Determining the high energy neutrino flavor ratio at the astrophysical source

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Abstract: We discuss the reconstruction of neutrino flavor ratios at astrophysical sources through the future neutrino-telescope measurements. We demonstrate by a statistical method the accuracies required in the measurements of energy-independent ratios R and S, where R is the number ratio between track and shower events. For energy below a few PeV, shower events consist of electron and tau neutrino events. However, for energies beyond, a tau-lepton behaves like a track similar to a muon. This motivates a new classification of event types and new definitions for R and S. We present our analyses in both types of measurements, R and S, at corresponding energy regime.

Keywords: Astrophysical neutrino, neutrino oscillation, neutrino astronomy.

1 Introduction

The operation of IceCube detector [1] and the R&D effort of KM3Net [2] are important progresses toward a km³-sized detection capability in the neutrino astronomy [3]. Furthermore the radio and air-shower detectors, such as ANITA [4] and Pierre Auger detector [5] respectively, are also taking the data. These detectors are sensitive to neutrinos with energies higher than those probed by IceCube and KM3Net. Finally, the radio extension of IceCube detector, the ARA [6], is also under consideration. It is expected to detect a score of cosmogenic neutrinos [7] per year.

Most of the astrophysical neutrinos are believed to be produced by decays of charged pions and subsequent decays of muons: \( \pi^+ \rightarrow \mu^+ + \nu_{\mu} \rightarrow e^+ + \nu_{\mu} + \nu_{\tau} + \bar{\nu}_{\mu} \) or \( \pi^- \rightarrow \mu^- + \bar{\nu}_{\mu} \rightarrow e^- + \bar{\nu}_{\mu} + \bar{\nu}_{e} + \nu_{\mu} \). This leads to the neutrino flux ratio \( \phi_0(\nu_{\mu}) : \phi_0(\nu_{\tau}) : \phi_0(\nu_{e}) = 1 : 2 : 0 \) at the astrophysical source where \( \phi_0(\nu_{\alpha}) \) is the sum of \( \nu_{\alpha} \) and \( \bar{\nu}_{\alpha} \) flux. Such a flux ratio results from an implicit assumption that the muon decays into neutrinos before it loses a significant fraction of its energy. However, in some source the muon quickly loses its energy by interacting with strong magnetic fields or with matter [8, 9, 10]. Such a muon eventually decays into neutrinos with energies much lower than that of \( \nu_{\mu} (\bar{\nu}_{\mu}) \) from \( \pi^+ (\pi^-) \) decays. Consequently this type of source has a neutrino flavor ratio \( \phi_0(\nu_{\tau}) : \phi_0(\nu_{\mu}) : \phi_0(\nu_{e}) = 0 : 1 : 0 \), which is referred to as the muon-damped source. The third type of source emits neutrons resulting from the photo-disassociation of nuclei. As neutrons propagate to the Earth, \( \nu_{e} \) are produced from neutron \( \beta \) decays [11], leading to a neutrino flavor ratio \( \phi_0(\nu_{\tau}) : \phi_0(\nu_{\mu}) : \phi_0(\nu_{e}) = 1 : 0 : 0 \).

A natural question in neutrino astronomy is then how well one can identify and distinguish these neutrino sources. The answer to this question depends on our knowledge of neutrino mixing parameters and the achievable accuracies in measuring the neutrino flavor ratio on the Earth such as \( R \equiv \phi(\nu_{\mu}) / (\phi(\nu_{\tau}) + \phi(\nu_{e})) \) and \( S \equiv \phi(\nu_{e}) / \phi(\nu_{\tau}) \). In this report, we shall provide an answer to this question with a statistical analysis.

2 Statistical Analysis

Before doing the statistical analysis, we should note that the observables \( R \) and \( S \) are energy-dependent since \( R \) is defined as \( N_{\text{track}} / N_{\text{shower}} \). In very high energy regime, i.e., \( E_{\nu} > 33 \text{PeV} \), the tau lepton originating from the tau neutrino behaves more like a track rather than a shower while the electron neutrino only gives rise to a shower signature. Therefore the more appropriate flux ratio parameters in such a case are \( R \equiv \phi(\nu_{\mu}) / (\phi(\nu_{\tau}) + \phi(\nu_{e})) \) and \( S \equiv \phi(\nu_{e}) / \phi(\nu_{\tau}) \). For simplicity and clarity, we denote the measurement in the low and high energy regimes as conditions I and II, respectively.

To reconstruct the neutrino flavor ratio at the source with a statistical analysis, we employ the following best-fit values and \( 1\sigma \) ranges of neutrino mixing angles [12]

\[
\sin^2 \theta_{12} = 0.23^{+0.07}_{-0.08}, \quad \sin^2 \theta_{23} = 0.56^{+0.17}_{-0.16},
\]

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\[ \sin^2 \theta_{13} < 0.012(0.047), \]  

for our analysis.

The statistical analysis is then performed with the following formula

\[
\chi^2 = \left( \frac{R_{\text{th}} - R_{\text{exp}}}{\sigma_{R_{\text{exp}}}} \right)^2 + \left( \frac{S_{\text{th}} - S_{\text{exp}}}{\sigma_{S_{\text{exp}}}} \right)^2 + \sum_{jk=12,23,13} \left( \frac{s_{jk}^2 - (s_{jk})_{\text{best fit}}^2}{\sigma_{s_{jk}}^2} \right)^2 \]  

with \( \sigma_{R_{\text{exp}}} = (\Delta R/R) R_{\text{exp}}, \sigma_{S_{\text{exp}}} = (\Delta S/S) S_{\text{exp}}, s_{jk}^2 \equiv \sin^2 \theta_{jk} \) and \( \sigma_{s_{jk}}^2 \) the 1\( \sigma \) range for \( s_{jk}^2 \). Here \( R_{\text{th}} \) and \( S_{\text{th}} \) are theoretical predicted values for \( R \) and \( S \) respectively while \( R_{\text{exp}} \) and \( S_{\text{exp}} \) are experimentally measured values. The values for \( R_{\text{exp}} \) and \( S_{\text{exp}} \) are generated from input true values of neutrino flavor ratios at the source and input true values of neutrino mixing parameters as given by Eq. 1. We assumes that both \( \Delta R \) and \( \Delta S \) are dominated by the statistical errors. In this case, they are related to each other by [13]

\[
\left( \frac{\Delta S}{S} \right) = \frac{1 + S}{\sqrt{S}} \sqrt{\frac{R}{1 + R}} \left( \frac{\Delta R}{R} \right). \]  

In our analysis, we can scan all possible neutrino flavor ratios at the source that give rise to a specific \( \chi^2 \) value. Since we have taken \( R_{\text{exp}} \) and \( S_{\text{exp}} \) as those generated by input true values of initial neutrino flavor ratios and neutrino mixing parameters, we have \( (\chi^2)_{\text{min}} = 0 \) occurring at these input true values of parameters. Hence the boundaries for 1\( \sigma \) and 3\( \sigma \) ranges of initial neutrino flavor ratios are given by \( \Delta \chi^2 = 2.3 \) and \( \Delta \chi^2 = 11.8 \) respectively where \( \Delta \chi^2 \equiv \chi^2 - (\chi^2)_{\text{min}} = \chi^2 \) in our analysis.

Let us take the accuracy for measuring \( R \) to be \( \Delta R/R = 10\% \) in both low and high energy regimes for the damped-muon source. The value for \( \Delta S/S \) can be calculated from Eq. 3. The reconstruction of neutrino flavor ratio with the above given \( \Delta R/R \) and \( \Delta S/S \) is discussed in the follow.

### 2.1 The reconstruction of initial neutrino flavor ratio by measuring \( R \) alone

It is instructive to see how well one can determine the initial neutrino flavor ratio by measuring \( R \) alone. We perform such an analysis by neglecting the second term on the RHS of Eq. 2. The 1\( \sigma \) and 3\( \sigma \) ranges for the reconstructed flavor ratios at the source are shown in Fig. 1. It is seen that, with \( \Delta R/R = 10\% \), the reconstructed 3\( \sigma \) range of the neutrino flavor ratio almost covers the entire physical region in low energy regime. Hence the possibility of a pion source cannot be ruled out given a true source of the damped-muon source. Clearly it is desirable to measure both \( R \) and \( S \) to rule out the pion source from the damped-muon source at low energies. On the contrary, we see that the pion source can be ruled out barely at high energies.
3 Discussion and Conclusion

In the tri-bimaximal limit of neutrino mixing parameters \[14\], \(\sin^2\theta_{23} = 1/2, \sin^2\theta_{12} = 1/3\) and \(\sin^2\theta_{13} = 0\), we find \(R^I = S^I/2\) and \(S^{II} = 1\). For further simplifications, let us consider astrophysical sources with negligible \(\nu_\tau\) fractions so that \(\phi_0(\nu_e) : \phi_0(\nu_\mu) : \phi_0(\nu_\tau) = \alpha : 1 - \alpha : 0\) with 0\(\leq\alpha\leq1\). Fig. 3 shows the flux ratios on the Earth as functions of the initial \(\nu_e\) fraction \(\alpha\). It is seen that \(S^I\) and \(R^{II}\) are more sensitive to \(\alpha\) while \(R^I\) and \(S^{II}\) is either less sensitive to \(\alpha\) or completely independent of this parameter.

![Figure 3: Neutrino flavor ratios on Earth for input sources with flavor ratios of \(\phi_0(\nu_e) : \phi_0(\nu_\mu) : \phi_0(\nu_\tau) = \alpha : 1 - \alpha : 0\) with 0\(\leq\alpha\leq1\). The thick and long dashed lines correspond to \(R^I\) and \(S^{II}\) respectively. The thin and short-dashed lines correspond to \(R^I\) and \(S^I\) respectively. Apparantly, \(S^I\) and \(R^{II}\) are more sensitive to \(\alpha\).](image)

In this report, we only show ternary diagrams for the assumed damped-muon source due to the limitation of pages. The occasion should vary for the pion source. We have also take into consideration nonzero \(\sin^2\theta_{13}\). Since the best-fit value for \(\theta_{13}\) is non-vanishing suggested by \[15\], the reconstructed flavor ratios should also depend on the CP phase. For the details of these studies, please see also \[16\] and \[17\].

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