Time Evolution of Cosmic-Ray Modified MHD Shocks

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We present initial simulation results for the time evolution of CR modified plane parallel shocks in magneto-hydrodynamical flows. The simulations utilize our new and very efficient “Coarse Grained finite Momentum Volume” (CGMV) transport scheme \cite{6}. The simulations aim to explore nonlinear feedback among the particles, the wave turbulence and the bulk flows. The calculations incorporate self-consistent treatment of the momentum-dependent CR diffusion convection equation and will soon be coupled with wave action equations for resonantly scattering Alfvén and fast mode waves, also treated through the CGMV scheme. Recent advances in MHD turbulence theory require a broadened outlook on scattering processes in CR modified shocks.

1. Introduction

The physics of strong, cosmic-ray modified shocks is complex and nonlinear. Through diffusive shock acceleration (DSA) cosmic-rays (CRs) can capture a major portion of the energy flux through the shocks, greatly modifying the shock dynamics and structures in the process (e.g., \cite{1, 3, 4, 5, 14}). CR propagation in and around the shocks is mediated by the presence of the large scale magnetic field and by resonant scattering on MHD waves, which are usually considered as Alfvén wave turbulence. Most, but not all, theoretical treatments of CR modified shocks assume a fixed CR scattering or diffusion law, and commonly the dynamical roles of the wave turbulence and the large scale magnetic field are ignored. On the other hand, a key feature of DSA is that the relevant wave turbulence is strongly amplified by streaming CRs near the shock \cite{2}. As a consequence, it can contribute a significant pondermotive force on the bulk plasma flow, and its dissipation upstream of the classical gas subshock structure can preheat the upstream plasma, which also modifies the character of the shock transition. Lucek and Bell \cite{10}, for example, have argued that the streaming instability can enhance the scattering waves so much that the scattering length is orders of magnitude less than expressed by Bohm diffusion using the upstream magnetic field. This has led several authors to argue that CR acceleration can be much more rapid than usually described (e.g., \cite{3, 12}). Furthermore, the scattering turbulence is likely to be anisotropic, which, among other things, can mean that the effective motion of the scattering centers will not match the motion of the bulk plasma, as usually assumed. For instance, Alfvén waves amplified by the streaming instability may be expected to propagate with respect to the bulk plasma along the large scale magnetic field at approximately the Alfvén speed. Since the Alfvén speed variations normal to the shock structure are not the same as the gas speed variations, this may modify the properties of DSA through the shock \cite{7, 11}. It is clearly important to carry out theoretical studies of DSA that can incorporate the physical processes just outlined. This paper reports initial steps in an effort to do just that.

2. Methods

For this study we have incorporated our new, efficient “Coarse Grained finite Momentum Volume” (CGMV) scheme for solving the CR diffusion-convection equation \cite{6} into our 1D TVD MHD code. This MHD code has been used effectively by us in the past to study DSA using conventional finite difference methods to evolve the diffusion-convection equation \cite{5, 9}. The results presented here are based on a prescribed spatial diffusion
coefficient, although we are in the process of incorporating a CGMV-based routine to evolve the wave action equation for Alfvénic turbulence, so that a self-consistent treatment of the full system can be carried out.

The CGMV scheme for evolving the CR distribution utilizes the first two momentum moments of $f(t, x, p)$ over finite momentum bins; namely, $\frac{\partial n_i}{\partial t} + u \frac{\partial n_i}{\partial x} = F_{n_i} - F_{n_{i+1}} - n_i \frac{\partial u}{\partial x} + \frac{\partial}{\partial x} \left( K_{ni} \frac{\partial n_i}{\partial x} \right) + S_{ni}, \quad (1)$

where $u = v + u_w$ is the net velocity of CR scattering centers, including the gas motion, $v$, and the mean wave motion, $u_w$ [6]. In addition, $F_{n_i} = \{ \dot{p}_i + q(p_i)D(p_i)/p_i \} p_i^2 f(p_i)$ is a flux in momentum space, with $\dot{p} = -p \frac{du}{dx}$. $K_{ni}$ and $S_{ni}$ are the spatial diffusion coefficient, $\kappa(x, p)$, and a representative source term, $S$, averaged over the momentum interval. $D(p)$ is the momentum diffusion coefficient. We henceforth express particle momentum in units of $mc$, where $m$ is the particle mass. In the present, preliminary study we assume the convenient spatial diffusion form, $\kappa(p) = \kappa_0 p^n$, and ignore momentum diffusion ($D = 0$). Generally, spatial CR diffusion is not expected to be isotropic with respect to the direction of the local magnetic field. The degree of anisotropy has been shown to have important consequences, especially when the magnetic field is quasiperpendicular to the shock normal [1, 5, 8]. For these initial simulations, however, we assume isotropic diffusion. We also assume a simple, common model for injection of low-energy CRs at the gas subshock; namely, that a fixed fraction, $\epsilon_{inj}$, of the thermal proton flux through the subshock is able to escape upstream at momentum $p_{inj}$ to join the diffusive CR population. This produces a source term in equation 1, for example, $S_{ni} = \frac{1}{4\pi} \sigma \epsilon_{inj} \phi(x)$, where $\sigma$ is the upstream plasma density, $\mu$ is the plasma molecular weight, $v_s$ is the plasma flow speed across the shock and $\phi(x)$ is a normalized weighting function to distribute the injection across the numerical shock.

These equations are coupled with the standard equations for compressible MHD by adding the pressure gradient of the CRs, $\frac{\partial P_c}{\partial x}$, into the MHD Euler equation and by including in the MHD energy conservation equation the term $-u \frac{\partial P_c}{\partial x}$ to account for work done on the bulk fluid by the CR pressure gradient. Note that this last contribution includes both the adiabatic compression of the plasma through the term $v \frac{\partial P_c}{\partial x}$, as well as a term representing dissipation of scattering wave energy generated through CR streaming [11, 7]. In addition, to account properly for thermal energy taken from the bulk plasma during the CR injection process, an energy sink term, $L = -(1/2)\epsilon_{inj} \phi(x) p_{inj}^2 \sigma v_s$ must be added to the MHD energy conservation equation.

3. Results and Discussion

CR-modified shocks are known to differ significantly from ordinary gasdynamical or MHD shocks. CR diffusion upstream produces a pressure gradient, adding adiabatic compression to the gas. This preheats the gas and substantially increases compression through the transition. It also weakens the gas subshock. Typically, in fact, the compression through a strong CR shock precursor dominates the total shock compression, so that most of the CR acceleration actually takes place in the precursor, rather than in the thin, weakened subshock. Furthermore, energy extraction from the thermal plasma by the CRs cools the bulk gas with respect to adiabatic gas shocks. Particularly if the CRs escape upstream this allows the shock transition to resemble a radiatively cooled shock transition, further amplifying the total compression through the transition, while reducing the downstream temperature and pressure compared to gasdynamical behaviors.

Previous theoretical studies of CR-MHD shocks have emphasized major differences with respect to gas dy-
namical models for CR-modified shocks. For weak to moderate strength oblique shocks, magnetic field compression restricts the enhanced compression mentioned above. That becomes a relatively small effect in very strong MHD shocks, however, once the total downstream pressure is dominated by the CRs [5, 14]. A much more significant influence in strong MHD shocks can be the difference between the motion of the bulk plasma and the motion of the CR scattering centers [7]; i.e., the drift of the CRs with respect to the fluid. If the upstream scattering is due primarily to Alfvén waves amplified by CRs streaming away from the shock, then one expects the mean motion of the scattering centers to propagate upstream along the mean magnetic field at approximately the local Alfvén velocity with respect to the fluid flow. What matters is the component of this velocity along the shock normal, since it parallels the flow and CR gradients. This Alfvén velocity component will scale as \( v_{A_w} \propto B_z/\sqrt{\rho} \), where \( B_z \) is the mean magnetic field component along the shock normal.

Figure 1. Three simulated MHD Mach 40 CR-modified shocks formed off a piston on the left boundary. The gas density, gas pressure and CR pressure spatial distributions are shown along with the CR momentum distribution at the subshock.

Since \( B_z \) does not change through the shock, \( v_{A_w} \propto 1/\sqrt{\rho} \), while the bulk flow speed varies as \( 1/\rho \), through mass conservation. Consequently, the effective rate of mirror convergence responsible for DSA \( \delta u/\delta x \), is reduced compared to a pure gas flow, reducing the rate of CR acceleration. Moreover, the magnitude of \( u \) is
increased by the upstream-facing drift, which increases the rate of precursor heating for a given CR pressure gradient. That reduces the strength of the shock transition, which also reduces the efficiency of DSA.

We illustrate these effects by comparing in Figure 1 three simulations involving a Mach 40 piston-generated shock. Each flow has an upstream magnetic field inclined 45 degrees from the shock normal with a magnetic pressure there equal to the gas pressure. The simple diffusion model assumed $\kappa_0 = 0.1$ and $\alpha = 0.51$. CR injection was included with $\epsilon_{\text{inj}} = 0.001$. Fourteen momentum bins were used in solving the diffusion-convection equation. The black, solid curves indicate behaviors when $u_w = 0$. This simulated shock is very similar to a gasdynamical shock discussed in [6] and illustrated in Fig. 2 of that paper. The shock quickly becomes CR dominated, but only approaches dynamical equilibrium at the last time shown. The CR momentum distribution at the subshock shows the strongly concave form typical of such simulated CR shocks. By comparison the other plotted results include effects of CR drift. The red, dotted curves represent the behavior when $u_w = v_{Aw}$, while the blue, dashed curves come from a simulation in which $u_w$ was included in the MHD energy equation, but not the momentum equation. That approach has been used by some authors e.g., [4] in non-MHD models to approximate the influence of MHD by allowing for dissipation of wave energy.

The differences between the gasdynamical solutions and the MHD solutions are obvious. The magnetic, Maxwell stresses have had little impact on the MHD solutions. However, as anticipated from the above discussion, compression through the shock is much reduced through CR drift, while the efficiency of CR acceleration is reduced by about one third. These two effects have also mostly eliminated the strong concavity in the CR momentum distribution, since the shock precursor is a much less important contributor to DSA. On the other hand, there is relatively little difference between the two CR-drift models. In this one strong shock case, at least, modeling the MHD shock by estimating Alfvén wave heating of the bulk plasma would provide a reasonable approximation to the more complete model.

4. Acknowledgments

This work by TWJ is supported in part by NASA grant NNG05GF57G and by the University of Minnesota Supercomputing Institute. HK is supported in part by KOSEF through the Astrophysical Research Center for the Structure and Evolution of Cosmos (ARCSEC).

References