Abstract: We are studying the constraints obtained on transport and acceleration mechanisms of galactic cosmic rays by using statistical tools in combination with the propagation package GALPROP and recent PAMELA data. Using only PAMELA data allows us to avoid inconsistencies between data sets from different experiments, minimise uncertainties on solar modulation parameters, and have a complementary and precise data set on (anti-)matter as well as primary and secondary nuclei over 3 orders of magnitude in energy. This allows us to simultaneously place strong constraints on cosmic-ray propagation and acceleration models. We describe our methodology and present some preliminary results in this paper.

Keywords: cosmic-rays; propagation models; statistics

1 Introduction

Cosmic rays (CR) were discovered one century ago and since then acceleration and propagation mechanisms have been much debated. Our understanding of this penetrating radiation from outer space comes primarily from the many balloon flights conducted during the last century. More recently, satellite projects such as PAMELA [1] have provided interesting results through precise measurements of the CR elemental fluxes. The PAMELA experiment can measure light nuclei CR fluxes (e.g., protons – p, helium – He, antiprotons – \( \bar{p} \)) and secondary-to-primary ratios (e.g., boron-to-carbon ratio – B/C, antiproton-to-proton ratio – \( \bar{p}/p \)) over a wide range and with excellent precision due to an exposure soon exceeding 5 years and the absence of residual overburden from Earth’s atmosphere.

Most of our knowledge of CR propagation comes from secondary CR nuclei, produced by interactions of primary cosmic rays with the interstellar medium (ISM). The diffusion model with possible inclusion of reacceleration and convection is widely believed to provide the most adequate description of CR transport in the Galaxy at energies below \( 10^{17} \) eV. Two main approaches have been employed to date: analytical (or semi-analytical) models and purely numerical models. Analytical solutions of the transport equation are derived assuming a simplified description of the spatial dependence of the galactic constituents (e.g., the distribution of gas) and transport parameters, and can take advantage of much faster computation methods than numerical models. Numerical methods are based on many simplified assumptions but also allow an easier implementation of more realistic transport processes. Some work has been done evaluating both methods to study CR transport (e.g., most recently [2, 3, 4, 5, 6]). Such results do not always present consistent best-fit parameter values however. For example, the published diffusion coefficient slope varies between 0.2 and 0.5 for diffusion-reacceleration models. In our work, we aim to reduce the inconsistencies arising from using data from different experiments by using recent PAMELA data and the public numerical code GALPROP [7, 8] to set constraints on propagation and acceleration parameters.

For a given species, the CR propagation equation can be written in the following general form

\[
\frac{\partial \Psi(\vec{r}, p, t)}{\partial t} = q(\vec{r}, p, t) \\
+ \nabla \cdot \left( D_{xx} \nabla \Psi - \vec{V} \Psi \right) \\
+ \frac{\partial}{\partial p^2} D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \Psi \\
- \frac{\partial}{\partial p} \left[ \rho \Psi - \frac{p}{3} \left( \nabla \cdot \vec{V} \right) \Psi \right] \\
- \frac{1}{\tau_f} \Psi - \frac{1}{\tau_r} \Psi,
\]

where \( \Psi \) is the CR density, \( q \) is the CR source, \( \nabla \) is the gradient operator, \( \vec{V} \) is the convection velocity, \( D_{xx} \) and \( D_{pp} \) are the diffusion coefficients, and \( \rho \) is the target density.
where $\Psi(\vec{r},p,t)$ is the CR density per unit of total particle momentum $p$ at position $\vec{r}$, in the form of a phase space density $f(\vec{r},\vec{p},t)$; $\Psi(p) = 4\pi p^2 f(\vec{p})$; $q(\vec{r},p,t)$ is the source term; $D_{xx}$ is the spatial diffusion coefficient; $\bar{V}$ is the convection velocity; $D_{pp}$ is the diffusion coefficient in momentum space; $\tau_f$ is the time scale for fragmentation and $\tau_d$ is the time scale for radioactive decay.

Before being observed at Earth, CRs penetrate the heliosphere and lose energy. The force-field approximation [9] which depends on a single parameter, the modulation potential $\Phi$, is used in this work to describe the effect of the solar modulation on the cosmic-ray energy spectrum.

2 Analysis and results

2.1 Parameter description

We focus our study on the propagation parameters characterising different processes during the transport of cosmic-rays through our Galaxy. Among those parameters, the spatial diffusion coefficient is defined as

$$D_{xx} = D_0 \beta \left( \frac{p}{p_0} \right)^\delta,$$

where $D_0$ is the normalisation at reference rigidity $p_0$, the factor $\beta = v/c$ is the particle velocity, and $\delta$ is the so-called spectral index of the diffusion coefficient. Hence, the free parameters concerning diffusion are $D_0$ and $\delta$. Earlier studies of the B/C ratio [7] introduced a break in the rigidity dependency of $D_{xx}$, defining $D_{xx}$ as $\beta D_0 (p/p_0)^{\delta_1}$ and as $\beta D_0 (p/p_0)^{\delta_2}$ below and above the reference rigidity $p_0$, respectively. In this preliminary study, we only consider models without a break. The normalisation of the diffusion coefficient is performed at 4 GV.

An energy gain through reacceleration is induced through diffusion in momentum space. The associated diffusion coefficient in momentum space $D_{pp}$ is taken from the model of minimal reacceleration by interstellar turbulence and is related to the spatial diffusion coefficient $D_{pp}$:

$$D_{pp} = \frac{4v_A^2 p^2}{3\delta (4 - \delta^2) (4 - \delta) D_{xx}},$$

where $v_A$ is the Alfvén velocity - the main free parameter related to reacceleration.

Finally, CRs can be transported in bulk away from the Galactic plane through convection mechanisms (Galactic winds). Here, the convection velocity $V(z)$ is assumed to vary linearly with the distance from the Galactic plane $z = 0$ as

$$V(z) = V(0) + \frac{dV}{dz} z.$$

If convection is considered in the propagation model, $dV/dz$ is the main free parameter and $V(0)$ is taken to be zero for a simplicity.

As well as transport processes, acceleration mechanisms of nuclei are crucial in order to describe CR data. For a CR species the injected density is assumed to be a power law in momentum, $q(p) \propto N p^\nu$, where $N$ is the normalisation at reference kinetic energy chosen to be $10^5$ MeV. As proposed in [10], a break similar to the one in the diffusion coefficient might be introduced in the CR injection spectrum at a reference rigidity to match the spectra of primary nuclei. In this preliminary study, no break is assumed. Since in the GALPROP code the normalisation is based on the proton injection spectrum, the free parameters related to source terms are the normalisation $N_p$ for proton flux and the injection index $\nu$.

To summarise, our free parameters are $D_0$, $\delta$, $v_A$, $dV/dz$, $\nu$ and $N_p$. For stable species, $D_0$ and the halo size of the Galaxy $z_h$ are degenerate. Therefore, we are not able to constrain both parameters simultaneously. We assume $z_h = 4$ kpc in agreement with earlier GALPROP studies of the $^{10}$Be, $^{26}$Al, $^{36}$Cl, $^{54}$Mn, and B/C ratio [11, 12] and to ease the comparison of the results.

Other GALPROP parameters are held at the conventional reacceleration configuration of [11, 13], tuned to reproduce the ACE isotopic abundances of [14]. The nuclear chain for proton and antiproton studies starts from $^4$He since primary helium nuclei and proton interactions with the ISM are dominant in the secondary production of protons and antiprotons.

2.2 Data

We use PAMELA data to study CR propagation. Usually, in order to cover a wide enough energy range to constrain diffusion parameters it is necessary to combine data sets from a variety of experiments. As pointed out in [6], errors might be underestimated for an experiment. In order to compensate the systematic discrepancies in the reported uncertainties from data sets, one can therefore introduce a set of nuisance parameters to rescale the reported errors. This can be avoided by using only PAMELA data, allowing the use of a smaller parameter set and minimising the computational time for parameter space scans. Another unavoidable problem caused by incorporating data sets from various experiments is that the modulation potential $\Phi$ based on the assumed interstellar spectrum of CR species may differ between experiments. Relatively poorly understood solar physics makes studies of CR transport in the Galaxy more difficult. Using only PAMELA data decreases therefore uncertainties on derived propagation parameters by including $\Phi_{\text{PAMELA}}$ as a nuisance parameter. There is, however, still a potential bias arising from the simplified approximation of solar modulation used. PAMELA achieves significantly better statistics and extends the energy range compared to previous experiments, especially on antiparticles, e.g., $p/\bar{p}$ ratio. PAMELA covers the energy range from 60 MeV to 180 GeV and improves the precision of data [15]. Such precise measurements enables us to put better constraints on CR transport parameters. Together with the $p/\bar{p}$ ratio, we account for the proton flux measured by PAMELA with great accuracy over a
Figure 1: B/C ratio for our best-fit parameters of PD-d, DC-d, DR-b, DRC-b models (as listed in Tab. 1) and PD model. Data points are from HEAO-3, ACE, CREAM-1, Spacelab-2 and AMS01 measurements [17, 18, 19, 20, 21].

broad energy range from 400 MeV to 1.2 TeV [16], to improve constraints on the primary injection spectrum. Meanwhile, the B/C ratio, one of the quantities most sensitive to the diffusion parameters, is expected to be measured from 100 MeV/n to 200 GeV/n by the PAMELA experiment in near future. In this preliminary work we use B/C data sets from pervious experiments [17, 18, 19, 20, 21] to provide constraints on the transport parameters.

2.3 Method

In this preliminary analysis, we study four configurations: the pure diffusion model, the diffusion-reacceleration model, the diffusion-convection model and the diffusion-reacceleration-convection model (referred to as PD, DR, DC and DRC, respectively). To investigate the best-fit parameters we use the χ² method. We interfaced GALPROP with the MINUIT library [22].

We use the GALPROP code to model the CR spectra and ratios in the Solar system, located at $r = 8.5 \text{ kpc}$ from the Galactic centre and in the Galactic plane ($z = 0$). We apply the force-field approximation to estimate CR fluxes at the top of the Earth’s atmosphere. Either the fluxes or the flux ratios derived from GALPROP are compared with data to calculate the χ². The χ² function is then used as an input to MINUIT which subsequently provides the best-fit values and their uncertainties.

2.4 Results

Table 1 summarises the best-fit parameters as well as the corresponding χ²/d.o.f for the above described propagation models, derived from a fit of the B/C ratio (labelled as “-b”) and from a simultaneous fit of the B/C ratio, p/p ratio and the proton spectrum (labelled as “-d”). The B/C ratios, p/p ratios and proton spectra obtained with the best-fit parameters of PD-d and DC-d models are shown together with the experimental data in Fig. 1, 2 and 3, respectively.

The spectral index $\delta$ is well constrained between 0.3 and 0.65 for all models considered. Whereas the PD models favour a Kraichnan turbulence spectrum of $\delta = 0.5$, the DC convection models favour a slightly higher value of $\delta \sim 0.64$ due to the positive correlation between the two low-energy processes. The Kolmogorov spectrum of turbulence is only recovered for the DR-b model and in agreement with earlier studies (e.g., [10, 6]), but including more observables, i.e., the p/p ratio and the p flux tend to disfavour reacceleration ($v_A \rightarrow 0$ for DR-d and DRC-d) since it is unable to reproduce the proton flux (not shown here). The Galactic wind velocities obtained for the DC models are about 10 km s$^{-1}$ kpc$^{-1}$ and in good agreement with other studies, e.g., [10]. The same is valid for the Alfven...
velocities of about 35-40 km/s for the DR and DRC-b models. The model reproducing the best the experimental data is the DRC-b model with a reduced \( \chi^2 \) of 1.58, since the models without reacceleration fail to reproduce the low-energy B/C ratio (see Fig. 1).

As it was already argued in [23], a simultaneous study of secondary-to-primary ratios and primary fluxes is necessary to obtain complementary information on transport and source parameters, but may underestimate the errors on the transport parameters, as well as being biased, and consequently not reliable. Indeed, primary fluxes are more prone to systematics and more sensitive to solar modulation than secondary-to-primary ratios. For these reasons, priors will be used on the source parameters in a subsequent Bayesian analysis.

In this study, we introduce a low-energy dependence of the diffusion coefficient: \( D_0 \delta^\eta (\rho/\rho_0)^\delta \), where \( \eta \) was added as a free parameter in the fit. This approach is different to an artificial break at a rigidity 10 GV used in [10, 6], since it acts only at low-energies and is physically motivated. As a test, we studied a pure diffusion model (hereafter referred to as PD) by a simultaneous fit of the B/C ratio, \( p/p \) ratio and proton spectrum. As shown in Fig. 1, 2 and 3, this model reproduces well all data which is also indicated by the \( \chi^2/d.o.f \) value of 1.23.

Finally, we want to stress that no model studied in this work can account for the “hardening” of the proton spectrum at \( \sim 200 \) GeV, as observed by the PAMELA experiment, since a break at high energy is neglected in this analysis.

### 3 Summary and Plans

A \( \chi^2 \) minimisation method was performed on \( p/p \) data and proton flux data reported by PAMELA and B/C ratio published by other experiments to evaluate CR transport and source parameters. This method was demonstrated to place strong constraints on parameters for four configurations: PD, DR, DC and DRC models. In addition, a pure diffusion model which includes a parameterisation of the low energy dependence on diffusion coefficient is also studied. This PD model can generally reproduce all the data. In future work, the upcoming PAMELA B/C ratio will be used to place more robust constraints on transport parameters. The low energy behavior will be understood further. We also plan to use a Bayesian approach to estimate the posterior probability density functions of the transport and source parameters. The credible regions in the parameter space can be naturally identified and thereby help us to understand the uncertainties and correlations between the parameters.

### References