On the shower age related characteristics of cosmic ray cascades in the atmosphere

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Abstract: The lateral distribution of the electromagnetic component of extensive air shower (EAS) at ground level can be parameterized by the shower age. We analyze EAS observables detected with the ground arrays which are essentially related to this parameter. Our aim is to reveal EAS characteristics to be used as the shower age estimator.

Keywords: extensive air showers of cosmic rays, age, data analysis

1 Introduction

The lateral distribution (LD) of EAS particles depends on the energy of the primary particle, $E_0$, its type, the depth of observation in the atmosphere, $x$, and zenith, $\theta$, and azimuth, $\phi$, angles. Some observable parameters of LD, detected with EAS arrays at ground level can be described approximately by the universal function of the shower age characterizing the cascade development stage. In other words, all dependencies on $E$, $x$, $\theta$, $\phi$ are parameterized by the shower age, in the limited interval of the core distance, $R$, to good approximation, neglecting fluctuations [1].

It is convenient to combine the depth and zenith angle dependencies: $x = x_0 \sec \theta$, where $x_0 = 1020$ g/cm$^2$ in Yakutsk, for instance. We neglect azimuthal dependence of LD in this paper. In cascade theory the electromagnetic shower age is approximated by $s_{EM} = 3x/(x + 2x_m)$, where $x_m$ is the shower maximum in the atmosphere. We are using here another approximation of the shower age: $s = x/x_m = x_0 \sec \theta/x_m$, which is related to the electromagnetic cascade theory definition: $s = 2s_{EM}/(3 - s_{EM})$.

In the following we are dealing with characteristics of LD$^1$ that are approximately the functions of shower age only. Examples of these are: RMS radius of LD and the lateral distribution function (LDF) slope$^2$, $\eta$, which were analyzed in our previous paper [1], and will be re-visited in sections 2 and 3. Another example is the shower front curvature; it will be analyzed in section 4. Conclusions are given in the postamble.

2 EAS observables depending on the shower age

There was a widespread notion in the previous decades that the LDF slope in EAS is related to $x_m$ in a model-independent manner. See for example [2, 3], and the references therein. Monte Carlo simulations with the CORSIKA code [4] demonstrated that yes, indeed, there are some hints of regular $\eta(x_m)$ dependencies, but at fixed zenith angles only (Fig. 1 from the presentation given by A.V. Sabourova at Quarks-2008 seminar, Sergiev Posad, 27 May,

1. at the EAS array observation level
2. at the periphery of the shower

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**Figure 1:** Correlation between LDF slope, $\eta$ and $x_m$ in EAS simulated with CORSIKA code, QGSJETII+FLUKA models implemented.
2008) [5]. There should be another shower parameter able to combine \( x_m \) and \( \theta \) dependencies.

In our previous paper [1] we argued that the shower age (cascade theory definition) is a parameter looked for. Actually, Nishimura and Kamata in 1958 proved [6] that the LDF shape of the electromagnetic cascade is the function of core distance scaled with Molière unit with the age parameter. In EAS, however, hadrons and muons are present in addition to electrons and photons, so the Nishimura-Kamata solution cannot be applied directly; the LDF shape is the function of energy and zenith angle, at least.

Nevertheless, there is a shower core distance and zenith angle interval for a given primary energy, where we can neglect the contribution of hadrons and muons to the density of charged particles; where the LDF shape is approximately the function of the shower age only. Fig. 2 from the paper cited demonstrates the parametrization of the LDF shape by the shower age: all the energies, zenith angles, and primary particles used in the samples result in LD slopes belonging to the same universal function, to good approximation.

Of course, this result concerns the average values only; fluctuations disperse the dependence, as shown by RMS deviations – but all the differences are less than 5%. The new definition of the shower age we are using here does not wipe out the parametrization.

In 1977 John Linsley proposed the so-called 'Elongation Rate theorem' [7]. Since then the relation \( x_m = \text{const} + D_{ER} \log E \) is commonly used, where \( D_{ER} = \Delta x_m / d \log E \) is 'Elongation Rate' parameter. It obviously demonstrates that \( x_m \) in EAS is independent of zenith angle. The physical reason is in the cascade curve around \( x_m \) formed mainly by the electromagnetic component, and in electromagnetic cascade equations which do not include \( \theta \). In another way it can be said that critical energy and radiation length are independent of \( \theta \).

This common feature of EAS development gives us an opportunity to study the shower age dependence of LD parameters even without having information on \( x_m \) position in the atmosphere. If we select showers of the same energy/\( x_m \) but with different zenith angles, then we can derive, say, RMS radius as a function of age, due to \( s = x_0 \sec \theta / x_m \), where \( x_0 \) and \( x_m \) are constant.

If the LDF slope dependence on \( E \) and \( \theta \) is parameterized with \( s \), then the derivatives are:

\[
\frac{\partial \eta}{\partial \cos \theta} = \frac{\partial \eta}{\partial x_m} = \frac{\partial \eta}{\partial \ln \frac{E}{E_0}} = \frac{\partial \eta}{\partial \ln \frac{E}{E_0}}
\]

Immediate consequences are:

\[
\frac{\partial \eta}{\partial \cos \theta} = \frac{\partial \eta}{\partial x_m}
\]

and

\[
\frac{\partial \eta}{\partial \ln \frac{E}{E_0}} = \frac{D_{ER}}{\Delta x_m} \frac{\partial \eta}{\partial \ln \frac{E}{E_0}}.
\]

Measuring \( \eta \) as a function of energy (zenith angle fixed), and zenith angle (fixed energy) we can get its dependence on \( x_m \).

3 Age dependence of the LDF slope measured in Yakutsk

The Yakutsk array detectors and experimental data concerning LD characteristics are described in Refs. [8] - [13]. Here we apply a procedure to these data, fixing \( E_0 \) in narrow bins, setting zenith angle limits as \( 0^\circ \leq \theta \leq 60^\circ \). The Yakutsk array arrangement is convenient for measuring LDF slope rather than the RMS radius. Thus, we focus on the former parameter in this section.

To estimate the shower age we need \( x_m \) in each event. We have used the experimental values, \( x_m(E_0) \), given in [14], approximated by the linear function with \( D_{ER} = 68 \text{ g/cm}^2 \), in order to calculate \( s = x_0 \sec \theta / x_m(E_0) \) as a crude approximation of the actual age in a particular shower.

Shower events detected with more than 8 scintillation detectors within the core distance interval (200,600) m are selected to estimate the LDF slope. Additional conditions are: the shower axis position within the array area; the stations in which the particle density detected is below the threshold value 0.5 m\(^{-2}\) are excluded from analysis.

The result is shown in Fig. 3. In general, the form of the observed function \( \eta(s) \) resembles what is expected in SIBYLL/UrQMD model [15, 16], but the LDF slope values are
different. Maybe, it is a distinctive feature of the model used, or the detector response is calculated with a systematic error.

4 Shower front curvature measured in the Pierre Auger Observatory

The Pierre Auger Observatory (PAO) collaboration publishes for the public domain every hundredth EAS event detected in the energy interval $E_0 \in (0.1, 41.1) \text{ EeV}$ [17]. The shower front curvature among other EAS observables is given. The radius of curvature is estimated basing on the data from water tank detectors of the observatory.

We have selected showers with energy $E_0 > 5 \text{ EeV}$, and where the number of surface detectors fired is greater than 3. As $x_m$ parameter in the particular event is not available in the published data, we have used the average maximum depth, $x_m$, related to the shower energy [18]:

$$x_m = 753.0 + (24 \pm 3)(\log(E) - 1.0),$$

at $E > 1.74 \text{ EeV}$.

The thickness of the atmosphere above Malargue is given by the approximation formula:

$$x(h) = a + b \exp(-h/c),$$

where $a, b, c$ are tabulated for 4 seasons basing on the weather-balloon data from Pampa Amarilla [19]. The radius of the shower front curvature as a function of the age estimated is shown in Fig. 4. Linear interpolation by the least-squares method results in the derivative:

$$\frac{dR}{ds} = 9.22 \pm 0.76, \text{ km}.$$
References