Escaping the accelerator: implications for an energy dependent composition?

LUKE O’C. DRURY

1Dublin Institute for Advanced Studies, School of Cosmic Physics, 31 Fitzwilliam Place, Dublin 2, Ireland
ld@cp.dias.ie

Abstract: The escape of charged particles accelerated by diffusive shock acceleration from supernova remnants is a more complex process than generally appreciated. The high-energy end of the spectrum can exhibit spectral breaks even with no formal escape as a result of geometrical dilution and changing time-scales. The bulk of the cosmic ray particles at lower energies must be produced and released in the late stages of the remnant’s evolution whereas the high energy particles are produced and escape early on; this may explain recent observations of slight compositional variations with energy.

Keywords: Shock acceleration, supernova remnants, chemical composition

1 Introduction

The idea that Supernova Remnants (SNRs) produce the bulk of the Galactic cosmic rays is now widely accepted. A key element in this general acceptance has been the development of a quite sophisticated theory of particle acceleration in shock waves, usually called diffusive shock acceleration (DSA), which provides a compelling explanation of how a small number (of order \(10^{-4}\) for conventional SNR parameters) of particles flowing into the shock can be accelerated to ultra-relativistic energies and become cosmic ray particles. Less attention has been paid to the question of how these particles then escape from the acceleration site and propagate into the interstellar medium although this is clearly an important question. However increasing attention is now being paid to this issue, in part because of theoretical developments relating to magnetic field amplification which require it to be taken much more seriously, and in part because of interest in the idea that escaping particles illuminating nearby molecular clouds might be sources of high energy gamma-rays and neutrinos [1, 2]. The purpose of this paper is to point out that the escape process can also very naturally produce an energy dependent composition which, at least qualitatively, appears to agree with recent observations.

2 Theoretical preliminaries

In the standard simple theory of DSA particles are accelerated while within a diffusion length of the shock, there is no escape of particles upstream whatsoever, and the only escape downstream is by advection with the bulk flow. As long as the shock can be treated as an essentially planar structure propagating at fixed speed into an upstream medium capable of scattering particles this picture is certainly correct. Asymptotically a plane surface moving at uniform speed, where the displacement is proportional to the time, will always overtake a randomly walking particle where the displacement grows only as the square root of the time. Thus at this level all particles are accumulated in the downstream region and there is no escape into the undisturbed far-upstream medium.

The problem of course is that a lot of simplifying assumptions have been made in this picture. A real SNR shock is spherical and not planar; decelerating and not moving at constant speed; and the upstream scattering will certainly depend on the effective magnetic field strength as well as the wave spectrum and may even be totally suppressed in partially ionized regions of the ISM. The picture is further complicated when the nonlinear reaction of the accelerated particles on the shock structure, and the resulting secular evolution of the shock structure, is included. Of course, and relevant to this paper, if the scattering is totally suppressed particles can escape upstream even from a uniformly moving and infinitely extended planar shock, but it is important to note that this is a singular limit. Letting the shock radius go to infinity at finite scattering and letting the scattering go to zero at finite radius are two mathematical limiting processes that do not commute, so the zero scattering and infinite shock problem is formally ill-posed. For the problem of escape from supernova remnants what we actually want to study is the case of a large but finite shock radius and an upstream medium (the ISM in our Galaxy) which has a non-zero even if small level of scattering (as required by cosmic ray propagation models).
Naively one may expect that, because $0.5$ is larger than $0.4$, a spherical blast wave expanding according to the Sedov scaling, $R \propto t^{0.4}$, between radius $R$ and time $t$ can be out-run by a randomly walking particle with displacement $D$ growing as $D \propto t^{0.5}$ so that escape has to be considered a real possibility. In fact a more sophisticated argument is required because the particle in the course of its random walk may return to the shock even though on average it moves further away. It is in fact quite easy to show [4] that a uniformly diffusing particle released at radius $R_1$ from the origin will enter a sphere centred on the origin of radius $R_0$ with probability $R_0/R_1$ and escape to infinity without ever encountering the sphere with probability $1 - R_0/R_1$. Thus a particle located even as far upstream as ten shock radii will have a slightly larger than 10% chance of returning to the shock (slightly larger because the shock is expanding; it would be precisely 10% for a stationary spherical surface). This is of course a mathematical result which among other things depends on uniform and isotropic spatial diffusion in three spatial dimensions and as such must be treated with appropriate caution, but it does show that escape is not quite as straightforward as one might think.

Further while escape is possible in three dimensions it is formally impossible in one dimension; a particle randomly walking on a line visits every point infinitely often even though the recurrence intervals grow ever longer. Thus if the cosmic rays were effectively tied to a single magnetic field line they could never escape (at least formally). In reality of course there is some element of cross-field diffusion and the three dimensional result is probably more relevant.

Even it there is a non-zero probability of return to the shock, for acceleration to continue this probability has to be very close to one (more precisely, it must be less than one by an amount of order $U/c$ where $U$ is the shock speed and $c$ is the velocity of light). Thus there can be a large intermediate range where particles are no longer being accelerated, but have not really escaped from the shock either (if one defines this as having negligible probability of coming back to the shock front). This can be shown more precisely using the so-called box model as was done in [3] and in more detail in [4] which should be consulted for a full discussion and references.

These calculations illustrate an important point. The break in the spectrum associated with the acceleration time scale becoming of order the dynamical time scale can be at a significantly lower energy than that at which particles are escaping from the shock and it is quite possible for a population of energetic particles to remain in the neighbourhood of the shock, and even return to it occasionally, without significant acceleration. This population will be geometrically diluted by the expansion of the shock and the increase in the upstream diffusion length (the region of space they fill continues to expand faster than they can be supplied by the on-going acceleration) and this process can rather naturally produce spectral breaks at the point where the acceleration is no longer in equilibrium with the expansion. The break in the spectrum reflects not just the geometrical dilution but also, and potentially of great interest, the time-history of injection at the shock.

This effect also explains an apparent paradox in the simple theory. If the maximum particle momentum is calculated as

$$p_{\text{max}} = \int_{0}^{t_{\text{acc}}} \frac{p}{t_{\text{acc}}(t')} dt'$$

(1)

where $t_{\text{acc}}$ is the acceleration time scale it is clearly a monotonic increasing function of time. However if one estimates the maximum momentum by arguing that the diffusion length has to be less than the shock radius, or equivalently that the local acceleration time has to be less that the dynamical time, this gives the condition

$$t_{\text{acc}} \approx 0.1 \frac{\kappa(p_{\text{max}})}{R^2} < \frac{R}{\dot{R}}$$

(2)

where $R$ is the shock radius and $\dot{R}$ its expansion speed. Thus on this condition the maximum momentum is implicitly determined by $\kappa(p_{\text{max}}) \approx 0.1 R \dot{R}$. For a Sedov blast wave the product $R \dot{R} \propto t^{-0.2}$ is a decreasing function of time (even if slowly) and as $\kappa$ is an increasing function of $p$ one expects that $p_{\text{max}}$ should actually decrease with time and not increase. The solution of course is that the spectrum has a break at the point given by equation (2) (where the acceleration is no longer able to be in equilibrium with the expansion), but continues to the higher energy given by the first condition of equation (1).

To some extent it thus comes down to semantics. One can have a perfectly consistent picture of active SNRs being surrounded by a halo of particles which were accelerated at earlier times to high energies, but which are no longer undergoing significant acceleration and are simply diffusing out into the Galaxy. But of course if this halo region were to become a large part of the Galaxy, and the probability of further interaction with the shock vanishingly small, then clearly it would make more sense to regard these simply as escaped particles. And of course once the shock becomes weak and the SNR effectively dies then all the particles, including those trapped in the interior, must eventually escape. Although very uncertain it is interesting to make some order of magnitude estimates. If the diffusion into the Galaxy occurs as indicated by studies of cosmic ray propagation, with a diffusion coefficient of order $10^{28}$ cm$^2$s$^{-1}$ for particles around a GeV and an energy dependence something like $E^{-0.5}$, then PeV particles from a thousand year old SNR would diffuse a distance of order $\sqrt{3} \times 10^{14}$ cm $\approx 180$ pc. Thus particles produced in a hypothetical early pevatron phase of a historical SNR will not have escaped into the general Galaxy but will fill a roughly spherical region around the SNR of radius a few hundred parsecs. This is of course the motivation for looking for high-energy emission from molecular cloud targets near SNRs.
3 Trapping inside the shock of low-energy particles

In fact for the bulk of the cosmic rays, and in particular for those which dominate the energy density and for which the composition is well determined, trapping until the SNR dies is undoubtedly the better picture. At these relatively low energies the planar shock approximation is good, the particles are accelerated at the shock and then swept downstream to accumulate in the interior of the remnant. Here of course they undergo adiabatic energy losses as the remnant expands, but the energy lost in this way goes to driving the shock and is thus recycled into the acceleration of new particles.

The acceleration depends crucially on having strong scattering in the neighbourhood (and in particular upstream) of the shock which produces a diffusion barrier at the shock preventing the escape of these low-energy particles. There are a wide variety of resonant and non-resonant instabilities which can produce the necessary magnetic turbulence and it is generally believed that the local diffusion coefficient at the shock can be driven down to values corresponding to Bohm scaling, that is a scattering mean free path of order the gyroradius. The sharp synchrotron rims observed in young remnants, originally in the radio and more recently in X-rays provide rather convincing direct observational evidence for such diffusion barriers associated with strong shocks. Were the diffusion typical of that inferred in the general ISM the rims would be much more extended and indeed as pointed out long ago Ginzburg shock acceleration would not be a viable mechanism for making the Galactic cosmic rays. Eventually however the shock will become too weak to sustain the required strong scattering, the diffusion barrier will collapse and at this point the particles in the interior will escape into the ambient medium as the shock (and by implication the SNR) dies.

3.1 Energy-dependent composition changes

This has the important consequence that at lower energies the cosmic ray composition should be dominated by particles accelerated just before the remnant dies (thereby suppressing any freshly synthesised component coming from the SN or its progenitor wind). This was originally discussed in a somewhat different context at the European Cosmic Ray Symposium held at Balatonfüred, Hungary, in 1994 [5]. The key point is that, as discussed above, the low energy particles trapped in the interior are subject to adiabatic losses on the dynamical time scale of the remnant, and that this energy is then recycled into producing freshly accelerated low-energy particles. Thus the dominant low-energy particle population is that accelerated within the most recent dynamical time-scale of the remnant. This energy argument is the more fundamental one, but in fact even if one simply looks at the amount of matter swept up, far more matter is swept up at late times when the remnant is large than the amount (of order the ejecta mass) swept up prior to the onset of the Sedov phase.

It is interesting to note that the picture is very different for the highest energy particles which have to be accelerated at the time when the shock is fastest (probably in the period leading up to the transition from free expansion to the Sedov phase) and it is tempting to speculate that this should produce slight compositional variations with energy. Indeed there appear to be such variations in recent experimental data, and we may well be seeing evidence of this effect. To put it succinctly, the composition at high energies should reflect acceleration early on prior to the mass sweep-up time when the ejecta interact with the same mass of swept-up material; the composition at low energies should reflect acceleration at the energy sweep-up time when the total thermal energy of the swept-up ambient material becomes comparable to the explosion energy (and the shock is weakening). In particular the apparently softer proton spectra compared to helium can be very easily explained by the older and weaker shocks propagating in a medium where more of the helium is neutral than in the strongly ionized environment of a young remnant where one also expects significant contamination by progenitor winds and mass loss.

References