Spectra of positrons and electrons and positron to electron ratio in the Galaxy

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Abstract: We report the results of new calculations of energy spectra of electrons and positrons and positron to electron ratio in the Galaxy. Fractional diffusion model was implemented to describe the particles propagation from sources. It is shown that a self-consistent description of the experimental data can be obtained if we assume that both positrons and electrons are injected into the interstellar medium by the sources with the same spectral exponent \( p \approx 2.85 \). We have also found that the positron to electron ratio increases to a constant value of \( \sim 0.22 \) for energies \( E > 100 \) GeV. This flattening of positron fraction obtained in our model can be examined in the near future by the PAMELA and the AMS-02 experiments.

Keywords: Energy spectrum, injection spectrum, fractional diffusion model, stable law, cosmic rays sources, positron to electron ratio.

Introduction

Results of the long-term terrestrial, balloon-borne and satellite experiments allowed to formulate the scenario of origin, acceleration and propagation of the cosmic rays. Its basic positions for the electron-positron component consist of the following.

1. The supernova remnants (SNR) are the sources of the primary electrons in the Galaxy; acceleration occurs on the shock front; the injection spectrum of the particles \( S(E) \sim E^{-p} \) with \( p \approx 2.4 \pm 2.5 \) [1].
2. Distribution of sources is stationary.
3. Propagation of the particles in the interstellar medium is described by the Ginzburg-Syrovatskii normal diffusion equation [1, 2]

\[
\frac{\partial N(\vec{r}, t, E)}{\partial t} = D(E)\Delta N(\vec{r}, t, E) + \frac{\partial (B(E)N(\vec{r}, t, E))}{\partial E} + S(\vec{r}, t, E);
\]

diffusivity \( D(E) = D_0 E^\delta, \delta \sim 0.3 \div 0.6, N \equiv \partial n/\partial E. \)
4. Positrons are the products of nuclear interactions of the primary cosmic rays nuclei with the interstellar medium; the positron fraction decreases as the energy increases.

However, the experimental results obtained in the last decade using new generation of the instruments (ATIC [3], PPB-BETS [4], PAMELA [5], H.E.S.S. [6, 7], Fermi-LAT [8]) contradict to the standard scenario predictions. For example, it was established that:

- the H.E.S.S. atmospheric Cherenkov telescope revealed a significant steepening of the electron plus diffuse photon spectrum above 600 GeV [6, 7] which is compatible with prominent spectral feature in the total electron spectra of ATIC and PPB-BETS [3, 4];
- the PAMELA satellite experiment [5] found that the positron fraction \( e^+/(e^+ + e^-) \) changes slope at around 10 GeV and begins to increase steadily up to \( E \sim 100 \) GeV. A similar trend was also indicated by earlier experiments HEAT [9] and AMS-01 [10].

On the basis of identified contradictions we offer the possible directions of an expansion for the standard scenario.

1. Replacement of the stationary sources model with the non-stationary model.
2. Inclusion of the primary positrons sources.
3. Taking into account of the nonlocal character of the particles propagation throughout turbulent (fractal-like) medium.

The main goals of this paper are calculation of the energy spectrum of the cosmic ray electrons and positrons.

Vol. 6, 267
in the fractal-like galactic medium under different assumptions about sources as well as establishment of conditions for which the theoretical results give the self-consistent description of the modern experimental data.

1 Fractional diffusion model

With account of the energy losses the equation for the concentration of particles of energy $E$, generated in a fractal medium by galactic sources with a distribution density $S(\vec{r}, t, E)$, has the form [11]

$$\frac{\partial N(\vec{r}, t, E)}{\partial t} = -D(E, \alpha)(-\Delta)^{\alpha/2}N(\vec{r}, t, E) + \frac{\partial (b(E)N(\vec{r}, t, E))}{\partial E} + S(\vec{r}, t, E).$$

(1)

Here $D(E, \alpha)$ is the anomalous diffusivity [12] and $b(E)$ is the mean rate of continuous energy losses. The fractional Laplacian (called ‘Riss operator’) $(-\Delta)^{\alpha/2}$ [13, 14] reflects a nonlocality of the diffusion process of particles in the interstellar medium.

The rate of change of the energy of electrons, as well as of positrons, $b(E)$, during their propagation in the medium is attributed to ionization, inverse Compton losses, bremsstrahlung, and synchrotron radiation. According to [15], we can write $b(E)$ as

$$b(E) = b_0 + b_1E + b_2E^2 \approx b_2(E + E_1)(E + E_2),$$

where $b_0 = 3.06 \cdot 10^{-16}n$ (GeV s$^{-1}$), $b_1 = 10^{-15}n$ (s$^{-1}$), and $b_2 = 1.38 \cdot 10^{-16}$ (GeV s$^{-1}$) (for the magnetic field intensity $B = 5 \mu$G and background photon density $\omega = 1$ (eV cm$^{-3}$), whereas $E_1 = b_0/b_1$ and $E_2 = b_1/b_2$.

The equation for Green’s function $G(\vec{r}, t, E; E_0)$ describing electrons and positrons diffusion under condition that the particles started from origin $\vec{r}_0 = 0$ at the time $t_0 = 0$ with the energy $E_0$ has the form [11]

$$\frac{\partial G(\vec{r}, t, E; E_0)}{\partial t} = -D(E, \alpha)(-\Delta)^{\alpha/2}G + \frac{\partial (b(E)G)}{\partial E} + \delta(\vec{r})\delta(t)(E-E_0).$$

(2)

The Green’s function of the problem was derived using standard substitutions [16] and Fourier transform:

$$G(\vec{r}, t, E; E_0) = \frac{g_3^{(\alpha)}(|\vec{r}| \lambda^{-1/\alpha})}{\lambda^{3/\alpha}} \times$$

$$\times \delta \left( E_0 - \left\{ \frac{E + E_1}{1 - b_1t(E + E_2)/E_2 - E_1} - E_1 \right\} \right) \times$$

$$\times H(1 - b_1t(E + E_2)/E_2 - E_1)H(t).$$

Here, $g_3^{(\alpha)}(r)$ is the probability density of three-dimensional spherically-symmetrical distribution [14, 17], and

$$E_0(t) = \frac{E + E_1}{1 - b_1t(E + E_2)/E_2 - E_1} - E_1,$$

$$\lambda(E, E_0) = \int_E^{E_0(t)} \frac{D(E', \alpha)}{b(E')} dE'.$$

2 Energy spectrum of electrons and positrons

A solution to equation (1) for a point impulse source with a power energy spectrum

$$S(\vec{r}, t, E) = S_E E^{-p} \delta(\vec{r})\Theta(T - t)\Theta(t),$$

which simulate generation of electrons in supernovae, has the form

$${\min}[t, 1/2b_2(E + E_2)]$$

$$(t - t_0)^{-3/\alpha} \times$$

$$(1 - b_2t(E + E_2))^{2 - g_3^{(\alpha)}(|\vec{r}| \lambda(t', E)^{-1/\alpha})}. (3)$$

For a point steady source

$$S(\vec{r}, E) = S_E E^{-p} \delta(\vec{r})$$

the solution is

$${\min}[t, 1/2b_2(E + E_2)]$$

$$(t - t_0)^{-3/\alpha} \times$$

$$(1 - b_2t(E + E_2))^{2 - g_3^{(\alpha)}(|\vec{r}| \lambda(t', E)^{-1/\alpha})}. (4)$$

According to [12, 15], the electrons intensity from all galactic sources was presented as

$$J(\vec{r}, t, E) = J_L(\vec{r}, t, E) + J_G(\vec{r}, E) = \frac{V}{4\pi} N(\vec{r}, t, E),$$

(5)

where

$${\min}[t, 1/2b_2(E + E_2)]$$

$$(t - t_0)^{-3/\alpha} \times$$

$$(1 - b_2t(E + E_2))^{2 - g_3^{(\alpha)}(|\vec{r}| \lambda(t', E)^{-1/\alpha})}. (4)$$

In (5) $J_L$ is the local component, i.e. the contribution nearby ($r \leq 1$ kpc) young ($t \leq 10^6$ yr) sources and $J_G$ is the global spectrum component determined by the multiple old ($t \geq 10^6$ yr) distant ($r \geq 1$ kpc) sources.

Distribution of the sources in the area $r > 1$ kpc (G-component) was described according to standard scenario.
(system of steady-state sources). At particles energies $E \lesssim 10$ GeV, the fluxes of both electrons and positrons observed in the Solar system are influenced by modulation effects; hence, we used the model proposed in [18], with the potential $\Phi = 600$ MV to take into account the solar modulation.

To calculate the electrons and positrons spectra from nearby young sources ($L$-component), simulation of the Poisson distribution parameter (average number of the sources in the local space) was chosen $\sim 10$. This estimation corresponds to a number of the well-known nearest supernova remnants and pulsars with $t \lesssim 10^6$ yr [19, 20]. Coordinates and times of birth of the sources were generated randomly and uniformly in the space region $r < 10^3$ pc and in the time interval $10^4 \lesssim t < 10^6$ yr. Duration of the particle generation by the local sources was assumed to be $T \approx 10^6$ yr.

3 Determination of model parameters

Estimation of one the key parameters of the model — exponent $\alpha$ — is based on the results of the investigations of particles diffusion in the cosmic and laboratory plasma. It is known that for the superdiffusion regime of propagation (parameter $1 < \alpha < 2$) the diffusion packet spreads with time as $\Delta x^2 \sim t^{\frac{\alpha}{2}}$. Interpretation of the data for magnetoosphere leads to conclusion that the width of the diffusion packet increases with the rate $\Delta x^2 \sim t^{1.4}$ [21]. Investigation of the particles propagation before the shock wave in the circumterrestrial plasma leads to dependence $\Delta x^2 \sim t^{1.19}$ [22]. Thus, these results allow to obtain the following estimate $\alpha \approx 1.4 \div 1.7$.

For the anomalous diffusivity $D(E, \alpha) = D_0(\alpha)E^\alpha$ the analysis of nuclear component of cosmic rays [12] gives parameters $D_0(\alpha) \approx 2 \cdot 10^{-4} \div 4 \cdot 10^{-2}$ pc$^2$/yr and $\delta \approx 0.27$. Note that the value of $\delta$ actually almost coincides with the value of $1/3$, accepted for the Kolmogorov turbulence [23].

The exponent of injection spectrum $p$ for both positrons and electrons changes from 2.6 (as established earlier from the analysis of the synchrotron radiation spectrum [24, 25]) to 2.85 [12] in the TeV energy region. The estimation for $p \approx 2.85$ in the sources of cosmic rays is supported by the number of recent results on SNRs: for the SNR RX J1713.7-3946 the estimate $p \approx 3.3$ was obtained [26]; the Fermi-LAT experiment reported $p \approx 2.87$ for $E > 69$ GeV for the SNR IC 443 [27, 28] and $p \approx 3.3$ for the SNR W44 [29].

Results and conclusions

We present results of calculations of the spectra of electrons and positrons under the assumption that observed spectra of the particles are formed by multiple old distant sources (spectra of primary particles from this group of sources are denoted as $(e^+_{pr})_G$, $(e^-_{pr})_G$ and calculated using equation (4)) and nearby young ones (their spectra are $(e^+_{pr})_L$ and $(e^-_{pr})_L$ calculated with (3)). To accommodate the contribution of secondary positrons and electrons ($e^+_{sec}$ and $e^-_{sec}$ respectively) produced in the collision of cosmic rays nuclei with the interstellar medium, the GALPROP code [30] was used with the parameters stated in our model.

In figure 1 we present results of calculation of the positron to electron ratio for the following model assumptions.

- For $E < 1$ GeV main contribution to observed spectrum of the particles is due to secondary particles as well as due to electrons and positrons injected by old distant sources

$$R \approx \frac{e^+_{sec} + (e^+_{pr})_G}{e^-_{sec} + (e^-_{pr})_G + (e^+_{pr})_G}, \quad (6)$$

- In the range $E \in [1 : 100]$ GeV the contribution of the nearby young sources becomes significant

$$R = \frac{e^+_{sec} + (e^+_{pr})_G + (e^-_{pr})_L}{e^-_{sec} + (e^-_{pr})_G + (e^+_{pr})_G + (e^-_{pr})_L}. \quad (7)$$

- For $E \gg 100$ GeV, as follows from the equation (7), observed spectrum of the positrons is formed by the local sources only and the positron to electron ratio increases to a constant value

$$R \approx \frac{(e^+_{pr})_L}{(e^+_{pr})_L + (e^-_{pr})_L} \approx \text{const}. \quad (8)$$

It can be seen that a self-consistent description of the experimental data can be obtained if we assume that both positrons and electrons are injected into the interstellar medium by the sources with the same spectral exponent $p$. We have also found that the positron to electron ratio increases to a constant value of $\sim 0.22$ for energies $E > 100$ GeV. This flattening of positron fraction obtained in our model can be examined in the near future by the PAMELA and the AMS-02 [31] experiments.

References

Figure 1: Comparison of the positron to electron ratio calculated within the frameworks of fractional diffusion model with the experimental data. Dash-dotted line correspond to the case (6). Full line is the result of the fractal diffusion model for the case (7). Results of calculations for the standard scenario with the GALPROP code [30] are also presented in this figure (dotted line). References to the experimental data are given in [25]