Electron spectrum with stochastic reacceleration

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Abstract. Based on the diffusion-halo model for cosmic-ray (CR) propagation, including stochastic reacceleration due to the collision with the hydro-magnetic turbulence, we study the behavior of the electron component. In the present work, we focus upon the average behavior of the diffuse electrons in the steady state, leaving aside those from nearby sources. The Galactic parameters appearing in these studies are essentially the same as those appearing in the hadronic CR components. We compare our numerical results with the recent experimental data on CR hadrons (secondary-to-primary ratio) and electrons, particularly remarking the most recent data by FERMI-LAT. Aside from the structure of the electron spectrum around 500 GeV, we conclude that the reacceleration process actually occurs during the passage of the CR in the Galaxy.

Keywords: CR electrons, propagation, reacceleration

I. INTRODUCTION

We have studied the three-dimensional CR propagation model for various hadronic components, stable primaries [1], secondaries such as borons and sub-iron (Z=21-23) [2], isotopes such as $^{10}$Be [3], and antiprotons [4] as well. Our model is based on the diffusion-halo model [5], taking the reacceleration process into account. We found that they are all consistent with the experimental data now available using the same galactic parameters, within the uncertainties in the experimental data and in various kinds of cross-sections.

In order to apply our model to the electron component and electron-induced diffuse-$\gamma$’s (D$\gamma$’s), we need information on the interstellar radiation field (ISRF), in addition to the interstellar matter (ISM), particularly their spatial gradients for the study of the ($l$, $b$)-distribution of D$\gamma$’s ($l$: longitude; $b$: latitude), while the smeared spatial distribution of the gas density in ISM, $n(r)$, and energy density in ISRF, $\epsilon(r)$, is applicable for the electron spectrum as discussed later. This is because the charged CRs are well mixed during their propagation in the Galaxy over a residence time of approximately $10^6$y, effectively smearing the local inhomogeneous structure of ISM and ISRF. However, we don’t touch the details of the interstellar environment of our Galaxy here, which are based on two reviews by Mathis et al. [6] and Ferriere [7], but present orally in the conference because of limited space.

II. ENERGY LOSS AND GAIN

The energy loss processes for the electron component are dramatically different from those for the hadronic components. There are four main processes: ionization (\(\equiv\) “ion”), electron bremsstrahlung (EB), synchrotron radiation (SY) and inverse Compton (IC). These each have quite different dependence on the incident electron energy, $E_e$: nearly constant for “ion”, in proportion to $E_e$ for EB (\(\equiv\) “rad”), and to $E_e^2$ for SY+IC (\(\equiv\) “sic”).

Let us summarize them explicitly in the following,

$$\frac{\langle \Delta E_e \rangle}{\Delta t}_{\text{ion}} = \bar{n}(r)w_{\text{ion}}(E_e), \quad (1a)$$

$$\frac{\langle \Delta E_e \rangle}{\Delta t}_{\text{rad}} = \bar{n}(r)w_{\text{rad}}[E_e/\text{GeV}], \quad (1b)$$

$$\frac{\langle \Delta E_e \rangle}{\Delta t}_{\text{sic}} = \bar{\epsilon}(r)w_{\text{sic}}[E_e/\text{GeV}]^2, \quad (1c)$$

typical values of $w_{\text{ion}}$, $w_{\text{rad}}$, and $w_{\text{sic}}$ are summarized in Table 1 in the case of $E_e = 1$ GeV. In practice we take into account the screening effect for EB [8, 9], $w_{\text{rad}} = w_{\text{rad}}(E_e)$, and Klein-Nishina cross-section for IC [10, 11], $w_{\text{sic}} = w_{\text{sic}}(E_e)$.

For the energy gain due to the reacceleration process (RA), we introduced a parameter related to the efficiency of the RA process, $\zeta_0$, corresponding to the cross-section for the occurrence of collision with the turbulence [2],

$$\zeta_0(r) \approx \frac{4}{9} \frac{v_{\text{rel}}^2}{n_0^2eD_0^2}, \quad (2)$$

and assumed it is independent of the position $r$. Thus the average energy gain and its fluctuation per unit time are written as [2]

$$\langle \frac{\Delta E_e}{\Delta t} \rangle_{\text{rea}} = \bar{\epsilon}(r)w_{\text{rea}}[E_e/\text{GeV}]^{1-\alpha}, \quad (3a)$$

$$\langle \frac{(\Delta E_e)^2}{\Delta t} \rangle_{\text{rea}} = \frac{1}{2} \bar{n}(r)w_{\text{rea}}[E_e/\text{GeV}]^{2-2\alpha}. \quad (3b)$$

with $w_{\text{rea}} = \epsilon_0[\text{GeV/cm}^3] = 1.5 \times 10^{-15} \text{ GeV/s}$, assuming $\zeta_0 = 50 \text{mb}$ [2].

Now, we introduce a parameter, $\eta_0$,

$$\bar{\epsilon}(r)/\bar{n}(r) \approx \langle \bar{\epsilon}(r)/\bar{n}(r) \rangle_{\text{rea}} = \eta_0, \quad (4)$$

TABLE I: Typical values of $w_{\text{rea}}$ at $E_e = 1$ GeV (\(\equiv\) “a”\(\equiv\) “ion”, “rad”, “sic”, “rea”) in units of $10^{-16}$ GeV/s.

<table>
<thead>
<tr>
<th>$w_{\text{rea}}$</th>
<th>$w_{\text{rad}}$</th>
<th>$w_{\text{sic}}$</th>
<th>$w_{\text{rea}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 10^3$</td>
<td>$3 \times 10^5$</td>
<td>$1 \times 10^4$</td>
<td>$1 \times 10^3$</td>
</tr>
</tbody>
</table>
corresponding to the effective ratio of the energy density of ISRF to the gas density of ISM. While in reality \( \eta_0 \) must have some dependence on \( r \), we assume it is independent of \( r \), and regard it as a free parameter. A typical value of \( \eta_0 \) is 2\( \sim \)10 eV for \( \langle \tilde{\alpha} \rangle \approx 0.2\sim 1 \) atoms cm\(^{-3} \) with \( \langle \tilde{e} \rangle \approx 2 \) eV cm\(^{-3} \), which should be determined so that the experimental data are consistently reproduced.

Using \( \eta_0 \), let us write down the total average energy-loss and/or the energy-gain per unit time all together,

\[
- \langle \frac{\Delta E_e}{\Delta t} \rangle = \tilde{n}(r)w(E_e),
\]

\[
w(E_e) = \eta_0 w_{\alpha} E_e^2 + w_{\text{m}} E_e + w_{\text{a}} - w_{\text{a}} E_e^{1-\alpha}.
\]

III. TRANSPORT EQUATION AND SOLUTION

The transport equation for the electron density, \( N_e(r; E_e) \), is given [12] by

\[
\frac{\partial}{\partial t} \nabla \cdot D(r; E_e) \nabla + \Delta E_e \cdot N_e = Q(r; E_e),
\]

\[
\Delta E_e = \frac{\partial}{\partial E_e} \left( \frac{\Delta E_e}{\Delta t} \right) - \frac{1}{2} \frac{\partial^2}{\partial E_e^2} \left( \frac{\Delta E_e^2}{\Delta t} \right) + \text{rea}.
\]

We assume following forms for the diffusion coefficient, the source spectrum, and the spatial distribution of the smeared gas density

\[
D(r; E_e) = E_e^\alpha D(r),
\]

\[
Q(r; E_e) = E_e^{-\gamma} Q(r).
\]

\[
D(r) \equiv D(r, z) = D_0 \exp\left[+ (r/r_\alpha + |z|/z_\alpha) \right],
\]

\[
Q(r) \equiv Q(r, z) = Q_0 \exp\left[- (r/r_\alpha + |z|/z_\alpha) \right],
\]

\[
\tilde{n}(r) \equiv \tilde{n}(r, z) = \tilde{n}_0 \exp\left[- (r/r_\alpha + |z|/z_\alpha) \right],
\]

where the source index, \( \gamma \), is expected to be 2.0\( \sim \)2.1 from current models based on the supernova shock wave acceleration [13, 14], while the recent experimental data on CR hadronic components suggest \( \gamma \approx 2.4 \) with \( \alpha = \frac{1}{2} \) (Kolmogorov-type spectrum in the hydromagnetic turbulence), based on our propagation model [1–4].

For the steady state \( \frac{\partial}{\partial t} = 0 \), the solution of equation (7) is written as

\[
N_e(r; E_e) = \int_{E_e}^{\infty} H(x; r) f(x; E_e) dx,
\]

where \( H \) and \( f \) satisfy

\[
\left[ \tilde{n}(r) \frac{\partial}{\partial x} \nabla \cdot D(r) \nabla \right] \cdot H = Q(r) \delta(x),
\]

\[
\left[ \alpha E_e^\alpha \frac{\partial}{\partial E_e} \frac{\partial w(E_e)}{\partial E_e} - \frac{\alpha c_0^4}{4} \frac{\partial^2 \alpha E_e^2}{\partial E_e^2} \right] f = 0,
\]

with \( f(0, E_e) = E_e^{-\gamma-\alpha} \). Needless to say, we can not apply the above procedure for electrons coming from a nearby source, but have to find an age-dependent solution, \( N_e(r; E_e, t) \), which has been studied by several authors [12, 15].

Since \( H(x; r) \propto e^{-\sigma \cdot x} \), where \( \sigma = D_0 \left[ \tilde{n}_0, c_{\text{eff}} z_\alpha \right] \) and \( D_0 \equiv D(r, 0) \) (see [2] for more complete form), the Laplace transform of \( f(x; E_e) \) with respect to \( x \), \( F_r(E_e) \), is sufficient for our purpose to get the electron density, \( N_e(r; E_e) \), having no need of explicit form of \( f(x; E_e) \) for us,

\[
F_r(E_e) = \int_0^{\infty} e^{-\sigma \cdot x} f(x; E_e) dx,
\]

thus the slab equation (12b) is rewritten as, after applying the above Laplace transformation,

\[
\frac{c_\sigma E_e^\alpha}{\alpha} \frac{d w(E_e)}{d E_e} - \frac{c_\sigma_0}{4} \frac{\partial^2 \alpha E_e^2}{\partial E_e^2} \cdot F_r = E_e^{-\gamma}.
\]

It is, however, difficult to obtain the analytical solution for the above equation, so that we give it first neglecting the fluctuation term, \( [E_e^{-\gamma} F_r(E_e)]^\prime \), and then we give the solution with the fluctuation term, regarding it as a perturbative contribution.

Omitting the fluctuation term, \( [E_e^{-\gamma} F_r(E_e)]^\prime \), and replacing \( F_r \) by \( F_{r, 0} \), equation (14) is rewritten as

\[
\left[ c_\sigma^2 \frac{d w(E_e)}{d E_e} \right] \cdot F_{r, 0}(E_e) = E_e^{-\gamma}.
\]

Here we have to take care of a critical energy, \( E_e \), at which \( w(E_e) = 0 \) (see eq. [6]), but we have consider here the energy region \( E_e > E_e \) for the simplicity.

Now we obtain immediately

\[
F_{r, 0}(E_e) = \frac{1}{w(E_e)} \int_{E_e}^{\infty} dE_0 E_0^{-\gamma} e^{-Y_r(E_0)},
\]

with

\[
Y_r(E_0) \equiv Y_r(E_0, E_e) = c_\sigma \int_{E_e}^{E_0} \frac{E_0^\alpha}{\alpha} \frac{d E_0}{w(E_e)} d E_0.
\]

As the magnitude of the RA efficiency is only as large as \( \zeta_0 \approx 50 \text{ mb} \) [2], we can regard the fluctuation term, \( [E_e^{-\gamma} F_r(E_e)]^\prime \), as the perturbative contribution in equation (14), so that we replace it by \( [E_e^{-\gamma} F_{r, 0}(E_e)]^\prime \), and put the solution, \( F_r(E_e) \), thus obtained as \( F_{r, 1}(E_e) \), where \( F_{r, 0}(E_e) \) is the solution without the fluctuation given by equation (16).

Then rewriting equation (14) as

\[
\left[ c_\sigma^2 E_e^\alpha \frac{d w(E_e)}{d E_e} \right] \cdot F_{r, 1} = E_e^{-\gamma} + \frac{c_\sigma_0}{4} [E_e^{-\gamma} F_{r, 0}]^\prime,
\]

the solution is immediately given by

\[
F_{r, 1}(E_e) = 1 + \frac{c_\sigma_0}{4} \int_{E_e}^{\infty} dE_0 \frac{[E_e^{-\gamma} F_{r, 0}(E_0)]^\prime}{w(E_e) F_{r, 0}(E_e)} e^{-Y_r(E_0)}.
\]

In Fig. 1, we present the contribution of the fluctuation in the RA process, \( F_{r, 1}/F_{r, 0} - 1 \), against \( E_e \), corresponding to the second term in the right-hand side of the equation (19) for \( \zeta_0 = 40, 50, 60 \text{ mb} \) each with three cases of \( \eta_0 \): 2, 6, 10 eV. Thus one finds that the contribution of the fluctuation of the RA is as large as 25 % for \( \zeta_0 = 50 \text{ mb} \), the magnitude of which reproduces most consistently the CR hadronic components as presented in
TABLE II: Summary of key parameters in the present paper.

<table>
<thead>
<tr>
<th>param.</th>
<th>typical values</th>
<th>physical meaning and/or interpretation for critical parameters appearing in the present work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = \gamma + \alpha$</td>
<td>2.6 - 2.8</td>
<td>asymptotic index of the CR-proton energy spectrum at SS in the high energy region, $E_p^{\gamma}$ with $E_p \gg 1$ GeV</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1/3 - 1/2</td>
<td>index of the rigidity-dependent diffusion coefficient, $D \propto R^\alpha$ (1/3: Kolmogorov-type, 1/2: Kraichnan-type)</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>2 - 10 eV</td>
<td>effective ratio of the energy density of the ISRF to the gas density of ISM, $\langle \eta(r) \rangle$</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>50 mb</td>
<td>reacceleration efficiency (or the effective cross-section for the occurrence of coll. with magnetic turbulence)</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>200 mb</td>
<td>inverse of the average path length, $1/\langle \tau \rangle$ (or the cross-section for the leakage from the Galaxy) at SS</td>
</tr>
</tbody>
</table>

For merely the normalization at the SS to estimate the absolute intensity of $D\gamma$'s, which appears in the next page of this volume [16]. The spatial inhomogeneities in the ISM and ISRF have no effect on CR charged particles, with the level of at most $10^{-8}$ in the anisotropy. This is the reason why the simple leaky-box model and/or the simplified diffusion model such as, for instance, constant gas density and constant diffusion coefficient without spatial gradient, reproduces the CR hadronic components so well (for instance see [12]).

Fig. 1: Contribution of the fluctuation in the RA.

Before proceeding to a comparison with the observational data, we summarize the key parameters in advance for the convenience of the following discussions, which are presented in Table II together with typical values and their physical meaning and/or interpretation.

One should keep in mind that in order to calculate the CR hadron and electron densities, $N_p(r; E_p)$ and $N_e(r; E_e)$, we use smeared exponential-type spatial distributions for both the gas density of the ISM, $\tilde{n}(r)$ (eq.[10c]), and the energy density of the ISRF, $\tilde{\epsilon}(r) = \eta_0 \tilde{n}(r)$ (eq.[4]).

One might be concerned about the simplification of the smeared spatial gradients with $\tilde{n}(r)$ and $\tilde{\epsilon}(r)$. However, one should recall that $N_p(r; E_p)$ and $N_e(r; E_e)$ obtained by the use of $\tilde{n}(r)$ and $\tilde{\epsilon}(r)$ basically play a role
Fig. 5: Electron energy spectrum obtained by recent experiments, together with the numerical curves for two cases, (a) $\beta = 2.7$ and (b) $\beta = 2.8$, where the vertical axis is multiplied by $E_e^3$. See [22–29] for the experimental data.

Aprst from the ATIC-excess around 500 GeV, our model with the reacceleration seems to reproduce the data in the higher energy region, $\gtrsim 10$ GeV, well, where the solar modulation effect is small. The parameter $\eta_0$ depends on the choice of the index $\beta$, namely $\eta_0 = 4 \pm 6$ eV for $\beta = 2.7$, and $2 \pm 4$ eV for $\beta = 2.8$, which are reasonable values.

Looking carefully Fig. 5, however, the experimental data seem to deviate slightly from the numerical curves, with $20\sim 30\%$ higher than expected around 500 GeV. FERMI team also points out that a conventional model using the GALPROP code [32] produces much soft spectrum in contrast to the FERMI measurement. We have no space to discuss these problems as well as the reacceleration problem, which is presented in another paper in this volume [16].

REFERENCES