A multiscale method for gamma/h discrimination in extensive air showers

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Abstract: We present a new method for the identification of extensive air showers initiated by different primaries. The method uses the multiscale concept and is based on the analysis of multifractal behaviour and lacunarity of secondary particle distributions together with a properly designed and trained artificial neural network. The separation technique is particularly suited for being applied when the topology of the particle distribution in the shower front is as largely detailed as possible. Here, our method is discussed and applied to a set of fully simulated vertical showers in the experimental framework of ARGO-YBJ, taking advantage of both the space and time distribution of the detected secondary particles in the shower front, to obtain hadron to gamma primary separation in EAS analysis. We show that the presented approach gives very good results, leading, in the 1-10 Tev energy range, to an improvement of the discrimination power with respect to the existing figures for extended shower detectors. The technique shows up to be very promising and its application may have important astrophysical prospects in different experimental environment of extended air shower study.

Keywords: Cosmic Rays; Extensive Air Showers; Multiscale Analysis; Wavelet Methods; Neural Networks

1 Introduction

The aim of this paper is to introduce an event by event separation technique based on the different topology of the showers produced by gamma and hadron-initiated showers, both as far as the space and time distribution in the shower front are concerned. A detailed picture of the shower is therefore mandatory for the method to be applied. The main idea is to exploit at best the differences in the multifractal behaviour of the space distribution and the differences in the arrival time distribution for showers initiated by different progenitors.

Our technique uses a multifractal-wavelet and lacunarity based analysis similar to the one used by Rastegaarzadeh and Samimi [1]. We improve Rastegaarzadeh and Samimi method by introducing a lacunarity parameter on time arrival of the showers and an artificial neural network that allows better separation. The concept of lacunarity quantifies the geometric arrangement of gaps in solid objects (lacunae). It can be extended to the description of distribution of data sets including, but not restricted to, those with fractal and multifractal distributions. We use lacunarity to describe gaps in time arrivals of showers.

The method applies at best in the experimental framework where a detailed picture of the shower front is given. We have been testing it using the experimental framework of the ARGO-YBJ detector [2]. ARGO-YBJ is a compact array of Resistive Plate Chambers (RPCs) with a sampling area of about 100x110 m^2, consisting of a central carpet of about 5800 m^2 with 92% coverage, surrounded by a guard ring with coarser sampling. Thus, in the central carpet surface a fully detailed map of the shower front can be worked out, with a space resolution for multiple hit counting given by 6.75x61.80 cm^2 (the RPCinduction strip size) and a time resolution of the order of 1 ns in a slightly larger unit, as wide as 8 contiguous ORed strips (55.6x61.8 cm^2).

2 Multiscale wavelet and fractal analysis

The wavelet transform is, by definition, the decomposition of a function on a basis obtained by translation and dilation of a particular function localized in both physical and frequency space. A wavelet analysis of a density field associates each point with a real number which represents the smoothed local density contrast at a given scale (see for example Pagliaro and Becciani [3], Gambera et al [4]). Fractal analysis, on the other hand, is concerned with the measurement of the local smoothness of the signals [5]. A deterministic fractal is defined using the concept of self-similarity. The fractal dimension D plays a central role: it is a measure of how the members are distributed in space.

Intuitively, the larger the fractal dimension, the rougher the texture [6]. Self-similar multifractals are geometrical objects invariant by dilation. Multifractality is not characterized by a single fractal dimension, but by a function.

The large numbers of interactions in showers from the same progenitor are self-similar. For this reason showers may be characterized by their fractal dimension. Moreover, the distribution of the shower particles near the core has a self-similar character different from those far away. Different physics is involved in production of different secondary particles. For these reasons, extensive air showers have a multifractal behaviour and more than one fractal dimension.

In our method, for a distribution of particles on a plane, the number of particles inside a radius R is computed. If a scaling law of the form $N(R) \propto R^D$ holds and D is a single non-integer value, the distribution has a fractal distribution with dimension D. Multifractal behaviour can be revealed by studying the scaling laws for secondary particles at different core distances. Holschneider [7] has shown that if a function f has scaling law with exponent D around x_0: $f(R(x_0, \lambda\epsilon)) \sim \lambda^D f(R(x_0, \epsilon))$ then the wavelet transform $W(s,t) = \int g(x; s, t) f(x) dx$, where s is the single dilation parameter (scale) and t is the translational parameter, has the same scaling exponent, if the wavelet g has a zero average and decays fast at infinity. Thus the local scaling behaviour is represented by $W(s,t) \sim s^{D(t)}$. Therefore for any distribution function f the slope of the plot of $\log W(s,t)$ versus $\log s$ will give the fractal dimension of the distribution around point t for the range of the scale s.

3 Lacunarity

Lacunarity is the deviation of a fractal from translational invariance and can be extended to the description of spatial distribution of real data sets, including those with multifractal distributions. Lacunarity is defined as $\Lambda = E(\frac{M}{E(M)} - 1)^2$ where M is the mass of the fractal set (defined as the total number of points in the image) and $E(M)$ is the expected value of the mass computed for the fractal. This measures the discrepancy between the actual mass and the expected value. Lacunarity is small when texture is fine and large when texture is coarse. The mass of the fractal set is related to the length by $M(L) = kL^D$.

Among the algorithms proposed to measure lacunarity, we adopt the gliding box method of Allain and Cloitre [8]. We assess time pattern lacunarity by gridding each time arrival array into squares as in McIntyre and Wiens [9]. In this protocol, a box is superimposed on the map. The number of full square contained inside the box is tallied. Then the box moves one unit up. The size of the box is initially four squares and then is enlarged adding squares until the box size equals the length of the array. A frequency distribution is then created. This frequency distribution is then converted to a probability distribution. Lacunarity is then computed from these probability values by determining the variance and mean of the number of full squares per gliding box: $\Lambda = [(variance/mean^2) + 1]$ and may assume any value between 1 and ∞. A pattern lacunarity 1 indicates that full squares are uniformly dispersed at a given scale. Values $\neq 1$ indicate non-uniformity in the distribution.

4 Simulations of extensive air showers

To test our separation technique we analysed simulated showers initiated both by gamma-rays and protons. The showers were simulated by means of the CORSIKA code with QGSJET model [10], and the ARGO-G code [11] to simulate the pattern of the shower front hit as detected in the ARGO-YBJ detector. A detailed description of the ARGO-YBJ detector performances is beyond our scope. A total number of 30000 events were chosen for the analysis. The showers have zenith angles $0° < \theta < 15°$ and primary energies from $E \sim 1 \cdot 10^{10} eV$ to $E \sim 1 \cdot 10^{14} eV$.

The spatial distribution of the secondary particles at ARGO altitude ($4300m$ a.s.l.) is used for the analysis. The time analysis is performed on the shower from time zero (first detection on the carpet of a secondary particle) to time 2000 ns. The output of the simulated showers includes the effect of the detector response as far as space and time distribution on the carpet is concerned.

5 Spatial separation technique

Our spatial separation technique uses the differences in wavelet based multifractal behaviours of showers of different progenitors. These are: the mean and the standard deviation of individual Gaussian fits to the distributions of multifractal dimensions.

For each shower, the dependence with respect to the distance from the shower core of the fractal dimension is almost linear. This has a consequence: we need to specify the mean and the standard deviation just for two different mean distances from the core. So we choose to analyze two regions: a circle of fixed radius r_C centred at the shower core (inner ring) and a ring with fixed inner and outer radii r_i and r_o (outer ring). In our analysis we choose the values: $r_C = 7$ m, $r_i = 9$ m, $r_o = 12$ m. The radii have been chosen such that on average the two rings contain about an equal number of hits. Our shower core is computed on the wavelet transform of the spatial map on the maximum scale ($s = 2^5$). On that scale, in this analysis, the shower core is found as the coordinates $xcore$ and $ycore$ of the maximum value of the wavelet coefficients. The multifractal behaviour of each individual shower is then specified by four quantities: the mean of the Gaussian fit in the inner and the outer ring (μ_I, μ_O) and the standard deviation in the inner and the outer ring (σ_I, σ_O). The set of scales are powers of two: $s = 2^r$ and the first scale always corresponds to the size of 1 pixel. The scale s may be considered as the resolution. We choose to investigate on the scales 2 to 32 that on the ARGO-YBJ carpet correspond to physical sizes $\approx$ 1.2 m and $\approx$ 20 m.

As pointed out in Section 2, if a function f has scaling law with exponent D around x_0: $f(R(x_0, \lambda\epsilon)) \sim \lambda^D f(R(x_0, \epsilon))$ then the wavelet transform $W(s,t) = \int g(x;s,t)f(x)dx$ has the same scaling exponent, and thus the local scaling behaviour is represented by $W(s,t) \sim s^{D(t)}$. We compute therefore a $log(W)/log(s)$ matrix for each scale s on the two regions selected (inner and outer ring). $D = log(W)/log(s)$ is the fractal dimension and has a Gaussian distribution. If μ_I, μ_O are the average values of the distribution of D in the Inner (I) and Outer (O) regions and σ_I, σ_O their standard deviations we compute average values of $\mu_I, \mu_O, \sigma_I, \sigma_O$ on all the scales and obtain our parameters: $\overline{\mu}_I, \overline{\mu}_O, \overline{\sigma}_I, \overline{\sigma}_O$.

All these quantities strongly depend on the nature of the progenitor. However, they show large fluctuations, prohibiting de facto discrimination. So, to get a high-resolution separation technique, we need at least one more parameter and then define and train an artificial neural network.

6 Separation technique on time arrivals

Our separation technique on time arrival of the shower use the lacunarity technique in the same two regions as the spatial separation: the inner and outer ring. First, we need to compute our time array as $T = T_{max} - T_{min}$ where T_{min} is the time arrival of the first secondary particle on the carpet and is set to 0 and T_{max} is the time arrival of the last secondary particle we include in our analysis. Maximum value of T is 2000 ns. Then we need to define the time scale on which we compute lacunarity. This is a crucial parameter. We call it t_{lac}.

A box of length t_{lac} is placed at the origin of the sets. The number of occupied sites within the box (box mass k) is then determined. The box is moved one space along the set and the mass is computed again. This process is repeated over the entire set, producing a frequency distribution of the box masses $n(k, t_{lac})$. This frequency distribution is converted into a probability distribution $Q(k, t_{lac})$ dividing by the total number of boxes $N(t_{lac})$ of size t_{lac}: $Q(k, t_{lac}) = n(k, t_{lac})/N(t_{lac})$

The first and second moments of the distribution Z_1 and Z_2 are computed: $Z_1(t_{lac}) = \sum_k k \cdot Q(k, t_{lac})$, $Z_2(t_{lac}) = \sum_k k^2 \cdot Q(k, t_{lac})$. The lacunarity is now defined as: $\Lambda(t_{lac}) = Z_2/Z_1^2$. Lacunarity is computed both in the inner and the outer ring (Λ_I, Λ_O). We find that 5 ns is a good choice for the t_{lac} parameter.

7 A standard three layer neural network

We assume therefore that the mass of the progenitor can be estimated with the use of an artificial neural network of seven variables. The neural network is a standard three layer perceptron with only one output neuron (1=hadron, 0=gamma). The neural network we choose is of the feed forward type. The input is made of the six neurons described before: (1,2) average of the fractal dimensions on the spatial scales in the inner and outer region: $\overline{\mu}_I, \overline{\mu}_O$; (3,4) average of the standard deviation of the fractal dimensions on the spatial scales in the inner and outer region: $\overline{\sigma}_I, \overline{\sigma}_O$; (5,6) lacunarity of the time arrivals arrays in the inner and outer region: Λ_I, Λ_O, plus, as seventh neuron, the time array (difference between first and last hit, see Section 6): $T = T_{max} - T_{min}$.

The hidden layer is made of four neurons, while the output vector is defined in a one dimensional space and it is trained to be 0 for gamma initiated events and 1 for hadronic ones. Network was implemented and optimized by using the Stuttgart Neural Network Simulator Tool (SNNS) [12]. SNNS is a simulator for neural networks developed at the Institute for Parallel and Distributed High Performance Systems at the University of Stuttgart. The network was trained by using 1000 events from indipendent samples for each multiplicity for a total of 10000 events for each progenitor.

Nhits	$< E_\gamma >$	$< E_h >$	ev(γ + h)	Q
$20 \div 99$	0.4 TeV	0.6 TeV	4000	1.32
$100 \div 299$	1.0 TeV	1.5 TeV	4000	1.84
$300 \div 599$	2.6 TeV	4.0 TeV	4000	1.96
$600 \div 999$	4.5 TeV	7.2 TeV	4000	2.37
$1000 \div 5000$	10.1 TeV	17.2 TeV	4000	2.98

Table 1: Q values for γ/h discrimination

8 Test run and results

The most important parameter in gamma/hadron discrimination is the Q (quality) factor. The Q factor is defined as $Q = \frac{\varepsilon_\gamma}{\sqrt{1-\varepsilon_h}}$ where ε_γ is the fraction of showers induced by photons correctly identified by the discrimination criterion and ε_h is the fraction of showers induced by protons correctly identified by the discrimination criterion so that $1 - \varepsilon_h$ is the background contamination.

In our test run Q values from 1.32 to 2.98 have been obtained (see Table 1). Q values strongly depend on the number of hits. In Figures 1 to 5 we present the results of applying our method to 20000 simulated showers of two different primaries (10000 γ and 10000 hadrons) with number of hits on the carpet between 20 and 5000. The neural network was trained on similar not overlapping sets of 2000 events for each multiplicity range on 5000 cycles. As it is seen in the histograms, the identification is achieved with a good resolution if a number of hits greater than 100 is provided.

We are presently working to obtain better results in the low energy ranges, i.e. the most populated one, as well as to extend the analysis to inclined showers.

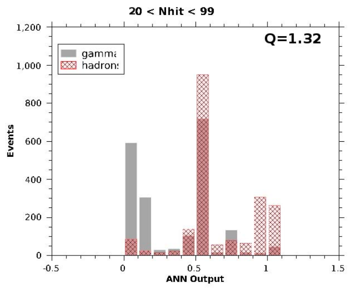

Figure 1: Gamma-hadron separation for 20 to 99 hits

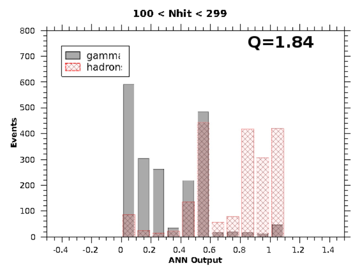

Figure 2: Gamma-hadron separation for 100 to 299 hits

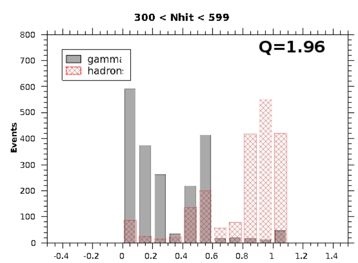

Figure 3: Gamma-hadron separation for 300 to 599 hits

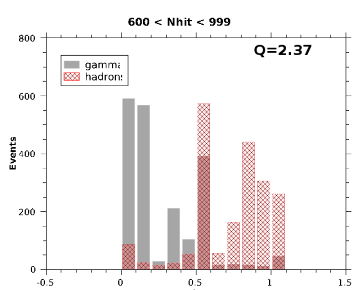

Figure 4: Gamma-hadron separation for 600 to 999 hits

9 Conclusions

A new technique for separating extensive air showers initiated by different progenitors, based on multifractal and

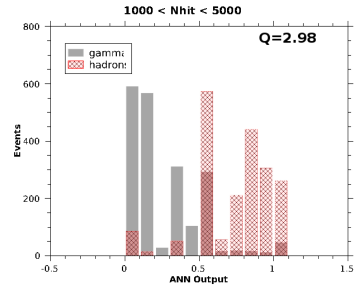

Figure 5: Gamma-hadron separation for 1000 to 5000 hits

lacunarity analysis, has been developed. The multifractal behaviour and lacunarity time structure of each shower has been represented by seven variables. Due to large shower to shower fluctuations, the differences in any single one of these variables have a very poor separation power [13]. So, on these seven quantities, a neural network analysis has been performed. Network were implemented and optimized by using the Stuttgart Neural Network Simulator Tool and trained by using events from indipendent samples.

It is well known that the most important parameter in gamma/hadron discrimination is the Q (quality) factor.

Our approach gives good results, leading to an improvement of the discrimination power with respect to the existing figures for extended shower detectors. The technique shows up to be very promising and its application may have important astrophysical prospects in different experimental environment of extended air shower study.

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