Multiband Spectrum of Tycho Supernova Remnant

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Abstract: We model a time-dependent non-thermal particle and photon spectra for a Type Ia SNR Tycho with radio, X-ray, and TeV emission in the frame of diffusive shock acceleration of the non-thermal particles in the shell of the SNR. We consider two possible cases: TeV photons are produced by inverse Compton scattering of relativistic electrons and by $\pi^0$-decay gamma-rays in proton-proton interaction, respectively. Our results indicate that (1) the observed multiband spectrum can be reproduced in these cases under reasonable range of model parameters; (2) there is remarkable difference in the energy range of $\sim 0.1 - 10$ GeV for predicted spectra in these cases, which should be discerned by observations; and (3) a magnetic field amplification in the SNR is required.

Keywords: acceleration of particles - gamma rays: theory - ISM: clouds - ISM: individual (Tycho) - radiation mechanisms: non-thermal

1 Introduction

Tycho’s supernova remnant (SNR) is a Type Ia SNR with an age of 438 yrs. The observations of this SNR at different wave bands have been made. At radio band, the images indicate a clear shell-like morphology with enhanced emission along the northeastern edge of the remnant [1, 2], the spectral index and the flux density at 1.4 GHz are 0.65 and 40.5 Jy respectively [3]. At X-ray band, the observations show that strong non-thermal emission are concentrated in the SNR rim [4, 5] and the detection of X-rays up to energies of 30 keV by Suzaku shows the presence of electrons up to at least $\sim 10$ TeV [6]. Recent observation at TeV band by VERITAS detect TeV photons whose spectrum can be described with a power-law $dN/dE = C(E/3.42\text{TeV})^{-\Gamma}$ with $\Gamma = 1.95 \pm 0.51_{\text{stat}} \pm 0.30_{\text{sys}}$ and $C = (1.55 \pm 0.43_{\text{stat}} \pm 0.47_{\text{sys}}) \times 10^{-14}$ cm$^{-2}$s$^{-1}$TeV$^{-1}$ [7]. Therefore the observed results provide important information to study multiband radiation of this SNR.

For Tycho SNR, there is uncertain whether the SNR is interacting with a molecular cloud (MC). Based on the fact that the northeast quadrant of the remnant is expanding at a lower rate than the rest of the object, it is pointed out that a part of SNR is interacting with a nearby high-density cloud [8, 9]. However, Tian & Leahy (2011) [10] analyzed the 21cm continuum, HI and 12CO-line data from the Canadian Galactic Plane Survey in the direction of Tycho and surrounding region and constructed HI absorption spectra to Tycho and three nearby compact sources. They concluded that Tycho is an isolated Type Ia SNR. On the other hand, the distance to the Tycho is uncertain, recent distance estimates are $3.8^{+1.5}_{-1.1}$ kpc [11].

In this paper, we assume that Tycho SNR is an isolated Type Ia SNR with a distance of 3.8 kpc, and study the multiband nonthermal emission from the SNR in the frame of a simple time-dependent model [12, 13]. In this model, the time-dependent non-thermal spectra of primary electrons and protons as well as secondary electron/positron ($e^\pm$) pairs can be calculated numerically by including the evolution of secondary $e^\pm$ pairs produced from proton-proton (p-p) interactions due to accelerated protons collide with the ambient matter in an SNR. The multi-wavelength photon spectrum for a given SNR can be produced through leptonic processes such as electron/positron synchrotron radiation, bremsstrahlung and inverse Compton scattering, as well as hadronic interaction due to the relativistic protons colliding with the ambient matter. Since the accelerated particles at shock reach their maximum energy near Sedov stage [14], and then both the electrons and protons obtain their most kinetic energies during more or less Sedov stage, so we make a revision for the model of Zhang & Fang (2007) [12], i.e. if the age $T$ of a SNR is larger than the time $t_{\text{Sed}}$ when Sedov stage begins, then the total amount of the kinetic energy contained in the injected particles has been completely converted into the kinetic energy of both the electrons and protons during the time $t_{\text{ci}} = T > t_{\text{Sed}}$.

2 Model and Results

In this model, the analytical model of the shock dynamics of an SNR with a explosion energy $E = E_51 \times 10^{51}$ erg expanding at a velocity $v_0 = v_9/10^9$ cm/s into a uni-
form ambient medium with density $n_0$ is used. The SNR evolves through the free expansion stage which ends at $t = t_{Sed} \approx 2.1 \times 10^4 (E_{51}/n_0)^{1/2} \times \nu_0^{5/2}$, the Sedov stage which ends at $t = t_{rad} \approx 4.0 \times 10^4 E_{51}^{4/17} \times n_0^{-9/17}$ yr, and the radiative stage in which the SNR begins to experience significant radiative cooling. The shock velocity $v_s(t)$ corresponding to these stages is $v_s(t) = v_0$ for $t < t_{Sed}$, $v_s(t) = v_0 \left[1/(t/t_{Sed})^{-5/3}\right]$ for $t_{Sed} \leq t < t_{rad}$, and $v_s(t) = v_0 \left[1/(t/t_{rad})^{-2/3}\right]$ for $t > t_{rad}$. The shock radius is given by $R_s(t) = \int_0^t v_s(t') dt'$.

The volume-averaged production rates of the shock-accelerated electrons and protons are assumed to be

$$Q_{i}^{\text{pri}}(E_i, t) = Q_i^0 G(t) \left[1 + 2 \eta_i c^2 \right]^{-[(\alpha + 1)/2]} \left(E_i / E_{i, \text{max}}(t)\right),$$

where $i = e, p$, $G(t)$ is a factor which relates to time, i.e. $G(t) = R_s(t_{Sed})/R_s(t)$ for $t \leq t_{rad}$ and $G(t) = 0$ for $t > t_{rad}$. $\alpha$ is the spectral index, $E_{i, \text{max}}$ and $E_{p, \text{max}}$ are maximum energies of accelerated electrons and protons, respectively. Factors $Q_i^0$ and $Q_p^0$ are normalized coefficients.

Assuming that an SNR interior is homogeneous, with a constant density $n_{SNR} = 4n_{ISM}$ and a magnetic field strength $B_{SNR} = 4B_{ISM}$, $n_e(E_e)$ and $n_p(E_p)$ are used to represent the differential densities of accelerated electrons and protons, respectively. The direction- and volume-averaged electron intensity $J_e(E_e, t) = (c/4\pi)n_e(E_e, t)$ and proton intensity $J_p(E_p, t) = (c/4\pi)n_p(E_p, t)$ at each moment during the SNR lifetime can be calculated by solving Fokker-Planck equations for both electrons and protons in energy space, which are given by [12]

$$\frac{\partial n_i(E_i, t)}{\partial t} = -\frac{\partial}{\partial E_i} \left[ E_i n_i(E_i, t) \right] + \frac{1}{2} \frac{\partial^2}{\partial E_i^2} \left[ D(E_i) n_i(E_i, t) \right] + Q_i(E_e, t) - \frac{n_i(E_i, t)}{\tau_i},$$

where $i = e, p$, the terms on the right-hand sides in Eq.(2) represents systematic energy losses, diffusion in energy space, the particle source function and catastrophic energy loss. It should be noted that the source term for electrons includes the evolution of secondary $e^\pm$ produced via p-p interaction when high-energy protons collide with the ambient matter in the SNR, i.e.

$$Q_e(E, t) = Q_e^{\text{pri}} + Q_e^{\text{sec}}(E, t) + Q_e^{\text{rec}}(E, t),$$

where

$$Q_e^{\text{rec}}(E, t) = 4\pi \mu_{ep} n_{SNR} \int \frac{dE_p J_p(E_p, t) d\sigma(E_{e, \infty}, E_p) / dE_{e, \infty}}{dE_{e, \infty}},$$

where $\mu_{ep}$ is an enhancement factor for collisions involving heavy nuclei in an SNR, $d\sigma(E_{e, \infty}, E_p) / dE_{e, \infty}$ and $d\sigma(E_{e, \infty}, E_p) / dE_{e, \infty}$ are the differential cross section for electrons and positrons produced via p-p interaction respectively [15]. We solve the equation (1) using a Crank-Nicholson finite difference scheme.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case I</th>
<th>Case II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>3.8 kpc</td>
<td>3.8 kpc</td>
</tr>
<tr>
<td>$T$</td>
<td>438 yr</td>
<td>438 yr</td>
</tr>
<tr>
<td>$\lambda_{\text{max}}$</td>
<td>$8.0 \times 10^{16}$ cm</td>
<td>$8.0 \times 10^{16}$ cm</td>
</tr>
<tr>
<td>$M_{ej}$</td>
<td>$1.2M_\odot$</td>
<td>$1.2M_\odot$</td>
</tr>
<tr>
<td>$v_0$</td>
<td>$1 \times 10^9$ cm s$^{-1}$</td>
<td>$1 \times 10^9$ cm s$^{-1}$</td>
</tr>
<tr>
<td>$\eta$</td>
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<td>2.2</td>
</tr>
<tr>
<td>$K_{\text{ep}}$</td>
<td>$0.01$</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$n_{\text{ISM}}$</td>
<td>$0.2 \text{ cm}^{-3}$</td>
<td>$0.2 \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>$B_{\text{ISM}}$</td>
<td>$19 \mu G$</td>
<td>$40 \mu G$</td>
</tr>
</tbody>
</table>

Table 1: Values of parameters used for the model (see text for details).

In above calculations, we need to determine the factors $Q_e^0$ and $Q_p^0$. In order to do so, we introduce a parameter $K_{\text{ep}} = Q_e^0/Q_p^0$, which is called the ratio of electrons to protons. Since the total amount of kinetic energy contained in both the injected electrons and the injected protons is $E_{\text{par}} = \eta M_{ej} \nu_0^2 / 2$, where $\eta$ presents the efficiency that the kinetic energy of the ejecta with initial mass $M_{ej}$ and initial velocity $v_0$ is converted into the kinetic energy of both the electrons and the protons. In the estimate of $E_{\text{par}}$. Assuming the total amount of the kinetic energy contained in the injected particles has been completely converted into the kinetic energy of both the electrons and protons during the time $t_{ci} = T$, we have

$$E_{\text{par}} = \int_0^{t_{ci}} dt V_{\text{SNR}}(t) \left[ \int_0^{E_{e, \text{max}}} dE Q_e(E, t) \right].$$

where $V_{\text{SNR}}(t) = 4\pi R_{\text{SNR}}^2(t)/3$ is the SNR volume.

After obtaining the electron intensity and proton intensity at each moment during the SNR lifetime, we can calculate the photon emission from the SNR. The non-thermal radiation processes of the accelerated particles involved in an SNR are synchrotron radiation, bremsstrahlung, inverse Compton scattering for leptons including electrons and positrons, and p-p interaction for protons, the formulae for various radiation processes see ref. [12].

In this model, model inputs include the distance $d$ and the age $T$ of the source, initial ejecta mass $M_{ej}$, initial shock velocity $v_0$, the maximum wavelength of MHD turbulence $\lambda_{\text{max}}$, conversion efficiency $\eta$, the factor $\xi$, electron/proton ratio $K_{\text{ep}}$, the spectral index $\alpha$, hydrogen density $n_{\text{ISM}}$ and magnetic field strength $B_{\text{ISM}}$ of the ISM. For the SNR considered here, Eq.(2) is respectively solved both with the parameters for the part of the shell evolving in ISM, then the multiband nonthermal spectra can be calculated.

We now apply this model to Tycho SNR which is assumed to be an isolated SNR. We consider two possible cases: (1) case I: TeV photons come from the inverse Compton scattering of relativistic electrons, and (2) case II: TeV photons are produced by the $\pi^0$ decay in p-p interaction. The
model parameters in these two cases are listed in Table 1. In our calculations, we use following parameters in two cases: $M_0 = 1.2 M_⊙$, $v_0 = 10^3$ cm/s, $\lambda_{\text{max}} = 8.0 \times 10^{16}$ cm, $n_{\text{ISM}} = 0.2$ cm$^{-3}$, and $\alpha = 2.2$. Note that $t_{\text{Sed}} \approx 338$ yr for the parameters used here, which is less than the SNR’s age $T = 438$ yr, indicating that both the electrons and protons obtain their most kinetic energies during more or less Sedov stage.

In Fig. 1, we show the isotropic intensities of both electrons and protons as well as nonthermal photon spectra for the Tycho SNR in the case I. In order to reproduce the observed spectrum, we need the values of parameters: $\eta = 0.05$, $K_{\text{ep}} = 0.01$, and $B_{\text{ISM}} = 19 \mu$G. In this case, TeV photons are mainly produced by the inverse Compton scattering of the relativistic electrons. In our calculations, the soft photon fields of the inverse Compton scattering include the cosmic microwave background (CMB) and dust-IR emission, the temperatures and energy densities of these photon fields are $2.7$ K, $25$ K and $U_{\text{CMB}} = 2.6 \times 10^{-7}$ MeV cm$^{-3}$, $U_{\text{IR}} = 2.0 \times 10^{-7}$ MeV cm$^{-3}$, respectively. From the bottom panel of Fig. 1, it can be seen that the contribution of $\pi^0$-decay gamma-rays in the energy range from $\sim 0.1 - 10$ GeV is important compared with that of the inverse Compton scattering.

Fig. 2 shows the isotropic intensities of both electrons and protons as well as nonthermal photon spectra for the Tycho SNR in the case II. The parameters involved in the calculation are shown in Table 1. In the case II, the model parameters which are different from those in case I are $\eta = 0.15$, $K_{\text{ep}} = 1.0 \times 10^{-3}$, and $B_{\text{ISM}} = 40 \mu$G. The observed spectrum can be reproduced and high-energy gamma-rays are mainly produced by the $\pi^0$-decay gamma-rays in the p-p interaction. Note that the conversion efficiency $\eta$ in the case II is larger than that in case I by a factor of 3, it is mainly due to the remarkable decrease of $K_{\text{ep}}$ compared to that in case I. On the other hand, a larger value of the interstellar magnetic field is required and it deduces the contribution of the inverse Compton scattering.

It seems that current observed spectrum can be reproduced in both cases, however predicted spectra in the energy range from $\sim 0.1 - 10$ GeV for two cases are remarkably different. From Fig. 1, the energy spectrum in this energy range consists of both $\pi^0$-decay gamma-ray and the inverse Compton scattering components in case I, but only $\pi^0$-decay gamma-ray component in case II. Such a difference should be discerned in observations.

3 Discussion and Conclusion

In this paper, we applied a time-dependent model of nonthermal photon emission from a shell-type SNR to Tycho SNR. We have considered two possible cases: one is that TeV emission is produced by leptonic process (case I) and another is that TeV emission is produced by hadronic process (case II). The model results in two cases can reproduce observed radio, X-ray, and TeV data well, but there is significant difference of predicted spectra in the energy range of $\sim 0.1 - 10$ GeV between case I and II (see Figs. 1 and 2).

Acciari et al. (2011) [7] used a simple model to reproduce the broadband spectrum of Tycho, where an isolated SNR was assumed and both leptonic and hadronic cases were
considered. They assumed a steady-state spectra of the accelerated particle of \( dN/dE \propto E^{-\alpha} \exp(-E/E_c) \) with same spectral index \( \alpha = 2.2 \) for the electrons and protons, but with different values of cutoff energy \( E_c \). After setting total supernova kinetic energy of \( E_{SN} = (1/2)M_{ej}v_{ej}^2 = 1.2 \times 10^{51} \) ergs, \( d = 4 \) kpc, and \( n_{ISM} = 0.2 \) cm\(^{-3} \), they calculated the model spectra in both leptonic and hadronic cases. In the leptonic case, they found electron-to-proton ratio \( K_{ep} = 10^{-2} \), the total particle energy \( E_{par} = 1.8 \times 10^{50} \) erg, and the magnetic field \( B = 78 \) \( \mu \)G inside the SNR. In the hadronic case, they found \( K_{ep} = 4 \times 10^{-4} \), \( E_{par} = 8 \times 10^{50} \) erg, and the magnetic field \( B = 230 \) \( \mu \)G inside the SNR. They found that the lowest magnetic field in these two cases is \( B \sim 80 \) \( \mu \)G and pointed out that this may be interpreted as evidence for magnetic field amplification.

In our calculations, \( M_{ej} = 1.2 M_\odot \) and \( v_{ej} = 10^9 \) cm/s are used, resulting in \( E_{SN} = 1.2 \times 10^{53} \) ergs. In the case I, \( E_{par} \approx 0.1 \times 10^{50} \) ergs and is dominated by protons because of \( K_{ep} = 10^{-2} \). Since the strong shock is assumed here, the magnetic field inside the SNR is \( B \approx 4B_{ISM} = 76 \) \( \mu \)G. In the case II, \( E_{par} \approx 1.8 \times 10^{50} \) ergs and \( K_{ep} = 10^{-3} \), the magnetic field inside the SNR is \( B \approx 4B_{ISM} = 160 \) \( \mu \)G. Therefore, our model parameters are roughly consistent with those of Acciari et al. (2011) [7].

It should be noted that the magnetic field amplification and the feedback of the accelerated protons are not included in this model, i.e. the nonlinear diffusive shock acceleration (or efficient diffusive shock acceleration) in SNRs (e.g. refs. [16, 17, 18]) is not considered here. According to diffusive shock acceleration of test particles, the magnetic field \( B \) in the SNR can be estimated as \( B = \sigma B_{ISM} \) and \( \sigma \) is the shock compression ratio which is 4 for strong shock. In our calculations, adopted values of \( B_{ISM} \) is significantly larger than the conventionally accepted value \( \sim 3 - 5 \) \( \mu \)G of \( B_{ISM} \).

Based on X-ray and gamma-ray observations, the magnetic field in the SNR is derived to be \( B \sim 200 - 300 \) \( \mu \)G (e.g., refs. [19, 20]), which means there is the magnetic field amplification in the SNR. Theoretically, in the nonlinear diffusive shock acceleration, the reaction of accelerated particles upstream of the shock results in the formation of a precursor, where the fluid speed decreases while approaching the shock, forming a subshock with a compression ratio \( \sigma_s \), the large pressure of the accelerated protons is produced and decelerates the incoming gas, leading to the total shock compression ratio \( \sigma \) is generally larger than 4 and is between \( 7 - 10 \) [14]. Meanwhile the magnetic field amplification in the SNRs can be self-generated through resonant streaming instability by accelerated particles [14] and is possibly the order of \( 10B_{ISM} \) or even larger [21]. Therefore, the magnetic field amplification in the SNR is required in our calculations if the conventionally accepted value of \( B_{ISM} \) is used. A simple method is to introduce an effective shock compression ratio \( \sigma_{eff} \) and the magnetic field in the SNR is estimated as \( B = \sigma_{eff} B_{ISM} \).