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Multiple Scattering of light in shower optical images - an analytical calculation

MARIA GILLER AND ANDRZEJ ŚMIAŁKOWSKI
[1] *Department of High Energy Astrophysics, University of Łódź, Poland*
maria.giller@kfd2.phys.uni.lodz.pl
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Abstract: One of the method for determining the primary energy of an UHE shower is to measure its instantaneous optical images as a function of the shower depth in the atmosphere. Such an image is composed of the direct light emitted by the shower towards the detector and of the light emitted by higher parts of the shower, scattered in the atmosphere and arriving at the same time as the direct light. The second component is often called the Multiply Scattered light suggesting that it is formed by photons scattered many times. We show that , at least in the case of Auger, this is a misnomer: the main contribution to it is made by photons scattered only once. We find analytically the image produced by these photons. We also propose a new parametrization of the ratio of the scattered to the direct light basing on our analytical solutions, separate for Rayleigh and Mie scatterings.

Keywords: UHE cosmic rays, air showers, fluorescence light, multiple scatterings

1 Introduction

One of the method for determining the primary energy of an UHE extensive air shower is to measure its instantaneous optical image as a function of the shower depth in the atmosphere, as it is done in the Fly's Eye, HiRes and Auger experiments. Such an image is composed of 1). the direct (not scattered) light emitted by the shower towards the detector at a time earlier by R/c (where R is the distance shower- detector), and 2). the light emitted by higher parts of the shower (i.e. earlier than the direct one), scattered in the atmosphere and arriving at the same time as the direct light. The second component is often called the Multiply Scattered light suggesting that it is formed by photons scattered many times. We show that , at least in the case of Auger, this is a misnomer: the main contribution to it is due to photons scattered only once. We call these photons the first generation, the photons scattered twice the second generation, and so on. We present approximate analytical calculations of the flux of the first generation and on the basis of these propose a new parametrization, separately for the Rayleigh and the Mie scatterings.

2 A point source in a real atmosphere

We assume, as it is usually done, the atmosphere to be a medium composed of molecules (causing Rayleigh scattering) and aerosols (Mie), each having the scattering mean free path growing exponentially with height. We adopt values describing approximately the conditions at the Pierre Auger Observatory site in Argentina: $\lambda_0^R = 18\,km$ and $\lambda_0^M = 15\,km$ (at the ground) with the scale heights: $H_R = 9\,km$ and $H_M = 1.2\,km$.
Basing on our earlier work [1, 2, 3], we have calculated the time distributions of the ratio of the second to the first generation within two angles ζ , for three values of k , the distance measured in units of λ_{eff}. In such an atmosphere the effective scattering length λ_{eff} can be expressed analytically [2]. Fig.1 shows that the contribution of the second generation to the flux arriving at short times after the arrival of the direct light is very small.

3 A moving point source - a shower

A distant shower exciting the atmosphere can be treated as a point source emitting light isotropically, moving with the light velocity c. As we described it in [3] the first generation photons , produced at some earlier point P (Fig.2) on the shower track, have to be scattered on the surface of the rotational ellipsoid with focal points at the detector and the point P. To obtain the total flux of the first generation simultaneous with the direct light produced at some point Q one has to integrate the contributions from all shower track elements above this point. We have calculated the ratio of the first generation to the direct light (zero generation) numerically as a function of several parameters. These parameters have been suggested by our analytical calculations (although approximate) which are the following:

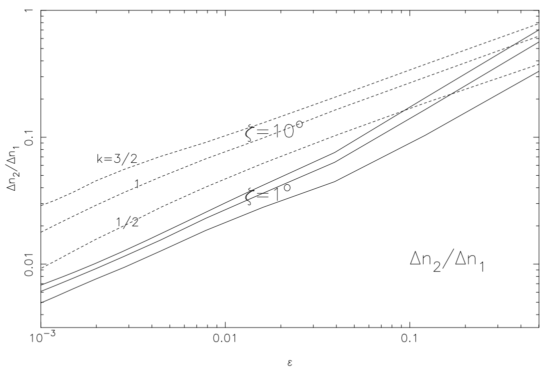

Figure 1: Ratio of the flux of the second to the first generation from flash ($t = 0$) of a point source in a real atmosphere as function of $\epsilon = ct/R - 1$. Source is at zenith angle 75°. Lines (solid for $\zeta = 1°$, dashed for 10°) are for $k = R/\lambda_{eff} = 0.5, 1.0, 1.5$ from bottom up.

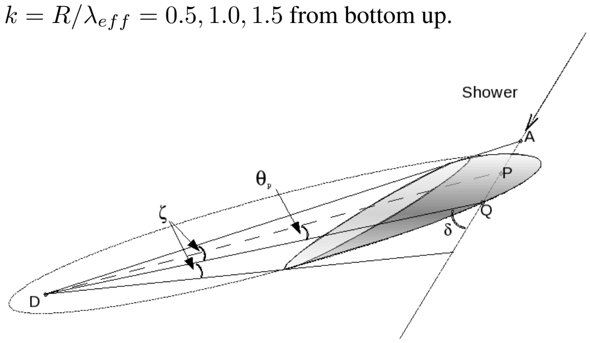

Figure 2: Fluorescence light produced at point P and scattered on the shaded part of the ellipsoid arrives at the detector D simultaneously with the direct light prod. at Q.

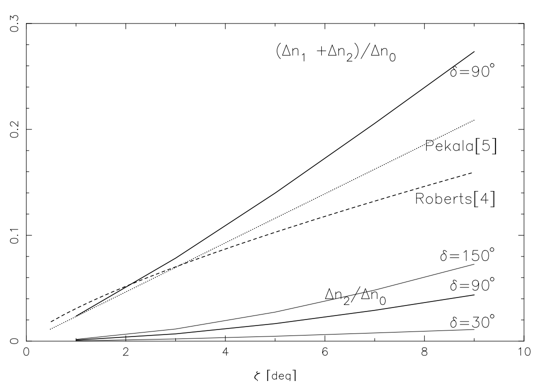

Figure 3: Ratio of the scattered to direct light $\Delta n/\Delta n_0$ integrated within angle ζ, for the shower 24 km away from the detector, with elevation angle of the centre of the field of view 10^o . Our results (first and second generation) are represented by the solid line for $\delta = \pi/2$. Bottom lines shows contribution of the second generation for three values of δ. Other lines are parametrisations from Monte-Carlo works: Roberts - dashed line and Pękala - dotted line.

3.1 Analytical calculations of dn_1^R/dt

As shown in our earlier work [1], after a flash of a point source the time distribution of the first generation arriving at the detector at an angle θ (per unit solid angle) equals:

$$\frac{d^2n_1(\theta,t)}{d\Omega dt} = \frac{c}{2\pi\lambda_S R^2} e^{-\frac{ct}{\lambda_{eff}}} f(\alpha) \frac{|\cos\theta|}{\tau^2 - 2\tau cos\theta + 1} \tag{1}$$

where R is the distance from the light source to the detector, λ_S is the scattering mean free path length at the scattering point S, $\tau = ct/R$, $f(\alpha)$ is the angular distribution of photons after scattering.

The detector field of view is centered on point Q on the shower with the opening angle ζ. We shall calculate separately the contribution to the first generation flux produced by the shower part within the field of view (section QA), and by that outside it (above point A) since we shall make different approximations in the two cases. Fig.2 illustrates the first case. The contribution from the light produced along a path length element dl at P at an angle θ_P can be found by integrating (1) over θ determined by the shaded part of the ellipsoid. Our approximation consists in assuming that the integration is over a symmetric (with respect to the ellipsoid axis DP) part, with $0 < \phi < 2\pi$ and $0 < \theta < \theta_{max} = \zeta$. We also see that most of the scatterings take place near point Q, so we assume that $\lambda_S = \lambda_Q$. Taking into account that angles θ are small we can find analytically the inner contribution for the Rayleigh scattered photons.
To calculate the contribution of the outer part of the shower we assume that all photons scattered on the shaded surface (now looking differently, rather as a saucer) arrive at the angle θ_P and that all scattering angles are equal δ. These approximations allowed us to find analytically the outer contribution. The sum of the inner and the outer parts gives the final flux of the first generation, dn_1^R/dt , within a given opening angle ζ . The ratio of it to the direct light which equals

$$\frac{dn_0}{dt} = \frac{Cc}{4\pi R^2} \cdot e^{-\frac{R}{\lambda_{QD}}} \frac{1}{1-\cos\delta} \tag{2}$$

where C is the number of fluorescence photons emitted per unit length by all shower electrons, gives the result:

$$\begin{aligned} \frac{\Delta n_1^R}{\Delta n_0} \;\; &\simeq \;\; \frac{3}{16}\frac{R}{\lambda_Q}\zeta \cdot \Big[3\delta' - \sin\delta' - 8\tan\frac{\delta'}{2}\ln(\sin\frac{\delta'}{2}) \\ &+ \;\; 2\frac{\sin\delta'(1-\cos\delta)(1+\cos^2\delta)}{1+\sin^2\delta - \cos\delta}\Big] \end{aligned} \tag{3}$$

where $\Delta n = dn/dt \cdot \Delta t$, Δt being a small time interval, $\delta' = \delta - \zeta$ (in radians) and the last term (out of the four in the sum) represents the outer contribution. We see that, with the approximations adopted, the ratio of the first generation to the direct light depends mainly on the ratio $k_Q = R/\lambda_Q$, reflecting the fact that most of the scattering takes place close to the point Q. The calculations also show that the ratio is (roughly) proportional to the opening angle ζ of the detector field of view, particularly when

$\delta >> \zeta$. For small ζ the dependence on δ, the inclination of the shower to the line of sight, separates from other parameters.
It is obvious that the dependence on δ is different for different scattering mechanisms, thus it will be different for the Mie scattering. We have not tried to calculate the latter analytically because the function $f^M(\alpha)$ is not well known and our approach is anyway approximate so that we have to rely on numerical calculations.

3.2 Numerical (exact) calculations of $\Delta n_1/\Delta n_0$ and $\Delta n_2/\Delta n_0$ and comparison with other works

The ratio of the first generation to the direct light arriving simultaneously is obtained by numerical integration of point contributions over l (shower track above point Q) and dividing the result by (2). Fig.4 shows the ratio for the Rayleigh (4a) and Mie (4b) scattering as a function of $k_Q = R/\lambda_Q$ for $\zeta = 1°, 3°$ and $5°$ and for several different distances R. It can be seen that for the Rayleigh case the ratio is proportional to k_Q and ζ, as it has been derived analytically. For Mie the dependence follows a power law, with the indices depending slightly on ζ. However, in each case there is practically no dependence on the distance R itself. These results are for $\delta = \pi/2$. The dependence on δ is shown in Fig.5a,b for $k_Q = 1$ and $\zeta = 1°, 3°$ and $5°$. One can see the difference of the behaviour of this dependence between Rayleigh and Mie. In Fig.5a there is also a comparison of our analytical calculations with the exact ones. The biggest difference reaches some 10% at large δ. There is no normalization there. We find this agreement quite satisfactory.
Nevertheless, as the agreement is not perfect and our analytical dependence on δ is a bit complicated we have parametrized it for the Rayleigh scattering so that the ratio $\Delta n_1/\Delta n_0$ can be expressed as

$$\frac{\Delta n_1^R}{\Delta n_0} = 0.024 \cdot k_Q \cdot \zeta \, g^R(x) \tag{4}$$

where $x = \delta/100°$, ζ is in degrees, and

$$g^R(x) = 0.112 + 1.86x - 1.33x^2 + 0.383x^3 \tag{5}$$

For Mie the factorization is not as complete as for Rayleigh, and our fit to the numerical results is the following:

$$\frac{n_1^M}{n_0} = 0.096 \cdot \left(\frac{k_Q}{0.7}\right)^{0.93-0.04\zeta} \cdot \left(\frac{\zeta}{3°}\right)^{0.93+0.05\frac{\delta}{90°}} \cdot g^M(x) \tag{6}$$

$$\begin{array}{lll} x \leq 0.6 & g^M(x) = x(7.76x^2 - 11.25x + 5.625) & \\ x > 0.6 & g^M(x) = 1 & (7) \end{array}$$

These fits are also shown in Fig.5. It is seen that they do not deviate from the exact values by more than a few percent. Finally, Fig.3 presents a comparison of our results with those of other authors, showing $\Delta n/\Delta n_0$ as a function of ζ. In our case $\Delta n = \Delta n_1 + \Delta n_2$ (Δn_2 also shown separately), in the case of Roberts [4] and Pękala [5], who performed Monte-Carlo simulations of this effect for showers inclined at various angles δ, it is a sum of many generations. Our results shown here are for $\delta = \pi/2$, corresponding more or less to an average value for Auger observed showers. It is seen that the contribution of the second generation to the total scattered flux is small for any δ.

4 Conclusions

We have presented an analytical solution to the so called 'multiple scattering' effect for the Rayleigh scattering imposing on the shower image in the direct light. It was possible by finding that practically the only important light is that scattered only once.
Our analytical solution, although somewhat approximate, describes well the exact numerical calculations. It shows on what parameters and how the image in the scattered light should depend. For Rayleigh scattering it is particularly simple. As $g(\delta)$ depends on the angular distribution at the scattering, quite different for Rayleigh and Mie, it is sensible to parametrize $\Delta n_1/\Delta n_0$ separately for these two scatterings, what has been done in this paper.
The same effect was calculated by Roberts [4] and Pękala et al [5] by Monte-Carlo simulations but they obtain a bit less than we do. Roberts was the first to notice that the amount of the scattered light depends on the scattering length at the observation point, λ_Q. However, his overall parametrization does not reflect well the dependence on the rest of the parameters. Our parametrization in [3] was not quite accurate as well. By separating the two processes we obtained a much better one in this work.

Acknowledgements

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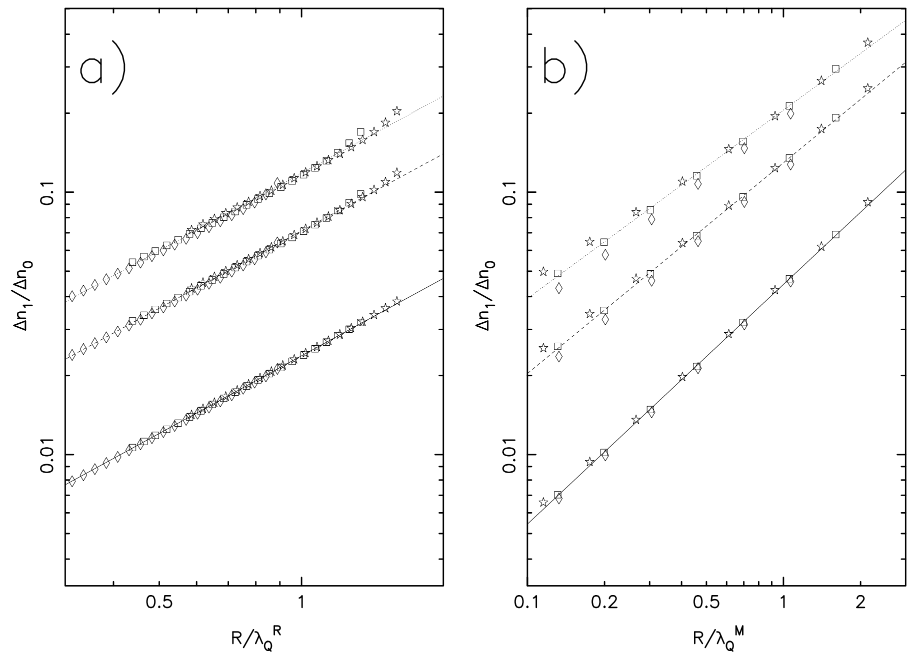

Figure 4: Ratio of the first generation to the direct light as a function of distance R to point Q on the shower in units of the scattering length λ_Q at that point. a) Rayleigh, b) Mie scattering. Curves correspond to $\zeta = 1°, 3°, 5°$ and $\delta = \pi/2$. Various point shapes refer to different distances R=12, 24, 32 km. Lines are power law fits.

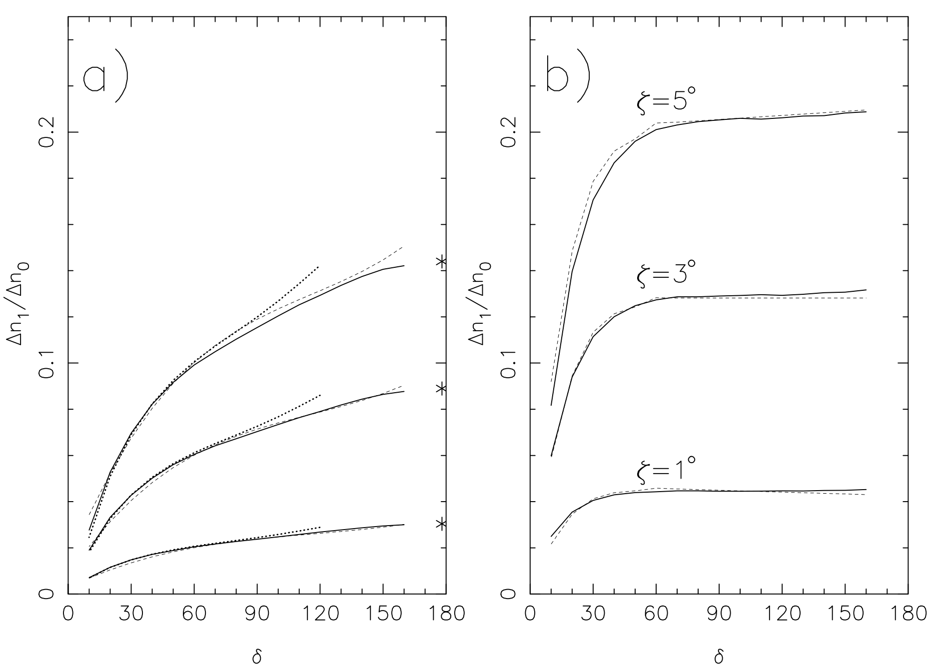

Figure 5: Ratio of the first generation to the direct light as function of angle δ between shower and line of sight. Solid lines - exact numerical calculations, dotted lines and stars in a) - analytical (approximate) calculations, dashed lines - our parametrization of the numerical curves (eq. 4-7). All curves are for $k_Q = R/\lambda_Q = 1$.