Abstract. Galactic Cosmic Rays (GCRs) entering the Heliosphere, pass trough the interplanetary magnetic field (IMF) to reach the Earth surface; they undergo diffusion, convection, magnetic drift and adiabatic energy loss, resulting in a reduction of particle flux at low energy (<1-10 GeV) that depend of solar activity and polarity (modulation). A quasi time-dependent 2D Stochastic Simulation of Solar Modulation is improved to describe with confidence this scenario. The feature of the program takes also into account the newest Solar Wind dependency found by Ulysses measurement. Our model is a good agreement with data from both solar periods, we were able to reproduce proton fluxes measured close to the solar minimum in positive periods (e.g. AMS-01) and we have reproduced also the BESS 2002 proton flux, measured in a negative period of solar maximum. We finally present the prediction of the proton flux for the AMS-02 period (year 2010-2012), in negative solar polarity and near the maximum solar activity.

Keywords: Heliosphere, Cosmic Rays, Solar Magnetic Field

I. INTRODUCTION

Accurate models of the Heliosphere should take into account all the features of the solar cavity in order to reproduce measured data. The final goal of such approach is to compare the simulation results with the increasing number of observed fluxes. We used the Parker field model for the heliosphere [1]. We have implemented a two dimensional (radius and heliocolatitude) drift model of GCR propagation in the heliosphere [2] that becomes time dependent due to the variation of the measured values of the solar wind velocity in the ecliptic plane ($V_0$), tilt angle ($\alpha$) and estimated diffusion coefficient ($K_0$). This model is including curvature, gradient and current sheet drifts, which are depending on the charge sign of particles and magnetic field polarity. In section III-C we present a study of the diffusion coefficients in relation to our approach, forward in time, in order to evaluate the time scale of the magnetic perturbation propagation with the solar wind.

II. STOCHASTIC 2D MONTE CARLO MODEL

Our model is based on the Fokker-Planck equation (hereafter FPE) for GCR transport in the heliosphere without drift terms ([4] and [5]):

$$\frac{\partial f}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 K_r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( K_{\theta \theta} \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{3r^2} \frac{\partial}{\partial r} \left( r^2 V \frac{\partial (\Gamma T f)}{\partial T} \right) - \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 V f \right)$$

(1)

Where $f$ is the cosmic ray number density per unit interval of particle kinetic energy, $t$ is the time, $T$ is the kinetic energy (per nucleon), $r$ is the heliocentric radial distance, $\theta$ is the heliocolatitude, $V$ the solar wind velocity and $\Gamma = (T + 2T_0)/(T + T_0)$ where $T_0$ is proton’s rest energy. Thanks to a rigorous mathematical proof by Ito [6] there is an exact equivalence between the FPE and a set of stochastic differential equations (SDE). These equations have been integrated with a montecarlo-stochastic technique in order to obtain the particle flux on Earth. The integration time step($\Delta t$), strictly related to the accuracy during each step of the integrating process, is taken to be proportional to $r^2$ to save CPU time[7]; in this way we were also able to increase the accuracy in the inner heliosphere, where our interest is focused, having a lower, but still acceptable, precision in the outer heliosphere.

The radial diffusion coefficient is $K_r = K_{r0} \cos^2 \psi + K_{r1} \sin^2 \psi$, where $\psi$ is the angle between radial and magnetic field directions [4]. The latitudinal coefficient is $K_{\theta \theta} = K_{\perp}$, the parallel and the perpendicular diffusion coefficients [8] are $K_{\parallel} = K_0 \beta K_P (B_0/3B)$ and $(K_{\perp})_0$ is the ratio between parallel and perpendicular diffusion coefficient, so $K_{\perp} = (K_{\perp})_0 K_0$ where $K_0 = 1 - 6 \times 10^{22}$ cm$^2$ s$^{-1}$, $\beta$ is the particle velocity, $P$ is the CR particle’s rigidity, $K_P (P)$ take into account the dependence on rigidity (in GV), $B_0 \sim 5$ nT is the value of heliospheric magnetic field at the Earth orbit, and $B$ is the Parker field magnitude [9]. In our model the solar wind speed $V(\theta)$ is a function of the heliocolatitude.
\[ V = V_0(1 + |\cos \theta|), \] for \(30^\circ < \theta < 150^\circ,\]

\[ V = 750 \text{ km s}^{-1}, \] for \(\theta \leq 30^\circ\) and \(\theta \geq 150^\circ.\]

Drift effects are included through analytical effective drift velocities: in this spiral field we evaluated the three components of drift (gradient and curvature drift plus drift along the neutral sheet, \([11]\)) that modify the integration path inside the heliosphere in this way:

\[
\Delta r_d = \Delta r + \left( v_g + \frac{\vec{B}}{r} \right) \Delta t
\]

\[
\Delta \mu_d = \Delta \mu - \left( \sqrt{1 - \mu^2} \right) \frac{v_\theta \Delta t}{r}
\]

(2)

\[ \Delta r \text{ and } \Delta r_d \text{ are the radial variation without and with drift effect, } \Delta \mu \text{ and } \Delta \mu_d \text{ are the latitudinal variation of the particle due diffusion and drift where } \mu = \cos \theta, \]

\[ v_g \text{ is the velocity of gradient drift, } v_\theta \text{ is the velocity of neutral sheet drift and } v_d \text{ is the velocity of curvature drift.} \]

The average drift velocity is:

\[ v_d = \frac{p}{3q} \left( \vec{B} \times \frac{\vec{B}}{B^2} \right) \]

We adopted the mathematical solution from Hatting and Burger \([12]\), the so-called wavy neutral sheet (WNS) model, for steady-state drift dominated modulation (our case). In this model the average of the drift velocity \((3)\) is taken over one solar rotation. As Local Interstellar Spectrum of protons (LIS) we used Burger’s model \([13]\).

III. MODULATION AND DATA SETS

A. Main parameters

We evaluated the dependence of the modulation effect from the main parameter, i.e. the diffusion coefficient value, in the quasi linear theory approximation (i.e. \(K_P = P/1GV\)). The modulation, as a function of the diffusion coefficient, follows the expected behaviour so, higher values of \(K_0\) corresponds to a lower modulation and vice versa. We also chose a fixed value for ratio between parallel and perpendicular diffusion coefficient, \((K_\perp)_0 = 0.05\). We used (see \([2]\) for a complete description) the tilt angle \(\alpha\) (of the heliospheric current sheet HCS) as a parameter for the level of the solar activity: the higher the value of \(\alpha\) the lower the expected GCR flux, for both solar field polarities. Tilt angles are computed using two different models: the “classic” model uses a line-of-sight boundary condition at the photosphere and includes a significant polar field correction. A newer, possibly more accurate, model uses a radial boundary condition at the photosphere, and requires no polar field correction. The classical model is used for periods of increasing solar activity (fex example 2007-2012, AMS-01 data, AMS-02 data), while the newer model fits better for period of decreasing solar activity (for example 2000-2007, BESS data\([3]\)).

The three drift components (see \([2]\)) do not depend from any kind of external parameter, except the solar polarity \((A_0, 0\) for positive period and \(A_0, 0\) for negative period \([2]\)). The general expression \((3)\) is locally unlimited for a quasi-isotropic distribution (see \([14]\) and \([9]\)), so we kept all drift components below \((\pi/4) v\) that is its spatially averaged maximum value. Although the WNS model is satisfactory to us we plan to test different approaches to particles drift in the near future, for example the so called Burger-Potgieter (BP) and the Potgieter-Moraal (PM) models (see \([12]\)). We also adopted the modification of the polar field suggested by Jokipii and Kota \([15]\) in order to reduce the drift contribution close to the poles and the related high radial gradients not observed in the data.

B. Data set

We choose CR proton data from 2 different experiments in order to compare and tune our model results. These are: AMS-01 \([16]\) and BESS \([17]\). The first experiment took data in a period of positive solar polarity, while the second one is in negative solar period. The corresponding periods for measured proton flux are: June 1998 (AMS), and August 2002 (BESS). Solar wind values for selected periods were obtained from omniweb \([18]\) choosing 27 days average, while tilt angles from the Wilcox Solar Laboratory \([19]\) as reported in Table I. Diffusion coefficient is needed by our model to evaluate the proton CR modulation in different conditions, so we get it from \([20]\) and \([21]\), (see Table I). These values are based on long term study of neutron monitors measurements and the Force Field model approach to the Heliosphere (see \([22]\)).

<table>
<thead>
<tr>
<th>Detector</th>
<th>(\alpha) (deg)</th>
<th>(V_{sw}) (km/s)</th>
<th>(K_0) (au^2/s)</th>
<th>(\langle K_\perp \rangle_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMS-01</td>
<td>30°</td>
<td>408</td>
<td>1.63 \times 10^{-4}</td>
<td>0.05</td>
</tr>
<tr>
<td>BESS</td>
<td>36°</td>
<td>422</td>
<td>0.87 \times 10^{-4}</td>
<td>0.05</td>
</tr>
</tbody>
</table>

C. Average parameters

Our model has been realized to reproduce the real behaviour of CR entering the heliosphere from its outer limit, the heliopause, located approximately (see \([23]\)) at 100 AU, and moving down to the Earth at 1 AU. Besides we started using fixed values of tilt angle \(\alpha\), solar wind speed \(V_{sw}\) and diffusion coefficient \(K_0\). The question is if this is a good approximation to reproduce the solar modulation of CR. First we evaluated the time \(t_{sw}\) needed by the Solar Wind to expand from the outer corona up to the heliopause. With an average SW velocity of 400 km/s it takes something like 14 months. On the contrary the period \(t_{sc}\) of the stochastic evolution of a quasi particle inside the heliosphere from 100 AU down to 1 AU is between a little more than 1 month (200 MeV) and few days (10 GeV). This scenario, where \(t_{sc} < t_{sw}\), clearly shows that fixed parameters are not enough for such a study. In fact if for example we are interested in the CR modulation of a period \(t_X\) at 1 AU,
this is related to a different condition at 100 AU where particles are injected: we can say that the conditions are those present at Earth almost 14 months before. As a first approach, we first decided to move from fixed values of $\alpha$, $V_{sw}$, $K_0$, to average values of these quantities 14 months back in the past, $\alpha^{\text{back}}$, $V_{sw}^{\text{back}}$, $K_0^{\text{back}}$ (see Table II).

Our goal is to reproduce the changing heliospheric scenario, so different values of $\alpha$, $V_{sw}$, $K_0$ for the same particle trajectory, in relation of the time spent by the particle interacting with the Sun magnetic field, i.e. depending on particle’s energy (for a preliminar approach see [24]). Then as a second approximation we started to have a dynamic diffusion coefficient: if we consider $\tau_{ev}$ negligible with respect to $t_{sw}$ we divide the heliosphere in regions of different $K_0$ value, in relation to the expansion velocity $V_{sw}$. So our set of parameters is made of $\alpha^{\text{back}}$, $V_{sw}^{\text{back}}$ averaged over 14 months back in the past, and 14 different values of $K_0$ for 14 different zones (as a function of the radius). Future steps will be to consider dynamical values for all the parameters as function of the radius and, later on, correlate the evolution of the heliosphere with the real time spent by the CR particle inside of it, depending on the stochastic path but also from the particle energy.

Our simulation code has been used not only to reproduce two sets of past data (like AMS-01 and BESS), but also for future measurements. The periodical behaviour of the Heliosphere allow us to predict with a certain level of precision, the future solar conditions and so the parameters needed for our simulation. In particular we choose the AMS-02 launch date (September 2010) and two different periods in the following years, close to the solar maximum (June 2011 and March 2012), still inside the 3 to 5 years of the AMS-02 mission.

For these periods we considered the prediction of Smoothed Sunspot Numbers from IPS (Ionospheric Prediction Service) of the Australian Bureau of Meteorology [25], then we looked for periods with similar condition, in the same solar polarity, so 22 years before, using SIDAC data (Solar Influences Data Analysis Center) [26]. We assume that the values of the diffusion coefficient, the tilt angle and the solar wind speed repeat themselves almost periodically and near-regularly between two consecutive 22 years cycles. Under this conditions we used the values measured in that periods as an estimation for future conditions of heliosphere.

Any way, this prediction are to be considered as preliminary since the Sun is in an unpredicted solar minimum that forced scientists to review all their estimations for next solar cycle [27].

IV. RESULTS

We started our simulations from the values in Table I choosing $(K_\perp)_0 = 0.05$ for both periods, so with $A > 0$ and $A < 0$. In parallel we also performed a simulation with a “dynamic” $K_0$ keeping, as stated in III-C, $\alpha$ and $V_{sw}$ fixed to average 14-months values. Results of these simulations are shown in Figures 1 and 2. For the period with positive solar polarity (AMS-01 data) it can be clearly seen that simulated fluxes with fixed values of the main parameters ($K_0$, $V_{sw}$ and $\alpha$) are different from measured data, in particular they are much lower. Simulated fluxes with dynamic values of $K_0$ show a better agreement with measured data, within the quoted error bars. For negative periods simulations (BESS) the behaviour is different: simulated fluxes with fixed parameters show a better agreement with measured data, while results with dynamic $K_0$ show a flux a little higher.

We must underline that the difference in this case is not as much as that one in the positive period, but still is larger than experimental error bars.

This difference may be due also to the fact that the back-time averages are too close to the magnetic field inversion (approx. at the beginning of year 2001). We are still investigating this possibility and the effective difference of $(K_\perp)_0$ values, quoted by many authors, between the two polarities. In Fig. 3 we show the prediction of GCR modulation for the AMS-02 mission. Because our study is still in progress, these fluxes are obtained for now with fixed parameter values; we plan to use also the dynamic model and study the difference between the fluxes we obtain.
V. CONCLUSIONS

We built a 2D (radius and heliocolatitude) stochastic model of particles propagation across the heliosphere. Our model takes into account drift effects and shows quantitatively good agreement with measured values. Proton spectra, as predicted by the model, are decreasing with increasing tilt angles and solar wind velocity. We implemented in the 2D model the possibility to calculate modulated proton spectra at different heliospheric distances. We compared our simulations with measured data from 2 different experiments, AMS and BESS. Starting from fixed values for the main parameters ($K_0$, $\alpha$ and $V_{sw}$) for the related period, we moved to averaged values of the same parameters back in the past, the first step, and then to a dynamic parametes, in order to reproduce the propagation of incoming CR through magnetic disturbances carried by the outgoing solar wind.

This approach seems to be better for positive periods at a first approximation, but quite satisfactory also for negative periods, and will be investigated deeply in relation to solar conditions and the particle energy, including other parameters. The dynamic approach to the heliosphere, as stated in Sec. III-C, and the forward approach, seems to be more close to the real physical propagation of GCR in the solar cavity. Introducing a dynamic evolution of the tilt angle $\alpha$, and the solar wind $V_{sw}$, together with long time data in the negative period, as AMS-02, will increase our knowledge of the complex structure and processes due to our closest star, and how to reproduce them.

Fig. 2. 2D model: CR modulation flux at Earth, BESS estimated parameters, $K_0$ dynamic, $V_{sw}$, and $\alpha$ from Table I and II

Fig. 3. 2D model: CR modulation flux at Earth for AMS-02

REFERENCES