Perturbation method for investigation of the galactic cosmic ray variations

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Abstract. Perturbation methods are applied in a mathematical modelling of propagation of the galactic cosmic rays (GCR) in the heliosphere. Perturbation theory is based on the possibility of approximate description (by power series expansions according to a very small parameter) of a differential equation’s solution using specially selected ‘easier’ solution which can be precisely studied. We use a nonlinear perturbation approach in connection with different terms of the GCR modulation phenomenon, i.e. convection, diffusion and energy changes of the GCR particles. The effects of these studies are compared with our previous results obtained based on the well known Parker’s transport equation [4].

In our first approach we treat perturbation in its physical and historical sense, more as disturbances [2] of some phenomenon, than a finding an asymptotical solution of a mathematical problem [1]. We take into account a heliolongitudinally disturbed parallel diffusion coefficient in a classical Parker’s transport equation in 3-D space [4] in a form:

\[ K_\alpha = K_\rho(\rho, \phi)K(\phi)K(R) = K_\rho(1 + 50\rho)(1 + 0.09\sin \phi + 0.07\sin 2\phi + 0.05\sin 3\phi)R^\alpha \]

where \( R \) is the particle rigidity, \((\rho, \phi, \theta)\) are spherical coordinates, \( K = 10^{23} \text{ cm}^2 \text{ s}^{-1} \) for \( R = 10 \) GV and \( \alpha = 0.5 \).

Results of our calculation obtained using difference scheme method presents Figure 1. It shows theoretical spectrum of the 27-day variation of the GCR intensity calculated at the helioequatorial plane at 1 AU distance (the Earth’s orbit). One can observe a quite good qualitative agreement with our experimental results [3] that the rigidity \( R \) power law spectrum of the amplitudes of the 27-day variation of the GCR intensity is harder in the \( A > 0 \) polarity period (1996-97) and is softer in \( A < 0 \) polarity period (1986-87). In the \( A > 0 \) epochs the theoretical power rigidity spectrum can be described as: \( A = 6.67 \left( \frac{R}{1 \text{ GV}} \right)^{-0.59} \)

and in the \( A < 0 \) epochs of solar magnetic cycle:

\[ A = 10.70 \left( \frac{R}{1 \text{ GV}} \right)^{-0.67} \]

Fig. 1: The expected spectrum of the 27-day variation of the GCR intensity \( A[\%] \) versus the rigidity \( R \) (GV) in the \( A > 0 \) (solid line) and \( A < 0 \) (dashed line) polarity periods of solar magnetic cycle calculated at the helioequatorial plane at 1 AU distance.

REFERENCES