Acceleration of Cosmic Rays in a System of Rotating Stars

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Abstract: We theoretically illustrate that the possible acceleration of cosmic rays passing through a kind of amplification channel induced by a system of rotating stars. Our analysis is based on the discrete kinetic model (considering Uehling-Uhlenbeck collision term) which has been used to study the propagation of plane (e.g., acoustic) waves in a system of rotating gases.

Keywords: Kinetic approach, wave amplification

1 Introduction

Cosmic rays : Radiation that enhances with altitude in the atmosphere were discovered by V F Hess in 1912, and it was clear by the early 1930s that this radiation comes from outer space. The all-particle energy spectrum of primary cosmic rays extends from 1 GeV (10^{12} eV) to above 10^{20} eV (100 EeV), the highest energies of known individual particles in the universe [1-3]. The highest-energy particles are so rare that they are detectable only by means of the giant cascades or extensive air showers they create in the atmosphere. Details of how these extensive air showers are observed and of how the parameters of importance are measured can be found in [3]. In spite of many efforts, we have still only a limited understanding of, where these particles are coming from, how they are accelerated to such extremely high energies, how they propagate through interstellar space [2].
The astrophysical environments that are able to accelerate particles up to such high energies, including active galactic nuclei, large scale galactic wind termination shocks, relativistic jets and hot-spots of Fanaroff-Riley radio galaxies, pulsars, magnetars, quasar remnants, starbursts, colliding galaxies, and gamma ray burst fireballs were discussed in [1]. Many researchers believed that cosmic rays are accelerated in a process called diffusive shock acceleration. Suitable astrophysical shocks occur in supernova explosions, and ionized nuclei gain energy as they are repeatedly overtaken by the expanding shock wave. Such mechanisms lead in fact to a power-law spectrum with a maximum energy of about $Z \times 10^{15}$ eV (Z : atomic number), which roughly agrees with the observed steepening (though the theoretically predicted spectrum proves to be steeper than actually observed) [2].

In this short paper, the discrete Uehling-Uhlenbeck equations [4] together with the model of free orientations (θs, the relative direction of scattering of particles w.r.t. to the normal of the propagating plane-wave front) are solved to study the diverse dispersion relations of plane waves in system of rotating stars (or disk-like gases). Anomalous amplification channels could occur for diffusion modes of propagating plane-wave fronts. We propose that the acceleration of cosmic rays passing through this kind of channel might happen.

2 Theoretical Formulations

We make the following assumptions before we investigate the general equations of our model :
(1) Consider a system of rotating stars (or disks) or a gas of identical particles of unit mass and a shape of a disk of diameter d, then each particle (or star) i, $i = 1, \cdots, N$, is characterized by the position of its center q_i and its velocity u_i. We also have the geometric limitations : $|q_i - q_j| \geq d$, $i \neq j$.
(2) Each particle (or star) moves in the plane with velocity belonging to a discrete set $\mathcal{V}$ of 4 velocities with only one speed in the plane. The velocity modulus c is a reference speed depending on the reference frame and specific distribution of particles. c is normally linked to the internal energy of the molecules in thermodynamic equilibrium.
(3) The collisional mechanism is that of rigid spheres, that is, the particles (or stars) scatter elastically and they change their phase states instantaneously, preserving momentum. Only binary collisions are considered, since a multiple collision here is a negligible events.
The collisions between two particles (or stars, say i and j)

take place when they are located at q_i and $q_j = q_i - d\mathbf{n}$, where $\mathbf{n}$ is the unit vector joining their centers. After collisions the particles scatter, preserving momentum, in the directions allowed by the discrete set $\mathcal{V}$. In other words, particles change according to

$$(q_i, u_i) \to (q_i, u_i^*), \qquad (q_j, u_j) \to (q_j, u_j^*).$$

The collision is uniquely determined if the incoming velocity and the impact angle ψ, $\psi \in [-\pi/2,\pi/2]$, are known, which is defined as the angle between u_i and $\mathbf{n}$ or $\mathbf{n}(\psi) = (\cos[\psi + (k-1)\pi/2], \sin[\psi + (k-1)\pi/2])$, $k = 1, \cdots, 4$ ($k = 4$ means during binary encounter there are, in general, two incoming velocities and two departing velocities). From the selected velocities we have two classes of encounters, i.e. (a) $\langle u_i, u_j\rangle = 0$ and (b) $\langle u_i, u_j\rangle = -c^2$, respectively.

(a). In the first class momentum conservation implies only : encounters at $\pi/2$ with exchange of velocities

$$u_i = u^k \to u_i^* = u^{k+1}, \qquad u_j = u^{k+1} \to u_j^* = u^k,$$

in the case $\psi \in [-\pi/2, 0]$, and

$$u_i = u^k \to u_i^* = u^{k+3}, \qquad u_j = u^{k+3} \to u_j^* = u^k,$$

in the case $\psi \in [0, \pi/2]$.

(b) Similarly, $\langle u_i, u_j\rangle = -c^2$;

(i) Head-on encounters with impact angle $\psi = 0$ such that

$$u_i = u^k \to u_i^* = u^{k+2}, \qquad u_j = u^{k+2} \to u_j^* = u^k,$$

(ii) Head-on encounters with impact angle $\psi \neq 0$ such that if $\psi \in [-\pi/2)0]$:

$$u_i = u^k \to u_i^* = u^{k+1}, \qquad u_j = u^{k+2} \to u_j^* = u^{k+3},$$

or if $\psi \in [0, \pi/2]$:

$$u_i = u^k \to u_i^* = u^{k+3}, \qquad u_j = u^{k+2} \to u_j^* = u^{k+1}.$$

For grazing collisions, that is $\langle \mathbf{n}, u_i\rangle = \langle \mathbf{n}, u_j\rangle = 0$, we put $u_i^* = u_i$, $u_j^* = u_j$. Schematic presentation is illustrated in Fig 1. $\mathbf{M}$ and $\mathbf{M}_1$ are the associated angular momenta which have opposite sign.

With general collision rules, we assume that the system of hard-disk gas (star) is composed of identical particles (stars) of the same mass. The velocities of these particles are restricted to, e.g., : $\mathbf{U}_1, \mathbf{U}_2, \cdots, \mathbf{U}_p$, p is a finite positive integer. The discrete number density of particles (or stars) are denoted by $N_i(\mathbf{x}, t)$ associated with the velocity $\mathbf{U}_i$ at point $\mathbf{x}$ and time t. If only nonlinear binary collisions are considered, and considering the evolution of N_i ($i = 1, \cdots, p$), we have

$$\frac{\partial N_i}{\partial t} + \mathbf{u}_i \cdot \nabla N_i = \sum_{j,k,l} A_{kl}^{ij} \left[N_k N_l (1 + \sigma N_i)(1 + \sigma N_j) - N_i N_j (1 + \sigma N_k)(1 + \sigma N_l)\right] N_i N_j), \qquad (1)$$

where (k, l) are admissible sets of collisions, $i \in \Lambda$ $=\{1, \cdots, p\}$, and the summation is taken over all $j, k, l \in \Lambda$, where A_{kl}^{ij} are nonnegative constants satisfying $A_{kl}^{ji} = A_{kl}^{ij} = A_{lk}^{ij}$: indistinguishability of the particles in collision, $A_{kl}^{ij}(\mathbf{u}_i + \mathbf{u}_j - \mathbf{u}_k - \mathbf{u}_l) = 0$: conservation of momentum in the collision, $A_{kl}^{ij} = A_{ij}^{kl}$: microreversibility condition. Here, σ is a Pauli blocking parameter [4], and, for $\sigma < 0$ (normally, $\sigma = -1$) we can obtain a gas of Fermi-Dirac particles (or stars); for $\sigma > 0$ (normally, $\sigma = 1$) we obtain a gas of Bose-Einstein particles (or stars), and for $\sigma = 0$, we obtain a gas of Boltzmann particles (or stars). The conditions defined for the discrete velocity above requires that elastic, binary collisions, such that momentum and energy are preserved.

Considering binary (two-disk encounter each time) collision only, with $\sigma = 0$, the equation of discrete kinetic models proposed in [4] is a system of $2n(= p)$ semilinear partial differential equations of the hyperbolic type : ($i = 1, \cdots, 2n$,)

$$\frac{\partial}{\partial t} N_i + \mathbf{U}_i \cdot \frac{\partial}{\partial \mathbf{x}} N_i = \frac{2cS}{n} \sum_{j=1}^{n} N_j N_{j+n} - N_i N_{i+n}, \quad (2)$$

where $N_i = N_{i+2n}$ are unknown functions, and $\mathbf{U}_i$ $=c(\cos[\theta + (i-1)\pi/n], \sin[\theta + (i-1)\pi/n])$; c is a reference velocity modulus, S is an effective collision cross-section for the 2-(rotating)disk system, θ is the free orientation parameter (the orientation starting from the positive x-axis to the U_1 direction and is relevant to the (net) induced scattering measured relative to the sound-propagating direction) which might be linked to the external field or the angular momentum or the rotation effects.

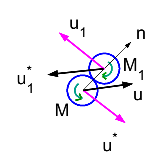

Figure 1: Schematic plot for a collision. $\mathbf{M}$ and $\mathbf{M}_1$ are the associated angular momenta which have opposite sign.

Since passage of the plane (e.g., acoustic) wave causes a small departure from equilibrium (Maxwellian type) resulting in energy loss owing to internal friction and heat conduction, we linearize above equations around a uniform Maxwellian state (N_0) by setting $N_i(t, x)$ =N_0 $(1 + P_i(t, x))$, where P_i is a small perturbation. (Maxwellian is referred to an equilibrium state here)

Firstly, we have, (say, i=m)

$$\frac{\partial P_m}{\partial t} + \mathbf{U}_m \cdot \frac{\partial P_m}{\partial \mathbf{x}} + 2cSN_0[(P_m + P_{m+n}) = \frac{cSN_0}{n} \sum_{k=1}^{2n} [(P_k + P_{k+n}], \qquad (3)$$

here, $m = 1, \cdots, 2n$. The linearized version of above equation is (set $f_p = 2cSN_0$)

$$\frac{\partial P_m}{\partial t} + \mathbf{U}_m \cdot \frac{\partial P_m}{\partial \mathbf{x}} + f_p(P_m + P_{m+n}) = \frac{f_p}{n} \sum_{k=1}^{2n} P_k. \quad (4)$$

In these equations after replacing the index m with $m+n$ and using the identities $P_{m+2n} = P_m$, then we have

$$\frac{\partial P_{m+n}}{\partial t} - \mathbf{U}_m \cdot \frac{\partial P_{m+n}}{\partial \mathbf{x}} + f_p(P_m + P_{m+n}) = \frac{f_p}{n} \sum_{k=1}^{2n} P_k. \quad (5)$$

Combining above two equations, firstly adding then subtracting, with $A_m = (P_m + P_{m+n})/2$ and $B_m = (P_m - P_{m+n})/2$, we can have $(m = 1, \cdots, 2n)$

$$\frac{\partial A_m}{\partial t} - c \cos[\theta + \frac{(m-1)\pi}{n}] \frac{\partial B_m}{\partial x} + 2 f_p A_m = \frac{2 f_p}{n} \sum_{k=1}^{2n} A_k, \quad (6)$$

$$\frac{\partial B_m}{\partial t} + c \cos[\theta + \frac{(m-1)\pi}{n}] \frac{\partial A_m}{\partial x} = 0. \quad (7)$$

Note that terms of $\partial P_m / \partial y = 0$ as P_m only varies along the wave propagating direction : x-axis direction. From $P_{m+2n} = P_m$, and with $A_m = (P_m + P_{m+n})/2$ and $B_m = (P_m - P_{m+n})/2$, we can have $A_{m+n} = A_m$, $B_{m+n} = -B_m$.

After some manipulations we then have

$$\left(\frac{\partial^2}{\partial t^2} + c^2 \cos^2 \left[\theta + \frac{(m-1)\pi}{n} \right] \frac{\partial^2}{\partial x^2} + 4cSN_0 \frac{\partial}{\partial t} \right) D_m = \frac{4cSN_0}{n} \sum_{k=1}^{n} \frac{\partial}{\partial t} D_k, \quad (8)$$

where $D_m = (P_m + P_{m+n})/2$, $m = 1, \cdots, n$, since $D_1 = D_m$ for $1 = m$ (mod $2n$).

Now we are ready to look for the solutions in the form of plane wave $D_m = a_m \exp{[i(kx - \omega t)]}$, $(m = 1, \cdots, n)$, with $\omega = \omega(k)$. This could be related to the dispersion relations of 1D forced plane-wave (e.g., ultrasound) propagation (of dilute monatomic hard-sphere gases) problem. So we have

$$\left(1 + ih - 2\lambda^2 \cos^2 \left[\theta + \frac{(m-1)\pi}{n} \right] \right) a_m - \frac{ih}{n} \sum_{k=1}^{n} a_k = 0, \quad (9)$$

where $\lambda = kc/(\sqrt{2}\omega)$, $h = 4cSN_0/\omega \propto 1/\mathrm{K}_n$ is the rarefaction parameter of the gas; K_n is the Knudsen number which is defined as the ratio of the mean free path of hard-disk gases to the wave length of the plane (e.g., acoustic) wave (here $m = 1, \cdots, n$).

Let $a_m = \mathcal{C}/(1 + ih - 2\lambda^2 \cos^2[\theta + (m-1)\pi/n])$, where $\mathcal{C}$ is an arbitrary, unknown constant, since we here only have interest in the eigenvalues of above relation. The eigenvalue problems for different $2 \times n$-velocity model reduces to $F_n(\lambda) = 0$, or

$$1 - \frac{ih}{n} \sum_{m=1}^{n} \frac{1}{1 + ih - 2\lambda^2 \cos^2 [\theta + \frac{(m-1)\pi}{n}]} = 0. \quad (10)$$

We solve $n = 2$ case, i.e., 4-velocity case. The admissible collision : $(1,3) \longleftrightarrow (2,4)$ for system of rotating disks during binary encounter is shown schematically in Fig. 2. The corresponding eigenvalue equations become algebraic polynomial-form with the complex roots being the results of λs.

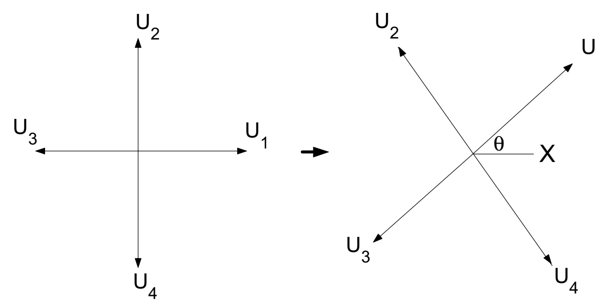

Figure 2: Schematic plot for the regular scattering and the orientational scattering. Plane waves propagate along the X-direction. Binary encounters of U_1 and U_3 and their departures after head-on collisions (U_2 and U_4). Number densities N_i are associated to U_i. θ is the free orientation parameter (the orientation starting from the positive x-axis to the U_1 direction and is relevant to the (net) induced scattering measured relative to the plane wave-propagating direction) which might be linked to the external field or the angular momentum or the rotation effects.

3 Numerical Results and Discussion

Using the mathematical or numerical software, e.g. Mathematica or Matlab, and after intensive validations, we can obtain the complex roots ($\lambda = \lambda_r +$ i λ_i) for the polynomial equations above. The roots are the values for the nondimensionalized dispersion (positive real part) and the attenuation or absorption (positive imaginary part), respectively. We plot those of $\theta = 0$ into Fig. 3. Curves of branch I follows the conventional dispersion relation of plane-wave (e.g., ultrasound) propagation in dilute monatomic hard-sphere gases [5]. We remind the readers that θ is relevant to the external field or the angular momentum or the rotation effects and possible effects of $\theta \neq 0$ could be traced in [3]. Curves of branch II, however, show a entirely different trend. The dispersion part seems to follow the diffusion mode reported in [5]. It increases but never reaches to a limit. The anomalous attenuation or amplification might be due to, if any, the intrinsic resonance (an eigen-oscillation) or the implicit behavior of angular momentum relation during 2-(rotating)disk (or star) encounter (each with opposite-sign rotating direction or angular momenta so that the total (net) angular momenta for this two-body encounter is zero) since the latter is absent or of no need in the formulation of 2-body collisions. We note that the direction of the rotational axis of each disk (or star) (ready to encounter) for the collision of 2-(rotating)disk system might be in opposite sign instead of the same sign. We don't know yet at present whether the former or the latter can favor the anomalous attenuation or amplification?

We noticed that some researchers argue that there must be some source of free energy to drive the growth. Meanwhile as argues in [6], the rotational energy of a young pulsar with period P that remains after the supernova explosion is estimated to be $2 \times 10^{50}(10 \text{ ms}/P)^2$ erg. It is an additional energy reservoir for particle acceleration (note that for the energy in a source capable of accelerating particles to 10^{20} eV, considering the energy in the magnetic field (< 0.1 G) : $\gg 10^{57}$ ergs [3]) and in particular it could be the source of very high energy electron-positron pairs. However, the cosmic ray passing through this plane-wave(e.g., acoustic)-amplified channel might be accelerated (neutrinos included)! In fact it was remarked in [7] : *Cosmic rays must be involved in the general Galaxy rotation* $\cdots$. Finally the results presented in Fig. 3 shows that the possible acceleration (due to amplification) is proportional to $h(\propto c\,S\,N_0)$ which is also proportional to Z (a nucleus charge number, considering the effective scattering cross-section S). The latter matches qualitatively with [8] : The PAO data strongly favor the nuclei composition getting progressively heavier at energy $\simeq (4-40)$ EeV.

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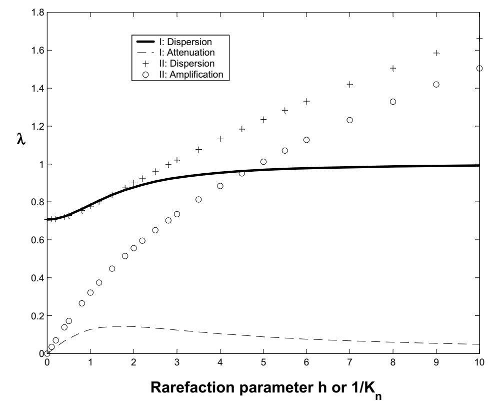

Figure 3: Dispersion relations of branches I and II over a range of h (rarefaction measure). λ_r : phase speed dispersion, λ_i : attenuation or amplification. $\lambda = \lambda_r +$ i λ_i. $\theta = 0$ here. Branch II (diffusion mode) shows anomalous amplification.

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