Isotopic Identification with the Geomagnetic Field for Space Experiments

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\textbf{Abstract:} The presence of the geomagnetic field shields earth against charged radiation by imposing a cutoff on their rigidity spectra. The existence of such a rigidity cutoff translates into a velocity threshold which depends on the particle mass. Depending on the orbiting position, particle incident angle and mass, isotopes of a given energy can reach the detector or being rejected due to the geomagnetic field. It is therefore possible to take advantage of the shielding power of the geomagnetic field to filter isotopes, making possible the reconstruction of the different isotopic fluxes. We present a preliminary study of a new method which consists in using the geomagnetic field to identify incoming isotopes from the cosmic radiation. We show that with this method, it the reconstruction of isotopic cosmic-ray fluxes up to an energy of $\sim 15$ GeV is feasible.

\textbf{Keywords:} Method, Cosmic-ray, Isotopes, Geomagnetic field.

1 Introduction

Propagation of cosmic rays through the Galaxy has been widely discussed for a long time since the discovery of the cosmic radiation. Measurements made on the GeV to TeV energy range as well as forthcoming data from the AMS-02 space experiment allow to study the cosmic-ray diffusion mechanism.

Most of our knowledge of cosmic-ray diffusion comes from secondary cosmic-ray nuclei, which are products of the interactions between primary cosmic rays and the components of the interstellar medium. The secondary-to-primary flux ratios are commonly used as probes of the diffusion coefficient through the $K_0/L$ ratio \[11\], $K_0$ being the cosmic-ray diffusion coefficient and $L$ the scale height of the cosmic-ray diffusive halo. However our lake of knowledge concerning the diffusive halo size limits our understanding of the cosmic-ray diffusion. Nevertheless, it is possible to remove the degeneracy between $K_0$ and $L$ from measurements of the abundance ratio between unstable and stable secondary isotopes generated by the same primary parent. Due to their finite life-time, radioactive-$\beta$ nuclei diffuse into the Galaxy over shorter distances compared to the stable nuclei. When these distances are smaller than the cosmic-ray diffusive halo size, the radioactive-nuclei diffusion become insensitive to the halo size.

To date, measurements of secondary unstable-to-stable flux ratios \[5, 6, 8--10, 12, 14, 15, to cite only the most recent\] are unfortunately restricted to low energy range, below 0.2 GeV/nuc, apart from the $^{10}$Be/$^{9}$Be ratio which counts two experimental points above 0.2 GeV/nuc. The absence of measurements at higher energies does not permit to remove the $K_0/L$ degeneracy and to disentangle different cosmic-ray propagation models. This lake of measurements above 0.2 GeV/nuc translates the challenge of accurately estimating the mass of incoming particles. As an example, the mass resolution $\Delta m/m$ require to separate $^{10}$Be from $^{9}$Be should be better than 3\%. Classical method used to estimate the mass of an incoming particle consists in combining independent measurements of its rigidity and velocity and thus, requires on both, the rigidity and velocity, precise measure.

To bypass this difficulty, we propose a new method which takes advantage of the Earth geomagnetic field to perform a natural filtering of the incoming isotopes. The geomagnetic field shields Earth against charged particles by imposing a cutoff rigidity bellow which particles of insufficient rigidity are not allowed to penetrate the geomagnetic field down to the Earth. This cutoff rigidity translates to a velocity threshold which is inversely proportional to the particle mass. As a consequence, when two isotopes impinge the Earth magnetic field at the same velocity, the heavier has indeed a higher rigidity and thus penetrates deeper into the geomagnetic cavity. Should the lightest isotope has a rigidity lower or equal to the cutoff rigidity, it will naturally be suppressed from the detected cosmic radiation. The use of the geomagnetic field as a purpose to filter isotopes was first suggested by Balasubramanyan \[1\] and Hubert \[7\].

The aim of this paper is to discuss an improved method of the works they started, allowing to identify isotopes on an event by event basis. For simplicity, in this paper we will
present this method under the Stœrmer approximation, assuming a dipolar geomagnetic field. A more detailed paper, accounting for the complex geometry of the Geomagnetic field is under preparation.

2 Geomagnetic Filter

In a cosmic-ray experiment, the number of detected particles depends on its the effective acceptance $\epsilon_{\text{acc}}$, its live-time $T_{\text{life}}$, and the transmission through the geomagnetic field $\epsilon_B$.

\[
N(R) = \phi(R) \Delta R \epsilon_{\text{acc}}(R) T_{\text{life}} \epsilon_B(R),
\]

with $R$ the particle rigidity, $\phi(R)$ the cosmic-ray flux arriving at Earth and $\Delta R$ the rigidity-bin width. The effective acceptance $\epsilon_{\text{acc}}$ is the product of the geometric acceptance by the detection efficiency of the experiment. Both, the acceptance and the live-time are intrinsic to the experiment and will not be further detailed in this paper. The geomagnetic transmission determines the fraction of particles passing through the geomagnetic shield that can be detected at the experiment.

Cosmic rays follow curved trajectories when they penetrate the geomagnetic field. Whether a cosmic ray can reach a point nearby Earth or not depends on its rigidity and direction as well as on the topology of the geomagnetic field. The minimal rigidity allowed for an incident cosmic ray to be detected is called the cutoff rigidity. Under the Stœrmer approximation [13], assuming a dipolar geomagnetic field, the cutoff rigidity is defined as:

\[
R_c = \frac{M_B}{2r^2} \frac{\cos^4 \ell_B}{(1 + \sqrt{1 - \cos^2 \ell_B \sin \theta_B \sin \phi_B})^2},
\]

where $M_B$ is the Earth dipolar magnetic moment, $r$ is the distance to the dipole center, $\ell_B$ is the magnetic latitude, $\theta_B$ is the cosmic ray zenith angle and $\phi_B$ is the cosmic ray azimuthal angle measured clockwise from the direction of the magnetic south. At a specific location, all incident cosmic rays having the same incident angles and the same charge sign have the same cutoff rigidity, regardless of their mass.

For a particle of charge $Z$ and mass $M = A m c^2$ (with $m c^2$ the atomic mass unit and $A$ the atomic mass number) the cutoff rigidity translates to a velocity threshold:

\[
\beta_c(A,Z) = \frac{R_cZ}{Amc^2} \frac{1}{\sqrt{\left(\frac{R_cZ}{Amc^2}\right)^2 + 1}},
\]

which now decreases with the increasing mass of the particles. Thus, when two particles of atomic mass $A_2 > A_1$ impinge the geomagnetic field with same velocity $\beta = \beta_c(A_1,Z)$ and same incident angles, the geomagnetic field shields the lightest particle ($A_1$) while the heavier ($A_2$), which has a velocity higher than its threshold velocity, is allowed to penetrate deeper into the geomagnetic cavity. Fig. 1 illustrates the velocity distribution of vertical incident $Z = 4$ charged particle with atomic mass number $A_1=9$ and $A_2 = 10$ expected to be measured at (from top to bottom) $50^\circ$, $55^\circ$ and $66^\circ$ of geomagnetic latitude.

The logic would be the same for $N$ isotopes having atomic masses $A_1 < A_2 < \ldots < A_N$ with the corresponding threshold velocity $\beta_c(A_1,Z) > \beta_c(A_2,Z) > \cdots > \beta_c(A_N,Z)$. Measurements performed at velocity $\beta > \beta_c(A_1,Z)$ would not allow to identify any of the isotopes since at such a velocity all isotopes are above the geomagnetic cutoff. Measurements performed between $\beta_c(A_{i-1},Z)$ and $\beta_c(A_i,Z)$ (with $i$ an integer ranging from 2 to $N$) would not allow to disentangle between isotopes $A_i$ to $A_N$. Nonetheless, none of the lighter isotopes ($A_1$ to $A_{i-1}$) would be present since they are shielded by the geomagnetic cutoff. Finally at a velocity ranging from $\beta_c(A_{N-1},Z)$ to $\beta_c(A_N,Z)$ only the heavier isotopes are find to be above the geomagnetic cutoff and measurements performed at such a velocity would allow to reliably identify the incoming particle as an isotope $A_N$. However, as can be seen from Fig. 1 only a small fraction of particles have velocity ranging between $\beta_c(A_{i-1},Z)$ and $\beta_c(A_i,Z)$ and thus, only a small fraction of the incoming heavier isotopes can be separated from the lighter one. This

Figure 1: Velocity distribution of vertical incident $Z = 4$ charged particle with atomic mass number $A_1=9$ and $A_2 = 10$ expected to be measured at (from top to bottom) $50^\circ$, $55^\circ$ and $66^\circ$ of geomagnetic latitude.
fraction corresponds to the geomagnetic filter efficiency

\[ \epsilon_{f,i,N}(\beta) = \frac{1}{\int_{-\pi/2}^{\pi/2} \frac{1}{T_{life}} \frac{dT_B}{d\beta} d\beta \int_{0}^{2\pi} \frac{1}{\phi_B} \int_{-1}^{1} \text{Acc}(\theta_B)d\cos\theta_B \sum_{j=i}^{N} \phi_j(\beta) d\beta} \]

\[ \times \int_{-\pi/2}^{\pi/2} \frac{1}{T_{life}} \frac{dT_B}{d\beta} d\beta \int_{0}^{2\pi} \frac{1}{\phi_B} \int_{-1}^{1} \text{Acc}(\theta_B)d\cos\theta_B \left\{ \int \{ H[\beta - \beta_c(A_i, Z, \ell_B, \theta_B, \phi_B)] - H[\beta - \beta_c(A_{i-1}, Z, \ell_B, \theta_B, \phi_B)] \} \sum_{j=i}^{N} \phi_j(\beta) d\beta \right\}, \]

where \( i \) is an integer varying from 1 to \( N \), \( H(x) \) is the Heaviside function. We defined \( \beta_c(A_0, Z) = \infty \) when \( i - 1 = 0 \).

Since \( R_c \), and subsequently \( \beta_c \), depends on the geomagnetic latitude, the geomagnetic filter efficiency also depends on the time spent by the experiment at different geomagnetic latitudes and subsequently, it depends on the orbit configuration. The term \( 1/T_{life} \times dT_B/d\beta \) in Eq.[4] gives the fraction of time spent by the experiment at different geomagnetic latitudes. Fig. 2 shows an example of geomagnetic filter efficiency for two isotopes, \( A_1 = 9 \) and \( A_2 = 10 \), with charge \( Z = 4 \) and for a space experiment having a circular orbit with inclination of \( 60^\circ \) with respect to the geomagnetic equatorial plane.

As can be seen in Fig. 2 up to \( \sim 10\% \) of the isotope \( A_2 \) can be filtered from the incoming cosmic rays. This plots shows the effects of both, the geomagnetic transmission – with an efficiency which decrease with decreasing energy – and the isotopic filtering which will now be discussed. For an \( 60^\circ \) orbit inclination, the minimal cutoff rigidity is \( \simeq 0.9 \) GV. Consequently, below a rigidity of \( 0.9 \) GV, all particles are shielded by the geomagnetic field and geomagnetic filter efficiencies are equal to 0. Around \( |\ell_B| \approx 60^\circ \), the difference in threshold velocity between isotopes \( A_1 \) and \( A_2 \) is maximal, as well as the exposure time, and thus, the geomagnetic filter efficiency of isotope \( A_2 \) (hereafter \( \epsilon_{f,2} \)) shows a peak at \( \simeq 1 \) GV. When the experiment move toward the equator \( \beta_c(A_1, Z) \) increases while the difference in threshold velocities \( \Delta \beta \) between isotopes \( A_1 \) and \( A_2 \) decreases. As consequence, the velocity range within which \( A_2 \) might be separated from \( A_1 \) diminishes and \( \epsilon_{f,2} \) decreases while the geomagnetic filter efficiency \( \epsilon_{f,1+2} \) increases. The increase observed around 10 GV is due to the geomagnetic transmission which allows more energetic particles to penetrate the geomagnetic cavity down to the experiment. Finally, above \( \simeq 15 \) GV, all particles are above the geomagnetic cutoff. It is no more possible to use the geomagnetic field to filter isotopes and \( \epsilon_{f,2} \) drop suddenly to 0 while \( \epsilon_{f,1+2} \) equal 1. Depending on the orbits configuration, the variation of the exposure with the latitude modulates the shape of the efficiencies, increasing or decreasing the amplitude of the peaks. Also, since orbits with lower inclination explore regions of higher cutoff rigidity, the peak found at 0.9 GV for a \( 60^\circ \) orbit inclination will shift to higher rigidity. On the other hand, orbit with higher inclination will shift the peak to lower rigidity. The cutoff rigidity also depends on the distance from the dipole center. Thus the shape of this curve will also depends on the orbit ellipticity.

Replacing \( \epsilon_B \) by \( \epsilon_{f,i,N} \) in Eq. 1 allows to reconstruct the cosmic-ray fluxes for different groups of isotopes \( i \rightarrow N \). With some additional calculation, it is then possible to estimate cosmic-ray fluxes for each individual isotopes:

\[ \phi_1(R) = \phi_{1\rightarrow N} - \phi_{2\rightarrow N}, \]

\[ \phi_2(R) = \phi_{2\rightarrow N} - \phi_{3\rightarrow N}, \]

\[ \vdots \]

\[ \phi_{N-1}(R) = \phi_{N-1\rightarrow N} - \phi_N, \]

\[ \phi_N(R) = N_N/\left[ \Delta R \text{Acc}(R) T_{life} \epsilon_{f,N}(R) \right]. \]

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Figure 2: Rigidity dependency of the geomagnetic filter efficiencies \( \epsilon_{f,1+2} \) (line) \( \epsilon_{f,2} \) (dashed line) of incoming isotopes \( A_1 = 9 \) and \( A_2 = 10 \). Efficiencies are obtained assuming a \( 60^\circ \) orbit inclination, assuming a acceptance of 100% for azimuthal incident angle ranging from 0 to \( \pi/2 \).
3 Prospect and Conclusion

In Section 2 we have seen that we can reliably identify incoming cosmic-ray isotopes based only on velocity measurements by tacking advantage of the shielding power of the geomagnetic field. From what we have seen, it is important that the experiment explore regions of different cutoff to be able to filter isotopes over a large velocity range. Thus, this method is more appropriate for space experiments which have a quasi-polar orbit rather than for balloon flights or experiments with equatorial orbit which remain mostly at the same geomagnetic latitude. It is obvious that this method can be applied only if the uncertainties on the velocity measurements are much lower than the difference in threshold velocities between the two groups of isotopes one wish to identify.

Fig. 3 compares the difference in threshold velocities of couples of isotopes of interest for studying the cosmic-ray diffusion to the expected velocity resolution of the AMS-02 Time of Flight and AMS-02 RICH sub-detectors [2, 3] as well as for the Time of Flight system of the PAMELA experiment [4].

The complementarity of the AMS-02 Time of Flight and RICH velocity measurements would allow to filter $^{10}$Be from $^{8}$Be and $^{7}$Be over a large rigidity range, from $\simeq$ 1 GV to $\simeq$ 15 GV. Also, thanks to the high velocity resolution of AMS-RICH ($\Delta \beta/\beta < 10^{-3}$), it will be possible to filter isotopes up to the Aluminium, in a more restricted velocity ranges. Because of the long lifetime of AMS-02, applying this method to the AMS-02 measurements would provide valuable information for the study of cosmic-ray propagation.

We now want to remind that in this paper we demonstrate the utility of this method under simple assumptions, estimating the threshold velocity from the Stœrmer approximation in a dipolar geomagnetic field. In reality, the geometry of the Earth geomagnetic field is more complex and varies with the solar activity. Stœrmer theory has the advantage to give a rough idea of the cutoff rigidity but it has a limited accuracy. Also, Stœrmer theory neglects the Earth shadow and penumbra which results in an underestimation of the cutoff rigidity. A proper way to estimate geomagnetic transmission and filter power would be to backtrace each incoming particle backward in a realistic geomagnetic field model to determine allowed and forbidden trajectories. A more detailed paper discussing of the implementation of the backtracking method to the isotopic filtering is under preparation. Nonetheless, even with a more detailed geomagnetic field model, preliminary study shows similar results than those presented in this paper with this simple approximation.

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References