A parameterized neutron monitor yield function for space weather applications

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Abstract: To determine the characteristics of galactic or solar cosmic ray flux near Earth by using neutron monitor measurements, the observation data must be converted by extensive calculations that are additionally burdened with inaccuracies (e.g. correction to sea level). However, for space weather applications a straightforward, fast, and possibly simple method is needed to allow data analysis in near real-time. The Geant4 simulation toolkit offers the possibility to simulate the interactions of cosmic ray particles with the atmosphere and the neutron monitor by the Monte Carlo method and therefore to determine the yield function of a specific neutron monitor in function of atmospheric depth and primary particle rigidity. The paper presents the results of such simulations for a NM64 monitor and includes a comparison with previously determined yield functions. The new yield function is parameterized and can therefore be adapted to a neutron monitor at any location. The value and the use of the new yield function are demonstrated with the analysis of the neutron monitor data of the worldwide network during the maximum phase of the ground level enhancement on December 13, 2006.

Introduction

Measurements performed by the worldwide network of neutron monitors (NM) are used to determine characteristics of the galactic (GCR) and solar (SCR) cosmic ray flux near Earth. Such analyses require a precise evaluation of both the atmospheric transport and the NM detection efficiency. These two characteristics are taken into account in the so-called yield function $S$ related to the NM count rate $N$ at a given time $t$ by the commonly used formula [4]:

$$N(P_c, z, t) = \int_{P_c}^{\infty} \sum_i S_i(P, z) \cdot J_i(P, t) \cdot dP$$

where

- $i$ primary particle type (proton or $\alpha$);
- $P$ primary particle rigidity;
- $P_c$ effective vertical cutoff rigidity;
- $z$ atmospheric depth of the NM;
- $J_i$ primary particle rigidity spectrum;
- $W_T$ total differential response function.

Most of the studies based on NM data use yield functions or parameterized response functions evaluated for specific conditions (e.g. sea level) that require extensive correction calculations. For some purposes, in particular space weather applications, a simpler, straightforward and fast method is needed to allow data analysis in near real-time. The approach presented here is based on the use of a yield function parameterized in function of the NM type, atmospheric depth and primary particle rigidity.

Using the Geant4 Monte Carlo code [1] we simulated the atmospheric cascades and the neutron monitor detection response. The resulting NM yield function is presented and compared with previously determined yield functions. The parameterization allows the adaptation of the yield function to a neutron monitor at any location. The value and the use of the new yield function are demonstrated with the analysis of the NM data of the worldwide network during the maximum phase of the ground level enhancement (GLE) during the solar flare on December 13, 2006.
**Computed NM yield function**

The yield function of a NM is a function of its geometry, environment, atmospheric depth as well as of the rigidity and type of primary cosmic ray particles. It can be evaluated as follows:

\[ S_i(P, z) = \sum_j \int \int A_j(E, \theta) \Phi_{ij}(P, z, E, \theta) \cdot dE \cdot d\Omega \]  

(2)

where

- \( j \) secondary type (neutron, proton, \( \mu^{\pm}, \pi^{\pm} \));
- \( A_j \) effective area (efficiency \times geom. area);
- \( \Phi_{ij} \) differential flux of secondaries per primary;
- \( E, \theta \) secondary energy and incidence angle.

**NM detection efficiency**

We used the Geant4 Monte Carlo code to determine the efficiency of both NM64 and IGY monitors to detect incident secondary particles. The NM geometries and materials were simulated with a maximum of details according to the descriptions given in [10, 14]. The simulation consisted in evaluating the NM response to a parallel beam of particles of different types, energies, and angles of incidence. Figure 1 shows the computed effective area of a standard 6-NM64 for vertically incident secondary neutrons and protons. A comparison with the results from Clem [3] and Hatton [10] shows good agreement.

**Particle transport through the atmosphere**

The spectra of secondary neutrons and protons generated in the atmosphere by cosmic ray protons were computed for discrete atmospheric depths between 50 and 1040 g/cm\(^2\) and primary rigidities between 0.1 and 100 GV. The simulation was performed with the Geant4-based Planeto-cosmics code [7, 8] for isotropically incident primaries at the top of the atmosphere.

**Parameterization of the yield function**

The 6-NM64 yield function\(^1\), \( S_p \), computed for discrete values of the primary rigidity and of the atmospheric depth was parameterized using a two-dimension and third-degree polynomial regression expressed by the following formula :

\[ \log S_p(P, z) = \sum_{m,n=0}^{3} C_{mn} \cdot z^m \cdot (\log P)^n \]  

(3)

where \( S, P, \) and \( z \) are respectively in m\(^2\) \cdot sr, GV, and g/cm\(^2\), and \( \log \) stands for decimal logarithm. Table 1 lists the \( C_{mn} \) coefficients.

<table>
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<th>( C_{mn} )</th>
<th>( n=0 )</th>
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<th>( n=2 )</th>
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<td>7.969E-9</td>
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Table 1: Standard 6-NM64 proton yield function coefficients \( C_{mn} \) evaluated with least squares method.

**Figure 1:** Effective area of a 6-NM64 for vertically incident protons and neutrons.

**Figure 2:** Computed (squares) and parameterized (solid lines) proton yield function of a 6-NM64 monitor.

1. Results from the simulation of a standard IGY monitor will be presented in a more comprehensive version of the paper.
As shown in Figure 2 the parameterization provides a good representation of the results obtained with Monte Carlo simulations in the relevant rigidity and atmospheric depth ranges, i.e. 0.7 GV < R < 80 GV and 300 g/cm² < z < 1040 g/cm².

Figure 3 shows a comparison of the computed yield function and derived differential response function (at solar minimum) with a set of references (all cited in [4]) for a 6-NM64 monitor at sea level. The total differential response function $W_T$ was calculated with the GCR spectrum from Castagnoli and Lal [2] by taking into account the contribution of α particles.

**Case study: GLE on December 13, 2006**

A first application of the newly determined yield function consisted of repeating the analysis of the ground level enhancement (GLE) on December 13, 2006. According to the method by Smart et al. [15] and Debrunner and Lockwood [5] a set of GLE parameters (i.e. apparent source position, solar particle intensity, and pitch angle distribution) was determined by minimizing the differences between the evaluated and observed count rate increases of 33 NM stations. A power-law dependence in rigidity was assumed for the solar proton spectrum intensity near Earth:

$$I(P, t) = A(t) \cdot \left( \frac{P}{1 \text{ GV}} \right)^{-\gamma(t)}$$

where $P$ and $I$ are expressed in GV and cm⁻²MV⁻¹sr⁻¹s⁻¹, respectively. Table 2 presents the GLE parameters determined with the parameterized yield function and those obtained using the yield function from Debrunner et al. [6]. The differences are marginal.

<table>
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<th>Debrunner et al. [6]</th>
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<td>Apparent latitude</td>
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<td>27.0° S</td>
</tr>
<tr>
<td>Apparent longitude</td>
<td>95.5° E</td>
<td>96.5° E</td>
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<tr>
<td>$A(t)$ [#/cm²/MV/sr/s]</td>
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<td>0.28</td>
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<td>$\gamma(t)$</td>
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Table 2: Determined parameters for the GLE maximum phase on December 13, 2006 (0305-0310 UT).

**Conclusions**

In this study we performed a new evaluation of standard NM yield functions by means of Monte Carlo simulations. Both the particle cascade in the atmosphere and the NM detection efficiency were determined with the Geant4 toolkit. The obtained results are in reasonable agreement with those from several previous studies in terms of primary proton yield function and differential response for a standard 6-NM64 at sea level. For fast and simple use, the computed yield function was parameterized in function of the primary rigidity.
and of the atmospheric depth. A first comparative analysis performed for the maximum phase of the GLE on December 13, 2006, demonstrates the consistency of the new approach with existing procedures.

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**References**


