Three Dimensional Modeling of the Recurrent Forbush Effect of Galactic Cosmic Rays

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We assume that the recurrent Forbush effect of the galactic cosmic ray (GCR) intensity (duration about 10 - 12 days with the symmetric time profile with respect to the minimum intensity and an expected amplitude of 3-4% for the rigidity range of 10 GV) is caused by the changes of the diffusion coefficient due to the considerable varying of the interplanetary magnetic field (IMF) turbulence. The basic changes take place in the energy range (<10^{-5} \text{ Hz}) of the IMF turbulence responsible for the diffusion of GCR particles to which neutron monitors and meson telescopes are sensitive. The increase of the exponent of the power spectral density in the energy range of the IMF turbulence leads to the hard rigidity spectrum of the GCR intensity during the main phase of the Forbush effect. The steady-state model based on the Parker’s transport equation is applied to describe the features of the recurrent Forbush effect of the GCR intensity.

1. Introduction

Elsewhere [2-5] was established that the change of the rigidity R spectrum \(\delta D(R)/D(R)\) of the Forbush effects of the GCR intensity is related with the changes in the energy range region of the IMF’s turbulence. So, it was supposed that a parallel diffusion coefficient \(K_\parallel\) of cosmic ray particles depends on the rigidity as, \(K_\parallel \propto R^{-\alpha}\) (\(\alpha = 2 - \nu\)) according to the quasi-linear theory of cosmic ray propagation [6]. Particularly, when an exponent \(\nu\) of the power spectral density (PSD) of the IMF turbulence \(PSD \propto f^{-\nu}\), where \(f\) is frequency) increases in the region of the frequency \(\sim 10^{-6} - 10^{-5}\) Hz, an exponent \(\gamma\) of the rigidity spectrum (\(\delta D(R)/D(R) \propto R^{-\gamma}\)) of the Forbush effect decreases [2-5]. In this paper we model the recurrent Forbush effect of the GCR intensity supposing that the parallel diffusion coefficient \(K_\parallel\) changes only due to the changes of the parameter \(\alpha\) during the recurrent Forbush effect.

2. Theoretical model

Generally short period change of the GCR intensity is non stationary process and for its describing the Parker’s time-dependent transport equation must be applied. An initial phase (rapid decreasing of the intensity) of the powerful sporadic Forbush effect belongs to these types of phenomena. However, the recovery period for the great of majority of the sporadic Forbush effect and the whole period of the recurrent Forbush effects can be considered as a steady-state process. The recurrent Forbush effect of GCR take place due to established corotating heliolongitudinal disturbances in the interplanetary space. The amplitudes of the recurrent Forbush effects of GCR intensity are rather small (\(\leq 3-4%\) in the energy range of 10 GeV) and a duration is reasonably large (10-12 days). We consider a steady-state \(\frac{\partial N}{\partial t} = 0\) Parker’s transport equation [1] (including diffusion, convection, and drift and energy change of the GCR particles in the diverged solar wind) to describe the recurrent Forbush effect of the GCR intensity:
\[ \nabla_i (K_{ij} \nabla_j N) - \nabla_i (U_i N) + \frac{1}{3} \frac{\partial}{\partial R} (N R) \nabla_i U_i = 0 \]  

(1)

Where \( N \) and \( R \) are density and rigidity of cosmic ray particles, respectively; \( K_{ij} \) is the anisotropic diffusion tensor of cosmic rays, \( U_i \) – solar wind velocity.

For the dimensionless density \( f = \frac{N}{N_0} \) and distance \( \rho = \frac{r}{r_0} \) (where \( N \) and \( N_0 \) are density in the interplanetary space and in the galaxy, respectively; \( r \) is the distance from the Sun and \( r_0 \) the region of modulation) equation (1) in the spherical coordinate system \((\rho, \theta, \phi)\) has the form:

\[ \begin{align*}
A_1 \frac{\partial^2 f}{\partial \rho^2} + A_2 \frac{\partial^2 f}{\partial \theta^2} + A_3 \frac{\partial^2 f}{\partial \phi \partial \theta} + A_4 \frac{\partial^2 f}{\partial \rho \partial \theta} + A_5 \frac{\partial^2 f}{\partial \rho \partial \phi} + A_6 \frac{\partial^2 f}{\partial \phi \partial \rho} + \\
+ A_7 \frac{\partial f}{\partial \rho} + A_8 \frac{\partial f}{\partial \theta} + A_9 \frac{\partial f}{\partial \phi} + A_{10} f + A_{11} \frac{\partial f}{\partial R} = 0
\end{align*} \]

(2)

The coefficients \( A_1, A_2, \ldots, A_{11} \) are functions of the spherical coordinates \( r, \theta, \phi \) and rigidity \( R \) of cosmic rays. To study a role of the changes of the IMF’s turbulence in the formation of the rigidity spectrum of the Forbush effect of cosmic ray intensity there were assumed that:

\[ K_{II} = K_0 \ K(r, \phi) \]

\[ K(R, \phi) = R^{\alpha(\phi)} \]

\[ K_0 = 4 \times 10^{22} \ cm^2 / s \]

\[ \alpha(\phi) = 2 - 1.1 \sin(3 \times (\phi - 150^\circ)) \]

\[ U = 400 \ km / s \]

Only the expression \( K(R, \phi) \) describes the changes in the energy range of the IMF’s turbulence in the vicinity of the interplanetary space being responsible for the recurrent type of the Forbush effect of GCR. The disturbed vicinity is restricted in the ranges of \( r < 15 \) AU, \( \phi \in (150^\circ - 210^\circ) \) and \( \theta \in (60^\circ - 120^\circ) \).

In the Figure 1 is presented the change of the parameter \( \alpha(\phi) = 2 - 1.1 \sin(3 \times (\phi - 150^\circ)) \) versus the heliolongitude. The Eq.2 was solved numerically using the method and the boundary conditions described in [2,3].

In the Figure 2 is presented the expected change of the density of GCR versus the heliolongitudes at the distance of 1 AU (Earth orbit). We consider that the change of the density of GCR versus the heliolongitude could be ascribed to the stationary time profile of density of cosmic rays caused by the relative motion of Earth in the disturbed vicinity of the interplanetary space. In the Figure 3 is shown the change of the expected rigidity spectrum exponent \( \gamma \) calculated based on numerical solution of the Eq. 2 as follows:
\[ \delta D / D(R) = 1/ f \frac{df}{dR} \propto R^{-\gamma} . \]

One can recognize (Figure 1 and Figure 3) a clear dependence between parameters \( \alpha \) and \( \gamma \) by the same token our assumption that the rigidity spectrum of the cosmic ray intensity is defined by the state of the IMF’s turbulence by parameter \( \alpha = 2 - \nu \); e.g. the exponent \( \nu \) of the PSD of the IMF turbulence for Bz component for the period of before the Forbush effect (1-15 Jun 2003) is equal to \( \nu = 1.3 \), during the Forbush effect (16-30 Jun 2003) \( \nu = 1.72 \) and after the Forbush effect (1-15 July 2003) \( \nu = 1.29 \)

3. Experimental Data

To compare results of the theoretical calculation of the rigidity spectrum with the experimental data of neutron monitors the recurrent Forbush effect of Jun 2003 was considered. The average intensity \( N_0 \) for each neutron monitor during the 14-16 Jun was accepted as a reference level (100%). The amplitudes \( \delta J_i / J_i \) of the intensity variation were calculated, as: \( \delta J_i / J_i = N - N_0 \), where \( N \) is daily average count rate of neutron monitor. The rigidity spectrum of the Forbush effect was supposed as \[7,8\].

\[ \begin{align*}
\frac{\delta D(R)}{D(R)} &= \begin{cases} 
AR^{-\gamma} & \text{for } R \leq R_{\text{max}} \\
0 & \text{for } R > R_{\text{max}} 
\end{cases} 
\end{align*} \]

(3)

Where \( R_{\text{max}} \) is the upper limiting rigidity beyond which the Forbush effect of GCR intensity vanishes; \( A \) is the amplitude of the Forbush effect in the heliosphere. The amplitude \( \delta J_i / J_i \) of the GCR intensity variation at any point of observation (by neutron monitor) with the geomagnetic cut off rigidity \( R_i \) and the average atmospheric depth \( h_i \) is defined as \[8\].

\[ \frac{\delta J_i}{J_i} = \int_{R_i}^{R_{\text{max}}} \frac{\delta D(R) W(R,h_i)}{D(R)} \frac{dR}{D(R)} \]

(4)

where \( W(R,h_i) \) is the coupling coefficient. Smoothed (3 days) time profiles of the GCR intensity variations \( \delta J_i / J_i \) based on R-Rome, I-Irkutsk, K-Kiel neutron monitors data for the period of 16-30 Jun 2003 are presented in figure 4a. For the power type rigidity spectrum (3) the expression (4) can be rewritten:

\[ \frac{\delta J_i}{J_i} = A_i \int_{R_i}^{R_{\text{max}}} W(R,h_i) \frac{dR}{D(R)} \]

(5)

The amplitude \( A_i \) of the Forbush effect in the heliosphere must be the same for the arbitrary neutron monitor in the scope of the accuracy of calculations. So, the temporal changes of the energy spectrum exponent \( \gamma \) was found by the minimization of the expression \( \phi = \sum_i (A_i - \bar{A})^2 \) with respect \( R_{\text{max}} \) and \( \gamma \)

(where, \( \bar{A} = \frac{1}{n} \sum_i A_i \), \( n \) is the number of neutron monitors with different cut off rigidities \( R_i \) ). Results of calculations of the rigidity spectrum exponent \( \gamma \) is presented in Figure 4b based on the data of eight (n=8)
neutron monitors with different cut off rigidities (Apatity - 0.65 GV, Climax - 3.03 GV, Irkutsk - 3.66 GV, Kiel - 2.29 GV, Moscow - 2.46 GV, Oulu - 0.81 GV, Thule – 0 GV and Yakutsk – 1.7 GV).

![Figure 4](image_url)

**Figure 4.** (a) Temporal changes of the GCR intensities for the period 16-30 Jun 2003, R-Rome, I-Irkutsk, K-Kiel; (b) rigidity spectrum of this Forbush effect.

It is seen from the Figure 4b that at the beginning of the Forbush effect the rigidity spectrum is soft ($\gamma \approx 1.5$), in the period of the minimum and near minimum intensity of GCR the energy spectrum becomes relatively hard ($\gamma \approx 0.9$); during the gradually recovery period of the intensity of GCR the energy spectrum steadily becomes again soft ($\gamma \approx 1.5$). One can state that there is a good agreement between the changes of the rigidity spectrum exponents expected from the theoretical consideration and experimental data.

4. Conclusions

We show that when the recurrent type Forbush effect is created by the changes of the parallel diffusion coefficient depending only on the rigidity of cosmic ray particles as $R^\alpha$ versus the heliolongitude, an expected rigidity spectrum exponent $\gamma$ is proportional to the parameter $\alpha (\alpha = 2 - \nu)$; when the exponent $\nu$ of the PSD of the IMF turbulence increases (in the range of $\sim 10^{-6}$ - $10^{-5}$ Hz) the exponent $\gamma$ decreases, i.e. the rigidity spectrum of the Forbush effect becomes hard. The analyze of the experimental data of the recurrent Forbush effect (16-30 Jun 2003) confirms the modeling results based on the Parker’s transport equation.

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References