/p(B-p_\theta/r)+v/r \quad (2) \\
\dot{r} & = p_r/p \quad (3) \\
\dot{\theta} & = p_\theta/rp \quad (4)
\end{align*}
\]

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where the particle momentum \( p \) assumes the role of the Hamiltonian

\[
p(p_r,r,t) = \sqrt{p_r^2 + \left(\Psi - \mathcal{P}\right)^2/r^2} \quad (7)
\]

where \( \mathcal{P}(p_r,r,t) = \mathcal{P}_0 - vt \). If \( v = 0 \), the relation \( p = \text{const} \) provides a complete solution of the problem, at least in the form \( t = t(r) \), seeing that \( p_r = p \dot{r} \). We assume that \( v \ll 1 \), i.e. the plasma gravitational infall is slow compared to the speed of light. Therefore, we may consider both the change in the particle “total energy” \( p^2 \) and the deformation of the “potential energy” of the Hamiltonian in eq.(7) \( U(r,t) = (\Psi - \mathcal{P})^2/r^2 \) being adiabatic.

Given the above considerations, the particle dynamics is easily understood in terms of the critical points (i.e., points where \( dU/dr = p_r = 0 \)) of the l.h.s. of eq.(5-6). One such point is where \( \Psi(r) = \mathcal{P} \), i.e. the coordinate of the particle guiding center \( r_d \), given by the following relation \( \Psi(r_d) = \mathcal{P}_0 - vt \). Apart from drifting with the velocity \( \dot{r}_d = -v/r_d B(r_d) \), particles oscillate in a potential well near its minimum, Fig.2. The adiabatic invariant can be calculated from eqs.(5-7)

\[
I = \oint pddr = \oint \sqrt{p^2 - U(r,t)}dr \simeq \pi p^2/B(r_d), \quad (8)
\]

The last relation in eq.(8) is valid in the guiding center approximation. Since \( \Psi \geq 0 \) for all \( r \), the solution for \( r_d \)
cesses to exist for \( t > \mathcal{P}_0/v \). Formally this means that the particle guiding center reaches the origin, but the particle itself does not. The full treatment of the dynamics (including the particle phase) is required in this case. In particular, before the minimum of \( U(r) \) at \( r = r_d \) disappears, another minimum of \( U \) emerges at large \( r = r_{\text{esc}} \gg r_d \), where \( r_{\text{esc}} \) stands for an 'escape' radius; the particle makes a long excursion from the filament when it moves around this minimum of \( U(r) \). In fact, it rotates in a weak magnetic field and has therefore a good chance to escape into the IGM. Naturally, a local maximum at \( r = r_s \) appears between the two minima and a separatrix, which crosses the point \((r_s,0)\), forms on the phase plane \((r,p_r)\), Fig.3. These two additional critical points emerge at \( t = t_s \), where the critical time \( t_s \) and the radius \( r_s \) are determined by the relations: \( \Psi''(r_s) = 0; \mathcal{P}(t_s) = \Psi(r_s) - r_s\Psi'(r_s) \). Starting from this moment, a particle, while still oscillating and climbing to higher energies in the left (narrow) potential well (see Fig.2) can also exercise finite motion in the shallow right potential well, provided that particle energy is low enough. The moment \( \mathcal{P}(t_s) \approx \mathcal{P}_0 - v_s \). These are two equations for the three unknowns \((p_r,r_s,\mathcal{P}_s)\) and the conservation of adiabatic invariant may be used as the third equation. However, when the orbit particle approaches the separatrix, the simplified drift theory approximation in eq.(8) becomes inaccurate. The integral representation of the adiabatic invariant is required for this final phase [3]. Note that this is the most efficient phase of the acceleration mechanism. What happens is that particles, while dwelling progressively longer at the hyperbolic point during their oscillations in the potential well, virtually circulate around the origin, being thus in a 'betautron' acceleration regime. The energy gain is very fast (explosive) at this stage, since the electric field of the accreting plasma is almost collinear to the particle velocity. At the same time the orbit radius decreases since the magnetic field increases fast enough along the particle orbit.

**Particle maximum energy** We specify the magnetic field profile \( B(r) \) as follows \( B(r) = r^{-\nu} + 1 \), with \( \nu = 1/(\gamma - 1) \), where \( \gamma \) is the adiabatic index (note that \( B(r) \) the density \( \rho(r) \) should behave similarly). To simplify the calculation of the adiabatic invariant \( I \) in eq.(8), we set the gas adiabatic index \( \gamma = 5/3 \), i.e. \( \nu = 3/2 \). For now, it is sufficient to evaluate the adiabatic invariant at the moment of separatrix crossing, Fig.2, that is for \( r_s \approx 1/(\nu^2_0) \), \( \mathcal{P}_s = 1/p_s \), and \( r_{\text{max}} = (\sqrt{2} - 1)^2 / p_s^2 \). From the conservation of adiabatic invariant we thus have \( I_0 = \pi p_0^2 / B_0 = I_s \approx 0.771/p_s \), where \( p_0 \) and \( B_0 \) are the particle momentum and the magnitude of the magnetic field at a certain point in the flow far away from the filament (where the drift approximation still applies).

From the last equation we obtain the following interesting (inverse-square) relation between the initial and the final (separatrix value) particle momentum \( p_s \approx 0.25 B_0/p_0^2 \). Since \( p_s \approx 1/\sqrt{r_i} \), the maximum momentum \( p_s \) is limited by the condition \( r_s \gtrsim r_i \), where \( r_i \approx 1 \) is the radius below which the flow changes its direction from radial (towards filament) to axial (towards node). Particles with sufficiently small gyro-radii are constricted along the z-axis out of the acceleration zone. This constitutes the re-acceleration character of the process and constrains the initial particle momentum \( p_0 \). A particle must enter the acceleration with the momentum \( p_0 > \sqrt{0.25 B_0 r_i^{-1/4}} \). This is a significant but not the prohibiting constraint on the initial particle momentum. The final particle momentum \( p_{\text{max}} \) is limited by the condition \( p_{\text{max}} \leq \min \left(r_i^{-1/2}, p_s \right) \). If \( p_s > r_i^{-1/2} \), the particle cannot be released from the accelerator and sinks into the filament.

As a proxy for \( r_i \), an estimate of magnetic field and/or density compression between the flow outside of the accretion radius and the filament axis can be used. So, \( r_i \sim 10^7 \) increase in \( B \) from the nanogas IGM field to the \( \mu \)G intracluster field appears to be reasonable. With the \( r^{-3/2} \) scaling of the magnetic field adopted above, this translates into \( r_s \sim 10^{-7} \), yielding, in turn, \( p_{\text{max}} \sim 10 \) for \( p_0^2 \sim 1/10 \). Therefore, if a particle enters the acceleration at a few \( 10^{12} \) eV it may reach \( 10^{21} \) eV by the moment of ejection. Note that \( r_s \) is then of the order of 1 Mpc (according to simple estimates from the three orders of magnitude density and magnetic field compression in the accreting flow) while the initial particle gyro-radius should be of the order of 10Mpc.
The final stage of the acceleration is the most important since the energy losses grow with the particle energy. The time dependence of the particle momentum short before the separatrix crossing can be written as \( p(t) = (\mathcal{P}_0 - vt)^{-1} \). The acceleration time reduces towards the end of acceleration \( \tau_a = p / \dot{p} = 1 / v_p \), which is in sharp contrast with the DSA, where the acceleration time grows linearly with \( p \) (at least for Bohm diffusion).

Energy losses Often, it is not the maximum energy of an accelerator which precludes the proton production with \( E \gtrsim 10^{20} \text{eV} \) but the losses (caused by strong photon and/or magnetic fields, surrounding the acceleration region) [4, 1]. Fortunately, the present acceleration mechanism rapidly speeds up towards the maximum energy, \( p = v_p^2 \), that is reached when a particle crosses the separatrix at \( p_{max} = p_s \). It is extremely beneficial for the UHECR production to terminate the acceleration process in such abrupt way; the synchrotron losses also drop abruptly as the particle is released into a void, where only a weak magnetic field is present.

More important than the Synchrotron-Compton losses are the photo-pion losses on the CMB [3], which is essentially a threshold process [4]. The photo-pion losses also strongly dominate the pair production losses above a few \( 10^{19} \text{eV} \), so we may ignore the latter in the energy range of our interest. In particular, for energies \( E \lesssim 3 \cdot 10^{20} \text{eV} \) the following simplified representation of the loss term can be used \( -E / \mathcal{E} \approx (c / l_E) \exp(-E_{th} / E) \), with \( E_{th} = m_p m_{\pi} c^4 (1 + m_\pi / m_p) / 2 kT \approx 3 \cdot 10^{20} \text{eV} \) and \( l_E \approx 10 \text{Mpc} \). Here \( m_\pi \) is the pion’s rest mass and \( T \) is the 2.7K CMB radiation temperature. For higher energies \( E \gg E_{th} \) a slightly lower value of \( c / l_E \approx 1.8 \cdot 10^{-8} \text{yr}^{-1} \) may be adopted but, this energy range can hardly be reached by this acceleration mechanism.

The dimensionless acceleration rate at the final stage of acceleration can be written as \( p^{-1} \dot{p} \sim v_p \). Combining this with the photo-pion losses and using the dimensionless variables, we obtain \( p / p = v_p - (R_B / l_E) \exp(-p_{th} / p) \), where \( p_{th} \) is the dimensionless photo-pion threshold momentum \( E_{th} / c \). The right hand side of this equation may either have two roots or none. Recalling that the particle gyroradius is normalized to \( R_B \), we may write the condition for the latter case \( (p / p > 0 \) for all \( p \) \) as follows \( v r_{\pi} (p_{th}) / R_B > e^{-1} \approx 0.37 \), where \( r_{\pi} (p_{th}) \) is the gyroradius of a proton with the momentum \( p \approx p_{th} \) in the \( B_0 \) magnetic field. Using an estimate \( v \approx 4C_{e\omega} / c \), the last condition rewrites \( C_{e\omega} / c > 3 \cdot 10^{-3} (R_B / 10 \text{Mpc})^2 (B_0 / \text{nG}) \). Most likely this inequality is marginally violated, so that the two roots of the expression for the energy losses do exist and significant photo-pion losses occur between these energies.

Overall, due to the fast energy gain in the betatron acceleration phase, the energy losses that are fatal for the DSA may be overcome. We therefore conclude that the maximum energy is likely to be determined by intrinsic limitations of the acceleration mechanism and not by the energy losses.

The intrinsic acceleration limit is set by either the separatrix crossing or by reaching the flow deflection inside the filament, whichever occurs first.

**Spectrum** The inverse-square relation between the initial and final particle momentum, suggests flipping the injected spectrum with respect to the fixed point \( p = (0.25B_0)^{1/3} \) of the map \( \rho_0 \leftrightarrow \rho_s \). If the injection spectrum has the form \( f_{inj} \propto p^{-q} \), and it should be taken in the interval \( 0.5 \sqrt{B_0 r_s}^{1/4} \approx p < (0.25B_0)^{1/3} \), then the accelerated particle spectrum will cover the interval \( (0.25B_0)^{1/3} \approx p < r_s^{-1/3} = p_{max} \), with an index \( q' = q / 2 \). However, as the particle momentum approaches \( p_{max} \), its orbit crosses and recrosses the circle of the radius \( r_s \), and the odds for particle convection with the flow towards one of the nodes increase. Moreover, the boundary between the radial and axial accretion at \( r = r_s \) is not sharp, so, one should not expect a sharp spectrum cut-off at \( p_{max} \) but rather its decline starting at lower momenta.

**References**