Cosmic Ray Acceleration at the Galactic Wind Termination Shock with Self-Excited Alfvén Waves

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Cosmic ray acceleration at the Galactic wind termination shock is considered. The generation of Alfvén waves by cosmic ray streaming is taken into account. The spectrum of Alfvén waves determines the cosmic ray diffusion. Applications are given for an explanation of the cosmic ray spectrum in the "knee" region.

1. Introduction

The existence of a Galactic Wind driven by cosmic rays (CR) in our Galaxy has been discussed extensively in recent years [5, 2, 3, 4, 14]. Cosmic ray sources in the galactic disk produce energetic particles which can not freely escape from the Galaxy but rather amplify Alfvén waves [13]. Such waves lead to an efficient coupling of the thermal gas to the energetic particles [11], and the pressure gradient of cosmic rays drives a Galactic Wind flow. Typically, the gas flow becomes supersonic at distances of about 30 kpc. This radially directed outflow should terminate in a termination shock at distances of several hundred kpc. It was recognized quite a while ago that this shock may reaccelerate cosmic ray particles [6, 7]. However this idea faces a serious problem. It is the difficulty of observation of accelerated particles inside the Galaxy. In fact, the condition of efficient acceleration coincides with the condition of strong modulation of particles in the Galactic Wind flow. This means that one cannot expect continuity of the CR spectrum between the reaccelerated particles and those produced by the disk sources.

In this paper we shall investigate this problem in more detail. The modulation in the Galactic Wind flow strongly depends on the CR diffusion that is mainly determined by Alfvénic turbulence. We shall take into account the generation of the Alfvén waves by CR streaming in the Galactic Wind flow. This mechanism is an important part of CR acceleration at astrophysical shocks [1].

We shall show that this self-consistent mechanism of the CR acceleration and propagation in the Galactic Wind produces an additional power-law CR component with energy index close to unity in the Galaxy, independent of the cut-off energy of the sources in the disk. These CRs are accelerated at the termination shock and modulated in the Galactic wind flow. Depending on the Galactic Wind parameters, the maximum energy of this component is of the order of \(10^{15} \div 10^{16}\) eV for CR protons.

2. Cosmic ray and Alfvén wave propagation in the Galactic Wind flow

Assuming azimuthal symmetry the isotropic part \(N(r, \theta, p, t)\) of the CR distribution function evolves according to the following equation

\[
\frac{\partial N}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 D_{\parallel} \cos^2 \alpha \frac{\partial N}{\partial r} - \frac{u}{r} \frac{\partial N}{\partial r} + \frac{2u \frac{\partial N}{\partial p}}{3r \frac{\partial p}{\partial r}}
\]

Here \(\alpha\) is the angle between the magnetic field and the radial direction. The CR distribution function \(N\) is normalized in the form \(n = 4\pi \int p^2 dp N\), where \(n\) is the CR number density. It was assumed that the Galactic
Wind flow is radial and that the wind velocity $u$ is constant. It is also assumed, that there is no reacceleration in the wind [12]. The magnetic field strength $B$ and the value of $\cos \alpha$ in the radial wind flow are given by

$$\cos \alpha = \frac{u}{\sqrt{\Omega^2 r^2 \sin^2 \theta + u^2}}, \quad B = B_\phi \frac{R_g^2}{r^2} \sqrt{1 + \frac{\Omega^2 r^2 \sin^2 \theta}{u^2}}. \quad (2)$$

Here $B_\phi$ is the poloidal magnetic field strength in the Galaxy, $\Omega$ is the frequency of the galactic rotation, $\theta$ is the galactic colatitude and $R_g$ is the galactic radius [10].

The boundary condition at the termination shock is given by the continuity of $N$ and of the flux density:

$$\left( D_\parallel \cos^2 \alpha \frac{\partial N}{\partial r} + u \left( 1 - \frac{1}{\sigma} \right) \frac{p}{3} \frac{\partial N}{\partial p} \right)_{r = R_s} = 0. \quad (3)$$

Here $\sigma$ is the shock compression ratio. We shall disregard CR diffusion beyond the shock.

The CR parallel diffusion coefficient $D_\parallel$ is determined by the spectrum of Alfvén waves:

$$D_\parallel = D_B / \epsilon(p). \quad (4)$$

Here $D_B = vr_\phi / 3$ is the Bohm diffusion coefficient of CR particles with gyroradius $r_\phi$ and velocity $v$, and $\epsilon(p) = 4\pi k [W(k)]_k = k_{res}$. $B^2$ is the dimensionless energy density of the Alfvén waves propagating along the magnetic field with a spectrum $W(k)$ and with wavenumber $k$ that equals the resonant wavenumber $k_{res} = r_\phi^{-1}$.

We used the following equation for the evolution of $\epsilon$:

$$\frac{\partial \epsilon}{\partial t} = -u \frac{\partial \epsilon}{\partial r} - \frac{\epsilon}{r} + \frac{v_a \cos \alpha 4\pi}{B^2/4\pi} \frac{p}{3} \frac{\partial N}{\partial r} - u \left( \frac{\partial k_{res}}{R_{res}^2} + \frac{1}{r} \right) \frac{p}{3} \frac{\partial \epsilon}{\partial p}. \quad (5)$$

This equation is valid at large distances from the Galactic disk, where the magnetic field is almost azimuthal. The third term in the Eq. (5) describes the generation of Alfvén waves by CR streaming [1]. The last term in this equation is zero because the decrease of the wavenumber $k$ in the expanding Galactic wind flow exactly corresponds to the decrease of the resonant wavenumber $k_{res}$ due to the drop of the magnetic field strength.

The distribution of the CR particles with energies smaller than the maximum energy accelerated at the termination shock and modulated in the Galactic wind flow can be found analytically using the following method. The Alfvén waves are produced effectively near the termination shock, where the CR gradient is large. The solution for the CR number and the Alfvén energy density in this region can be found from Eqs. (1) and (5) using a plane shock approximation [1]:

$$\epsilon(r, p) = \frac{v_a \cos \alpha 4\pi}{uB^2/4\pi} \frac{p}{3} \frac{\partial N}{\partial r}, \quad N(r, p) = \frac{N_R(p)}{1 + \frac{4\pi}{3} \frac{v_a(R_0 - r)}{D_\parallel \cos \alpha B^2/4\pi} \frac{p}{3} \frac{\partial N}{\partial r}}, \quad R_s - r << R_s. \quad (6)$$

Here $N_R(p)$ is the CR distribution at the shock front. Using the boundary condition (3) one finds that $N_R(p)$ is a power-law function of momentum and its index is $\gamma = 3\sigma / (\sigma - 1)$. It is interesting that at large distances from the shock the CR distribution does not depend on the distribution at the shock front $N_R$ and the corresponding CR diffusion coefficient is energy independent. This means that inside the Galactic Wind flow we should observe a CR component with a power-law $\propto p^{-3}$.

The generation of waves is not strong in this region, the CR diffusion coefficient is large and the CR distribution is almost independent of $r$. We can then find using Eqs. (1) and (5):
\[ D_\parallel \cos^2 \alpha \frac{\partial N}{\partial r} = -u \frac{p}{3} \frac{\partial N}{\partial p}, \epsilon = \epsilon_b(r) \exp \left( -\frac{4\pi}{9} \frac{\partial N}{\partial p} \int_{R_g}^r \frac{v_a v_p^4}{ \frac{D_B \cos \alpha B^2}{4\pi}} \right) \tag{7} \]

where \( N_g \) is the CR distribution in the Galaxy at \( r = R_g \) and \( \epsilon_b \propto r^{-1} \) is the background level of the Alfvén wave energy density in the Galactic wind. The transition from this solution to the solution (6) takes place at distances close to \( R_g \) where Eqs (6) and (7) are marginally valid. Equating these solutions we obtain the equation for the momentum distribution in the Galaxy \( N_g(p) \):

\[ \ln \left( \frac{v_a \cos \alpha}{u e_B B^2/4\pi} \frac{4\pi}{3} v_p^4 N_g(p) \right) = -\frac{4\pi}{9} \frac{\partial N_g}{\partial p} \int_{R_g}^r \frac{dR_s}{\frac{D_B \cos \alpha B^2}{4\pi}} v_a v_p^4 \]  

\[ \tag{8} \]

The all quantities here are taken at \( R = R_g \). It is clear from this equation that the CR distribution in the Galaxy is close to \( p^{-3} \). Performing the integration over \( r \) in Eq. (8) we obtain with logarithmic accuracy:

\[ N_g = \frac{3}{4\pi} \frac{u}{\Omega \sin \theta R_g} \frac{r_g B^2/4\pi}{R_s v_a p^4} \ln \left( \frac{x(p)}{\ln^{-1}(x(p))} \right), \quad x(p) = \frac{3D_b(p)}{u R_g} >> 1. \tag{9} \]

Here \( D_b(p) = D_B \cos^2 \alpha / e_b \) is the background CR diffusion coefficient. The CR distribution in the Galaxy at small energies does not depend on the CR distribution at the termination shock. However the maximum energy of CR particles does. The maximum energy of CR particles accelerated at the termination shock can be estimated from the condition of the equality of \( N_g \) and \( N_R \).

We shall express the CR distribution at the termination shock in terms of the dynamical pressure of the Galactic Wind flow \( \rho u^2 \), where \( \rho \) is the wind density. We shall assume further that \( \sigma = 4 \) and \( \gamma = 4 \). For this case

\[ N_R = 3 \xi_{CR} \rho u^2 H(p - p_{\min}) H(p_c - p) \frac{4\pi p^4}{\ln(p_c/p_{\min})} \tag{10} \]

Here \( H(p) \) is the step function and \( \xi_{CR} \) is the ratio of the CR pressure at the shock front to the dynamical pressure of the wind. \( p_c \) and \( p_{\min} \) are the cut-off and minimum momentum of CR particles accelerated at the shock. From equality of Eq. (6) and (7) we find the equation for the cut-off momentum \( p_c \):

\[ \ln \left( \frac{p_c}{p_{\min}} \right) \ln \left( \frac{x(p_c)}{\ln^{-1}(x(p_c))} \right) r_g(p_c) = \xi_{CR} R_g \frac{u \Omega \sin \theta R_g}{v_a c} \]  

\[ \tag{11} \]

The distance to the termination shock \( R_g \) is determined by the kinetic power of the Galactic Wind flow \( \dot{\varepsilon} = 2\pi \rho u^3 R_g^2 \) and the intergalactic pressure \( P_{IG} \) via the condition \( P_{IG} \sim \rho u^2 \). Eqs. (9) and (11) then read:

\[ 4\pi p^4 \epsilon N_g = 10^{-28} \text{erg cm}^{-3} \text{ GeV} \frac{p_c}{300 \text{ km} \text{s}^{-1}} \frac{10^{41} \text{ erg} \text{s}^{-1}}{5 \cdot 10^{-16} \text{ s}^{-1}} \frac{1}{\Omega \sin \theta} \left( \frac{P_{IG}}{10^{-15} \text{ erg cm}^{-3}} \right) \]  

\[ \frac{\dot{\varepsilon}}{\left( \frac{P_{IG}}{10^{-15} \text{ erg cm}^{-3}} \right)^{1/2}} \tag{12} \]

\[ \ln \left( \frac{p_c}{p_{\min}} \right) \ln \left( \frac{x(p_c)}{\ln^{-1}(x(p_c))} \right) \frac{c p_c \xi_{CR}^{-1}}{3 \cdot 10^8 \text{ GeV}} = \frac{\dot{\varepsilon}}{10^{41} \text{ erg} \text{s}^{-1}} \frac{300 \text{ km}}{u} \frac{\Omega \sin \theta}{5 \cdot 10^{-16} \text{ s}^{-1}} \left( \frac{10^{-15} \text{ erg cm}^{-3}}{P_{IG}} \right)^{1/2} \tag{13} \]

The numerical solution of Eqs. (1) and (5) is shown at Fig.1. We take \( u = 300 \text{ km s}^{-1} \), \( p_c / p_{\min} = 10^3 \) and \( \Omega \sin \theta = 5 \cdot 10^{-16} \text{ s}^{-1} \). It was assumed that the termination shock is modified by the CR pressure and \( \xi_{CR} = 0.5 \). We use the value of the intergalactic pressure \( P_{IG} = 10^{-15} \text{ erg cm}^{-3} \) that is close to the value of the gas pressure of a warm-hot intergalactic medium derived from the observations of X-ray and ultraviolet line absorption [9]. The analytical solution (12) is in a very good agreement with the numeric solution. The maximum momentum found numerically is a factor of 3 smaller than the cut-off momentum (13).
3. Conclusions

We have found that self-consistent CR acceleration at the Galactic Wind termination shock produces a power-law $p^{-3}$ CR component in the Galaxy. The magnitude and maximal energy of this component depend on the Galactic Wind flow parameters. Depending on the kinetic power of the wind that determines the distance to the termination shock, the maximum energy of this component is $10^{15} \div 10^{16}$ eV for CR protons. Its intensity is close to the observable CR intensity in the “knee” region. This is because the intergalactic pressure is comparable with the energy density of the “knee” particles. Heavier nuclei related to this component should broaden this “knee” region.

References