Dependence of the energy reconstruction precision on shower arrival direction and core location in a regular surface detector array

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In high energy cosmic ray surface detector arrays, the primary energy is traditionally determined from the value of the lateral distribution function (LDF) at a fixed distance $r_0$ from the shower core. Depending on the array spacing, this distance is normally chosen such that the bias on energy introduced by composition and shower fluctuations is minimized. However, the core position uncertainty and the fluctuations in the signals at the different detectors also influence the precision in the energy determination. We show how the optimum distance $r_0$ at which the LDF should be evaluated for energy determination depends on the position of the shower core with respect to the array and on the shower direction.

1. Introduction

A widely used method to estimate the primary energy of extensive air showers with a surface detector array consists in fitting the observed particle densities to an assumed lateral distribution function, and then using the signal density interpolated at a certain distance from the core as an energy estimator. Typically, at least three non-aligned detectors are used at first to estimate the incoming direction of the event and, therefore, the shower plane. The projection onto this plane of the stations with their signals is used to search for a trial core position that best fits an assumed lateral distribution function (LDF) like, for example, the Nishimura-Kamata-Greisen function [1]:

$$S(r) = k \left( \frac{r}{r_1} \right)^{-\alpha} \left( 1 + \frac{r}{r_1} \right)^{-(\eta-\alpha)}$$

Once an optimum fit is achieved by either maximum likelihood or minimum chi-square, the signal density $S(r_0)$ is inferred at a fixed distance $r_0$. The technique was suggested in [2] and [3] and is an attempt to deal in an optimum way with shower-to-shower statistical fluctuations in the development of the cascade. Shower-to-shower fluctuations are a function of the distance to the core and go through a minimum at a distance between a few hundred meters and around two kilometers depending on the identity and energy of the primary nucleus. The characteristic distance $r_0$ is then chosen inside this range in a compromise between minimal fluctuation and optimal interpolation for a given array spacing. Traditional experiments like Haverah Park, Yakutsk and AGASA have adopted $r_0 = 600$ m and the respective conversion relations show that the primary energy scales almost linearly with $S(600)$ [3, 4, 5, 6, 7] which means that the energy and $S(600)$ spectra closely resemble each other.

In the case of the Southern site of the Auger Observatory, for example, the preliminary adopted distance is $r_0 = 1000$ m [8], where fluctuations of the signal show a broad minimum, while the trial lateral distribution function used for core location is a power law.

The adoption of a characteristic distance far from the core, around 1000 m, has an additional advantage. Near to the core, the signal depends strongly on fluctuations on the depth of the first interaction while, in
principle, at large distance statistical fluctuations dominate the size of the signal. However, there are other
sources of error that stem from the uncertainty in the form of the LDF and imprecision in the determination
of the exact core position, with respect to which the LDF fitting is performed and the \( r_0 \) is defined in
practice. It can be shown, however, that there exists a point \( r_0^* \) at which, regardless of the adopted LDF and
the error involved in the estimation of the core position, the signal can be measured more accurately.

In the present work we show that, for a given array architecture, the exact position of \( r_0^* \) depends
systematically on the location of the core relative to the detectors, on incoming direction and on energy, in a
way which could introduce systematic effects in the energy spectrum if not taken into account properly.

2. Numerical approach and results

We use a simplified numerical approach to the simulation of extensive air shower detection in a surface
array. Our ideal detector is an infinite array of equally spaced stations distributed in elementary triangular
cells of 1500 m of side. We analyze showers with core positions inside a given elementary cell.

In what follows we model shower events and their corresponding array response by assuming a LDF of the
form:

\[
S(r_{km}, E_{EeV}) = \left[ \frac{7.53E_{EeV}^{0.95}2^{2\beta(\theta)}}{\sqrt{1 + 11.8[\sec(\theta) - 1]^2}} \right] \times r_{km}^{-\beta(\theta)} \times \left(1 + r_{km}^{-}\right)^{-\beta(\theta)}
\]  

(2)

where \( \beta = 3.1 - 0.7 \sec(\theta) \) and we have taken the relation between \( S(1000) \) and energy from [8]. The signal
expected at each station is then combined with Poissonian noise and recorded if it is above a threshold of 3.2
VEM [8]. In order to mimic the reconstruction procedure we adjust the signals at the triggered stations with
a trial LDF of the form:

\[
\log S_f(r_{km}) = a_1r_{km}^{-\epsilon_2} + a_3,
\]

(3)

which is as acceptable a fitting to the data as eq. (2), since the \( S(1000) \) method shouldn’t depend on the
knowledge of the exact functional form of the LDF. For each event, 100 fits using eq. (3) are performed
shifting the reconstructed shower cores by 100 meters in arbitrary directions. Poissonian weights for the
signals (in VEM) are used in the fitting process. The point \( r_0 \) is defined as the one with the lowest dispersion.

Figure 1 exemplifies the procedure, corresponding to an event of 30 EeV, zenith angle 45° and core at 0.5
km from the nearest station. It can be seen that there is a well defined point, \( r_0 \), illustrates the fact that
the previous result also depends on energy (a single triangular cell is shown in each inset).

![Figure 1](image-url)

**Figure 1.** A simulated event of \( E=30 \) EeV, zenith angle 45° and core at 0.5 km from a station. The diamonds represent
the observed signal at the actual distance to the axis of the shower. The dotted line is the real LDF used to generate the
signal (eq. 3) and the remaining curves are fits to the data using eq. (4) for different reconstructed cores. The point where
all the curves cross is \( r_0 \).
Figure 2. Calculated values of $r_0$ for vertical events of $E=10$ EeV, and random core position. The large dots give the location of the stations. The x-y axis are space coordinates in km for this patch of the simulated array.

Although not shown, there are also effects with zenithal angle and azimuth. It is clear that systematic effects are present depending on the location of the core, and that those effects change in a predictable pattern depending on energy and event geometry relative to the grid of detectors.

3. Conclusions

We showed that the core position uncertainty and the fluctuations in the signals at the different detectors influence the precision in the energy determination. We conclude that the optimum distance $r_0$ at which the LDF should be evaluated for energy determination, depends on the

Figure 3. Energy dependence of $r_0$, as a function of core location, for vertical showers. The triangle represents an elementary cell with detectors at its vertices (contours outside the triangle are extrapolations and must be disregarded).
position of the shower core with respect to the array and on the shower direction and should be taken into account properly, on an event by event basis if possible, for an accurate determination of the energy spectrum. The inclusion of saturation of stations near the core enhances this effect.

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References