Constraints on the origins of the ultra-high energy cosmic-rays using the IceCube diffuse neutrino limits : An analytical approach

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Abstract: Astrophysical neutrinos are expected to be produced in the interactions of the ultra-high energy cosmic-ray protons with surrounding photons. Therefore astrophysical neutrino model fluxes are highly dependent on characteristics of the cosmic-ray sources, such as their cosmological distributions and the photon field in each sources. We study possible constraints on the properties of cosmic-ray sources in a model-independent way using the recently obtained upperlimit on the diffuse flux of neutrinos with energies above 100 PeV by the IceCube detector. The semi-analytic formula is derived to estimate the fluxes of cosmogenic neutrinos as functions of source evolution parameter and source extension in redshift. Then the obtained formula enables us to convert the upperlimits on the neutrino fluxes into the constraints on the cosmic-ray sources. It is found that the recently obtained upperlimit on the cosmogenic neutrinos by IceCube tightly constrains the very high source evolution scenarios. The derived analytic formula also shows that the future limits from the 1km\textsuperscript{3} scale detectors such as IceCube and KM3NET are able to further constrain the the ultra-high energy cosmic-rays sources with evolutions comparable to the cosmic star formation rate.

Keywords: Ultra-high energy cosmic rays; neutrinos

1 Introduction

The origin of the ultra-high energy cosmic-rays (UHE-CRs) has still been a long-standing puzzle in astro-particle physics. The recent observations by the Auger collaboration \cite{1, 2} and by the HiRes collaboration \cite{3} indicated that the most energetic population seems propagating from extra-galactic space, but identification of the astronomical object classes responsible for UHECR emission is far from being established. A part of the difficulties arises from our poor knowledges about mass composition of UHECRs and (inter-)galactic magnetic field configuration, both of which strongly influences UHECR particle trajectories from their origin to the earth. From this view point, neutrinos secondary produced by UHECR nucleons provide complementary informations on the UHECR origin. They are penetrating over cosmological distances without being deflected by cosmic magnetic field. The intensity of the “cosmogenic” neutrino \cite{4} produced by the GZK mechanism, the collisions of UHECR nucleons with the cosmic microwave background (CMB) photon via photo-produced $\pi$ meson decay as $\pi^{\pm} \rightarrow \mu^{\pm} \nu_{\mu} \rightarrow e^{\pm} \nu_{e} \nu_{\mu}$, carries information on distribution of the UHECR sources \cite{5}. Measurement or upperlimit of these ultra-high energy neutrino flux leads to direct constraints on UHECR origin without relying on uncertain estimation of magnetic fields and extra-galactic background light (EBL), in contrast to the constraints by the diffuse photon flux \cite{6}. Estimations of the neutrino fluxes with the entire parameter space of cosmogenic neutrinos are able to convert the observationally obtained upperlimits of neutrino flux into the constraints on the UHECR sources.

In this work, we derive the analytical formula to approximately calculate intensities of the secondary produced neutrinos such as the cosmogenic neutrinos. The comparison of the predicted intensity assuming various parameters on the UHECR source distribution such as its evolution to the recently published upperlimit \cite{7} and the future sensitivity \cite{8} on detection of cosmic neutrino flux in energy range around EeV ($= 10^9$ GeV) by the IceCube neutrino observatory then conduces to model independent constraints on the UHECR origins. Our approach aims at bounding the UHECR source evolution and its redshift dependence in as much comprehensive way as possible without introducing any models on specific astronomical objects such as the star formation evolution. Using the analytical formula is especially suitable for this purpose as it allows to calculate neutrino intensities in the full phase space of the source evolution without an intensive computational task. It also provides an easy-to-handle tool to approximately calculate neutrino intensity as benchmark estimation to interpret the present and future results of ultra-high energy cosmic neutrinos by IceCube and KM3NET \cite{9}.


2 Analytical function to calculate $\nu$ flux

2.1 Formulation

The neutrino flux per unit energy $dJ_\nu/dE_\nu$ is generally obtained by

$$
\frac{dJ_\nu}{dE_\nu} = \frac{c}{H_0}n_0 \int_0^z dz (1+z)\sqrt{\Omega_m (1+z)^3 + \Omega_k} \int_0^z d\nu^\prime \frac{dN_{\nu^\prime}}{dE_\nu dL} (E_\nu (1+z^\nu), z^\nu, z) \frac{d\nu^\prime}{dz^\nu}.
$$

(1)

The first integral adds neutrinos generated by cosmic ray protons emitted from sources at redshift $z$. $\psi(z)$ represents the cosmic evolution of the spectral emission rate per comoving volume. The second integral calculates total neutrino flux originated in a single source at redshift $z$ by summing up neutrinos generated by the UHECR interactions at redshift $z_0 (\leq z)$. $dN_{\nu^\prime}/dE_\nu dL$ is the yield of produced neutrinos with energy $E_\nu$ per unit UHECR propagation length. It is determined by the UHECR intensity from the source at $z$, the CMB photon density, and the photo-pion interaction dynamics, for example, in case of the cosmicogenic neutrino production. $H_0$ is the Hubble constant, $c$ is the speed of light, $n_0$ represents the UHECR local source density at the present universe, i.e., $z = 0$, which is determined by the observed UHECR flux in the end. Following our policy of the present analysis valuing comprehensiveness and simplicity, we parameterize $\psi(z)$ as $(1+z)^m$ so that the parameter $m$ can represent “scale” of the cosmic evolution often used in the literature.

The integrations in Eq. 1 are found to be semi-analytically computable for the cosmogenic neutrino model when the following approximations are introduced.

1. neglect the contribution of UHECR colliding with IR/O background.
2. consider photo-pion production only from the $\Delta$ resonance region.
3. simplify the kinematics of the photo-pion production as a single pion production.
4. approximate the energy attenuation length, $\lambda_{\text{GZK}}$, of UHECRs as a constant with energies above $10^{20}$ eV.

Then we get

$$
\frac{dJ_\nu}{dE_\nu} = (\alpha - 1) E_{\text{GZK}}^{-1} F_{\text{CR}} c \frac{k_B T}{H_0} \frac{s_R - m_r^2}{4 k_B T} \frac{(s_R - m_r^2)^{-(\alpha - 2)}}{(s_R + m_r^2 - m_\pi^2)^2 - 4s_R m_r^2} \frac{\Delta S_R}{4 k_B T} \sqrt{(s_R + m_r^2 - m_\pi^2)^2 - 4s_R m_r^2} 1 - r_x \zeta.
$$

(2)

as the neutrino all-flavor intensity. Here $F_{\text{CR}}$ is the UHECR intensity with energy above the reference GZK energy $E_{\text{GZK}} = 10^{20}$ eV, $\alpha$ is the spectral index of UHECRs, $k_B$ is the Boltzmann constant, $T$ is the CMB temperature, $s_R = 1.47$ GeV$^2$ is the Lorentz-invariant Mandelstam variable, the square of invariant mass at the $\Delta$ resonance, $\Delta S_R = 0.6$ GeV$^2$ is the width of the $\Delta$ resonance, $\sigma_{\nu p}^R = 2.1 \times 10^{-21}$ cm$^2$ is the photo-nucleon production cross section of channel $\gamma p \rightarrow n\pi^+$, $r_x = m_R^2/m_\pi^2 \simeq 0.57$ is the muon-to-pion mass squared ratio. $\zeta$ is the term to account the evolution dependence and given by

$$
\zeta = e^{-2} \frac{1}{\gamma_e} \frac{\Omega_{m0}^{2/3}}{\nu (\Omega_{m0} (1+z_0)^3 + \Omega_k)} \left( \ln \left( \frac{\nu}{\eta_{R}} \right) \right) \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right) \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right) + \frac{2}{\gamma_e} \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right) \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right)
$$

(3)

$$
\gamma_m = \frac{2(\alpha + m - 3)}{2}
$$

$$
\eta_{\pm} = \frac{\eta_{\pm} \eta_{R}}{\eta_{R}}
$$

(4)

$$
\frac{x_
u}{x_\nu} = \frac{1}{x_\nu} \left( \frac{\nu}{\eta_{\pm} \eta_{R}} \right) \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right) \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right) + \frac{2}{\gamma_e} \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right) \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right)
$$

Here $x_\nu = 0.275$, and $x_\nu = 0.16$ are the numerical constants. The bounds of the redshifts, $z_{\text{up}}$, and $z_{\text{down}}$ are associated with $z_{\text{max}}$ in Eq. 1, the maximum redshift of the UHECR sources, but depend also on neutrino energy $E_\nu$ due to kinematics of the $\pi$ decay and the redshift energy loss. They are given by

$$
\begin{align*}
\eta_{\pm} & = \frac{\ln \left( \frac{\nu}{\eta_{\pm} \eta_{R}} \right) + \frac{\nu - 1}{\nu}}{\nu} \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right) + \frac{2}{\gamma_e} \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right) \left( \frac{\ln \left( \nu \right) + \frac{\nu - 1}{\nu}}{\nu} \right) \\
\end{align*}
$$

(5)

The UHECR intensity $F_{\text{CR}}$ in Eq. 2 is taken from the measurement of the HiRes experiment [3] in the present study and obtained to be $1.76 \times 10^{-21}$ cm$^{-2}$ sec$^{-1}$ sr$^{-1}$. Note that the case when the evolution function $\psi(z)$ becomes constant above a certain redshift can be also analytically calculated with a minor modification into $\zeta$.

2.2 Validity assessment of the analytical method

The introduced approximations to derive the analytical formula induce a certain inaccuracy. Nevertheless we illustrate a key point that the formula reasonably describes the flux in energy regime from $\sim 100$ PeV up to $\sim 10$ EeV, the central energy range of the IceCube cosmogenic neutrino search [7, 10]. The contribution of neutrino generation by UHECR colliding IR/O becomes only sizable in energy region below 100 PeV [11]. The neutrinos from
photo-produced pions above the \( \Delta \) resonance are mostly visible only in the lower energy range below 100 PeV [12], and the single pion production is the most dominated channel in the \( \Delta \) resonance. The detailed behavior of UHECR proton propagation in extra-galactic space is not a deciding factor in neutrino intensity below 10 EeV because these neutrinos are mostly generated at cosmological distances away, which is substantially much longer than the UHECR proton energy attenuation length in the CMB field. This fact deeply involves the well known findings that the cosmogenic flux above 10 EeV is sensitive to \( E_{\text{max}} \), the maximal injection energy of UHECR protons from their sources, and the spectral index of UHECR spectrum \( \alpha \), but the flux below 10 EeV is rather insensitive to these parameters, but mostly determined by the source evolutions which describes history of the UHECR emissivity in the cosmological time scale [5, 13]. Reference [11] has extensively scanned many parameter space of the cosmogenic neutrinos with a numerical Monte-Carlo method and shown that the intensity around 1 EeV is indeed robust against \( E_{\text{max}} \) and models of transition between the Galactic and extra-galactic cosmic-ray components. Reference [6] has also calculated \( \nu \) fluxes under the various parameter space that is consistent with the UHECR observed spectrum and exhibited that the neutrino intensities below 10 EeV for the similar source evolutions agree each other well within a factor of two in various \( \alpha \) and the different transition scenario unless one does not make extreme assumption (for example, very hard UHECR spectrum with transition to extra-galactic component in higher energies). These observations are also suggested by the fact that the neutrino intensities at O(EeV) predicted by the relatively old works [5, 12] assuming harder UHECR spectrum of \( \alpha = 2.0 \) and higher \( E_{\text{max}} \), and those by the recent works [6, 11] giving the nod to \( \alpha \sim 2.5 \) are consistent within a factor of two for comparable scale of the source evolution.

For further demonstration, we compared the cosmogenic neutrino integral flux above 1 EeV obtained by the full numerical calculations with that given by the present analytical formula with the same/comparable source evolution parameters. The results are listed in Table 1. We use \( \alpha = 2.5 \) in the analytical formula. They are comparable each other for wide range of parameters mostly within a factor of two. The similar agreement is found in the intensity at 100 PeV. It is confirmed that the present formula provides a reasonable estimate of the neutrino flux from \( \sim 100 \) PeV to \( \sim 10 \) EeV with uncertainty of factor of two. This uncertainty reflects the possible range of intensity of the extra-galactic UHECR component allowed by the observed UHECR spectrum, and the accuracy of the approximations in derivation of the analytical formula.

### Table 1: Cosmogenic \( \nu \) fluxes predicted by the model-dependent full numerical calculations and those given by the present analytical formula with the corresponding parameters on source evolution.

<table>
<thead>
<tr>
<th>( \nu ) Flux Model</th>
<th>Integral Flux ( F(E_{\nu} \geq 1 \text{EeV}) ) [cm(^{-2}) sec sr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahlers et al. [6]</td>
<td>( m = 2.0, z_{\text{max}} = 2.0 ) ( 1.85 \times 10^{-18} )</td>
</tr>
<tr>
<td>The analytical formula</td>
<td>( m = 2.0, z_{\text{max}} = 2.0 ) ( 4.50 \times 10^{-18} )</td>
</tr>
<tr>
<td>Kotera et al. [11]</td>
<td>SFR1 ( m = 3.4(z \leq 1.0) ) const. ( 1.02 \times 10^{-17} )</td>
</tr>
<tr>
<td>The analytical formula</td>
<td>( m = 3.4(z \leq 1.0) ) const. ( 1.02 \times 10^{-17} )</td>
</tr>
<tr>
<td>Ahlers et al. [6]</td>
<td>( m = 4.6, z_{\text{max}} = 2.0 ) ( 3.39 \times 10^{-17} )</td>
</tr>
<tr>
<td>The analytical formula</td>
<td>( m = 4.6, z_{\text{max}} = 2.0 ) ( 3.88 \times 10^{-17} )</td>
</tr>
<tr>
<td>Kalashev et al. [13]</td>
<td>( m = 5.0, z_{\text{max}} = 3.0 ) ( 7.38 \times 10^{-17} )</td>
</tr>
<tr>
<td>The analytical formula</td>
<td>( m = 5.0, z_{\text{max}} = 3.0 ) ( 1.04 \times 10^{-16} )</td>
</tr>
<tr>
<td>Kotera et al. [11]</td>
<td>FR II ( m = 5.02(z \leq 1.5) ) const. ( 5.04 \times 10^{-17} )</td>
</tr>
<tr>
<td>The analytical formula</td>
<td>( m = 5.02(z \leq 1.5) ) const. ( 5.04 \times 10^{-17} )</td>
</tr>
</tbody>
</table>

### 3 Constraints on UHECR origin with the IceCube diffuse neutrino flux limit

We estimate expected event rate with the IceCube neutrino observatory by using the derived formula. The analytical function is valid in the IceCube cosmogenic neutrino detection energy range distributed around 1 EeV [7]. Convolution of Eq. 2 and the IceCube neutrino effective area [7, 8] gives the event rate for the entire phase space of the evolution parameter \( m \) and the maximal redshift \( z_{\text{max}} \). The Feldman-Cousins upper bound [14] then defines the excluded region on the \( m-z_{\text{max}} \) plane at a given confidence level. Figure 1 displays the resultant constraint. The shaded region represents the factor of two uncertainty in the analytical estimation discussed in the previous section. The IceCube 2008-2009 observation has already started to limit UHECR source class with strong evolution \( m \geq 4.5 \). This bound may be still weaker than that by the Fermi diffuse \( \gamma \)-ray flux measurement [6], but the limit by neutrinos is more solid against uncertainty of the assumptions of \( E_{\text{max}} \), inter-galactic magnetic field, and the EBL intensity, while the Fermi limit strongly depends on these parameters and it is more difficult to extract constraint on the UHECR source evolution. The full IceCube 5 year observation would certainly probe the most interesting region of the source evolution phase space where the powerful astronomical objects like radio galaxies and GRBs are included.

### 4 Discussion

The present analysis indicated that five year observation by the IceCube observatory would fully scan the parameter space on the source evolution in which many of the
proposed UHECR astronomical sources are distributed. A null neutrino observation would imply that either UHECR sources are local with \( z_{\text{max}} \lesssim 1 \), or only very weakly evolved with \( m \lesssim 3 \), or the UHECRs are not proton dominated but heavier nuclei like irons after all. The first two possibilities may lead to speculation about the highest energy particle radiation from an entirely different and probably dimmer class of objects. The last possibility has already been suggested by the measurement of the depth of maximum of airshowers by the Auger collaboration [15].

Neutrino search in ultra-high energies provides a complementary constraint on the proton fraction of UHECRs in this case [16].

The largest uncertainty in the present analytical formula at the lower energy range (\( \sim 100 \) PeV \( \lesssim E_{\nu} \lesssim 1 \) EeV) arises from our neglect of the IR contribution to the cosmogenic neutrino production. While photo-produced pions from UHECRs interaction with the IR background are major origin of neutrinos with energies below 10 PeV, they are relatively minor contribution in the higher energies we have mainly discussed from aspect of the cosmogenic neutrino detection by IceCube. Size of the IR contribution has been different among the various numerical calculations in the literature, indicating that prediction of the IR contributions above 100 PeV is not very straightforward. For example, the calculation in Ref. [11] exhibits higher contribution of the IR background, while the effect is very suppressed in Ref. [6] mainly due to the introduction of the minimal energy of extra-galactic UHECR population. The variance is consistent with the factor of two uncertainty we assigned to the intensity predicted by the analytical formula. Note that the neglect of the IR background always leads to conservative constraint on the UHECR source evolution and that the IR background yield itself is not firmly understood [17].

References

[8] A. Ishihara for IceCube Collaboration, these proceedings.