Studies of non-standard oscillations of atmospheric neutrinos at Super-Kamiokande

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Abstract: Mass-induced oscillations between muon neutrinos and tau neutrinos have become the standard theory accounting for the zenith angle distributions of atmospheric neutrinos. However, some non-standard effects can also induce neutrino oscillations. In this talk, the standard neutrino oscillation is compared with three different categories of non-standard ones. By comparing the minimum chi-square values of these different models, we find that the standard oscillation fits the atmospheric neutrino observation better than the non-standard ones significantly. We further study the constraints on the non-standard oscillation effects.

Introduction

The standard theory of explaining the atmospheric neutrino zenith angle distributions observed experimentally is the $\nu_\mu - \nu_\tau$ oscillation due to the neutrino mass eigenstate mixing. However, there are other non-standard effects which can also cause neutrino oscillations and they are of both theoretical and experimental interest. In this talk, we study three different neutrino oscillation models inspired by different theories. The first one is the oscillation between muon neutrinos and sterile neutrinos ($\nu_s$); the second is the $\nu_\mu - \nu_\tau$ oscillation in the context of Mass Varying Neutrino (MaVaN) model; the third is the oscillation caused by violations of Lorentz invariance (LIV) and CPT symmetry (CPTV).

The data we use in this study are Super-Kamiokande (SK) atmospheric neutrino data. For detailed information about the SK experiment and the detector, see Ref. [1].

The zenith angle oscillation analysis

Both oscillation analysis presented in Ref. [6] and Ref. [1] are using the least chi-square technique and the particular method adopted is called the “pull method.” In this method, the chi-square has two parts: the data bin chi-square term and the systematic uncertainty chi-square term.

$$
\chi^2 = \sum_{i=1}^n \chi^2_i + \sum_{j=1}^m \frac{\epsilon_j^2}{\sigma^2_{sysj}}, \quad (1)
$$

where $i$ is the bin index and $j$ is the systematic uncertainty index; $m$ is the number of bins and $m$ is the number of systematic uncertainties; $\chi^2_i$ is the chi-square of the $i$th bin; $\sigma^2_{sysj}$ is the $j$th systematic uncertainty and $\epsilon_j$ is the “pull” to it.

In the analysis framework of Ref. [1], the data bin chi-square terms are calculated in the following way:

$$
\chi^2_i = \frac{(N^\text{exp}_i - N^\text{obs}_i)^2}{\sigma^2_i}, \quad (2)
$$

here, $\sigma^2_i$ is the statistical variance of the $i$th bin which will be approached by an iteration process [1].

In the analysis framework in Ref. [6], due to the fining binning, some bins have very small number of entries thus the data bin chi-square terms are calculated using the likelihood ratio approach:

$$
\chi^2_i = 2(N^\text{exp}_i - N^\text{obs}_i + N^\text{obs}_i \ln \frac{N^\text{obs}_i}{N^\text{exp}_i}), \quad (3)
$$

In both Eq. 2 and Eq. 3, the effects of the systematic uncertainties on the expected number of entries
of the $i^{th}$ bin is considered in the following way:

$$N_i^{\text{exp}} = (1 + \sum_{j=1}^{m} f_j \epsilon_j) N_i^{\text{exp0}},$$

where $N_i^{\text{exp0}}$ is the expected number of events of the $i^{th}$ bin without considering the systematic uncertainties and $f_j$ is the response coefficients of the $i^{th}$ bin to the $j^{th}$ systematic uncertainty.

The best-fit parameters of a particular model are estimated by minimizing the chi-square in Eq. 1. To minimize the chi-square in Eq. 1, we prepare a grid in a pre-selected parameter space and minimize the chi-square at each grid point with respect to the pulls by solving a linear equation set which is obtained by taking $\frac{\partial \chi^2}{\partial \epsilon_k} = 0$ [1]. In the case of Eq. 3, we only keep the linear terms of $\epsilon_j$ and the higher orders are considered as corrections during an iteration process [6].

**Sterile neutrinos**

The $\nu_\mu - \nu_s$ oscillation has two signatures compared to the $\nu_\mu - \nu_\tau$ oscillation: 1. the depletion of neutral current (NC) events; 2. the matter effect. Thus, besides the charged current (CC) samples used in the standard $\nu_\mu - \nu_\tau$ oscillation analysis, in this analysis, the sub-GeV NC-enhanced sample is enriched by applying the cuts in Ref. [5] and the multi-GeV NC-enhanced sample consists of the leftover events after enriching the CC $e$-like events in Ref. [6]. For the typical atmospheric neutrino mass squared splitting and mixing angle, our simulation shows that the neutrino oscillation probabilities around and above 10 GeV are significantly suppressed due to the matter effect.

The best-fit results of two models are shown in Fig. 1 for some of the data samples. The minimal chi-square value at the best-fit points are 971 and 1024 for the $\nu_\mu - \nu_\tau$ and the $\nu_\mu - \nu_s$ oscillations respectively. The degree-of-freedom is 853. This chi-square difference corresponds to approximately 7 $\sigma$ exclusion level for the $\nu_\mu - \nu_s$ model. Although the $\nu_\mu - \nu_s$ model is excluded at a very high confidence level, the involvement of sterile neutrinos in the atmospheric neutrino oscillations is not completely excluded. Sterile neutrinos might get involved through an admixture. Using the

model presented in Ref. [4] based on a $2+2$ mass hierarchy, we are able to constrain the admixture level of sterile neutrinos in atmospheric neutrino oscillations. In this case, muon neutrinos are oscillating into a superposition state of tau neutrinos and sterile neutrinos: $|\nu_+\rangle = \cos \xi |\nu_\tau\rangle + \sin \xi |\nu_s\rangle$.

In this case, $\sin^2 \xi$ is the admixture level of sterile neutrinos. By assuming maximal mixing between $\nu_\mu$ and $\nu_+$, we find the best-fit prefers $\sin^2 \xi = 0$, which recovers the $\nu_\mu - \nu_\tau$ oscillation.

Shown in Fig. 2, we see that at 90% confidence level, 23% of sterile neutrino admixture is allowed.

**Mass varying neutrinos**

In this analysis, SK-I atmospheric neutrino data and the binning in Ref. [1] is used. The chi-square
\[ \Delta m^2 = \Delta m_0^2 \left( \frac{\rho}{\rho_0} \right)^n, \] (4)

where \( \Delta m^2 \) is the mass squared difference without matter density dependence, \( \rho_0 = 1 \text{ mol/cm}^3 \) and \( \rho \) is the matter density and \( n \) represents the dependence of \( \Delta m^2 \) on the matter density. When \( n = 0 \), we recover the standard oscillation case.

With the oscillation analysis which is presented in Ref. [1], we compare the minimal chi-squares of different cases with the one of the standard oscillation and find that it is unlikely for MaVaN models to explain the SK atmospheric observation. We further perform a study of a more general case by treating \( \Delta m_0^2 \) and \( n \) as the model parameters by assuming maximal mixing. Shown in Fig. 3, SK atmospheric data clearly prefer the standard oscillation case. The best-fit values are: \( n = -0.04 \) and \( \Delta m_0^2 = 1.95 \times 10^{-3} \text{eV}^2 \). The result can be understood as that at 90\% confidence level, the dependence on the matter density is allowed to be \(-0.12 < n < 0.1 \).

### LIV and CPTV

A theory which modifies the dispersion relation can as well induce neutrino oscillation even without neutrino masses. The model we adopt in this study is the minimal Standard Model Extension (SME) by Kostelecky and collaborators [8]. Some phenomenological results are based on Ref. [3] and Ref. [2].

The Lagrangian of the minimal SME has one interaction term breaking the Lorentz invariance and one breaking the CPT symmetry. We study the rotational symmetric cases by keeping only the temporal components of those two interaction terms. Flavor eigenstates can be superpositions of Maximal Attainable Velocities (MAV) [3] eigenstates defined by the LIV interaction term. Thus neutrino oscillations are induced. The survival probability for muon neutrinos are [3]:

\[ P_{\nu_\mu \rightarrow \nu_\mu} = 1 - \sin^2 2\theta_c \sin^2(c^{TT} LE/2), \] (5)

where \( \theta_c \) is the mixing angle between two MAV eigenstates and \( c^{TT} \) is the difference between two MAV’s. Similarly, the oscillation caused by the CPTV term takes the form [2]:

\[ P_{\nu_\mu \rightarrow \nu_\mu} = 1 - \sin^2 2\theta_a \sin^2(\Delta a L/2), \] (6)

where \( \theta_a \) is the mixing angle between two eigenstates defined by the CPTV interaction term and \( \Delta a \) is the difference between two eigenvalues of the matrix \( (a_L)_{AB} \) in the two generation case.

A more general formula of this form which considers the energy dependences of the LIV effect is as follows:

\[ P_{\nu_\mu \rightarrow \nu_\mu} = 1 - \sin^2 2\theta_c \sin^2(\kappa LE^{-\alpha}). \] (7)

Where real number \( \alpha \) is from the energy dependence effect of LIV and \( \kappa \) has dimension of energy to the power of \( \alpha - 1 \).

Our analysis using Eq. 7 shows that the oscillations caused by either LIV or CPTV are strongly disfavored by the SK observation compared to the standard oscillation case which is \( \alpha = 1 \). By considering that LIV and CPTV induced oscillations as the sub-dominant effects, we can put constraints on the symmetry-breaking parameters \( c^{TT} \) and \( \Delta a \).

Using the survival probability expressions in Ref. [3], we can study the limits on the LIV parameter \( c^{TT} \). We assume the best-fit values for
the mass eigenstate mixing and study two cases: $\Delta \phi = 0$ and $\Delta \phi = \pi$, where $\Delta \phi$ is the phase difference between the MAV mixing matrix and the standard mixing matrix. Shown in Fig. 4, at 90% confidence level, $\epsilon_T$ is allowed to be up to $1.2 \times 10^{-24}$ in the $\Delta \phi = 0$ case and up to $1.3 \times 10^{-24}$ in the $\Delta \phi = \pi$ case.

Based on Ref. [2], by assuming maximal mixing, the allowed limit for the CPTV parameter $\Delta a$ is shown in Fig. 5. At 90% confidence level, the allowed limit for $\Delta a$ is $1.05 \times 10^{-23}\text{GeV}$.

Conclusions

Using Super-K atmospheric neutrino data, we compare the standard $\nu_\mu - \nu_\tau$ oscillation model with three types of non-standard ones: the $\nu_\mu - \nu_\tau$ oscillation, the $\nu_\mu - \nu_\nu$ oscillation in the context of the MaVaN model, and neutrino oscillations induced by LIV and CPTV. We find that the $\nu_\mu - \nu_\tau$ oscillation induced by mass eigenstate mixing is the best model accounting for the Super-K atmospheric zenith angle distributions. Furthermore, by integrating these non-standard phenomena and the standard oscillation into one general forms, we are able to constrain the limits of non-standard effects.

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References