Ion Acceleration and Alfvén Wave Excitation at Interplanetary Shocks

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The selfconsistent theory of ion diffusive shock acceleration and the associated generation of Alfvén waves are presented. The wave intensity satisfies a wave kinetic equation and the ion distribution function satisfies the diffusive transport equation. These quasilinear non-stationary equations are solved numerically for a given speed of the shock, traveling through the inner heliosphere.

1. Introduction

Numerous measurements accomplished in interplanetary space indicate that intensive acceleration processes take place in the vicinity of interplanetary shock fronts. Some aspects of energetic particle generation and the associated generation of Alfvén waves can be understand in the frame of simplified plane-wave approach of the diffusive shock acceleration theory [1].

At the same time one can expect that the geometrical factors (finite, increase shock size, adiabatic cooling in the expanding solar wind) can essentially influences the acceleration [2]. Nonstationary character of the interplanetary shocks essentially influences also Alfvén wave generation by the accelerated particles due to so-called overacceleration effect [3].

In this paper we study the acceleration of solar wind protons by interplanetary shocks with an account of geometrical and adiabatic factors. We adopt here Alfvén wave grows rate according to Gordon et al. [4], which differs from that one derived by Lee [1] and used in our previous study [2].

2. Model

The front of the interplanetary shock, created during the solar flare, has a complicated nonspherical form. One can expect that the most effective acceleration takes place at the front part of the shock, where the shock velocity is the highest and the interplanetary magnetic field (IMF) has a small angle with the shock normal in the inner heliosphere ($r \leq 1$ AU). This part of the shock will be considered as a part of the sphere of radius $R_s$ which increases in time with the constant speed $V_s = dR_s/dt$. We assume that IMF $B$ as well as the solar wind speed $w$ are of the radial direction. As far as the transverse size of the acceleration region $L_\perp$ is large enough ($L_\perp \sim R_s$), and fast particles are strongly magnetized ($\kappa_\parallel \gg \kappa_\perp$), the spherical approximation can be used. In this case the diffusive transport equation for the particle (proton) distribution function $f(r, v, t)$ has a form

$$\frac{\partial f}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( \kappa_\parallel r^2 \frac{\partial f}{\partial r} \right) - w \frac{\partial f}{\partial r} + \frac{2w}{3r} v \frac{\partial f}{\partial v} - \frac{f}{\tau_\perp},$$

where $\kappa_\parallel (\kappa_\perp)$ is the parallel (perpendicular) diffusion coefficient and $v$ is the particle velocity. The last term in the Eq. (1) effectively describes particle leakage from the acceleration region due to the diffusion across the IMF lines with the mean time $\tau_\perp = L_\perp^2 / \kappa_\perp$.

We neglected the shock modification by the pressure of accelerated particles. Therefore the shock front is treated as discontinuity at which the medium speed relative to the shock front $u = V - w$ undergo a jump from the value $u_1$ at $r = R_s + 0$ to $u_2 = u_1 / \sigma$ at $r = R_s - 0$, where $\sigma = 4[1 + (3/M_e^2)(\sigma_e/R)]^{-1}$ is the
shock compression ratio, \( M = u_1/c_s \) is the Mach number, \( c_s \) is the sound speed, the subscript \( e \) corresponds to \( r = r_e \equiv 1 \) AU. At the shock front the distribution function fulfills the boundary condition

\[
\frac{u_1 - u_2}{3} \frac{\partial f}{\partial v} = \left( \kappa_{||} \frac{\partial f}{\partial r} \right)_1 - \left( \kappa_{||} \frac{\partial f}{\partial r} \right)_2 + Q_0,
\]

where \( Q_0 = [u_1 N_{mj}/(4\pi v_{mj}^2)] \delta(v - v_{inj})H(t - t_0) \) is the source term, which provides the injection of some part \( \eta = N_{mj}/N_1 \) of medium particles \( N_1 \) into the acceleration process; \( v_{inj} = 4c_s \) is the velocity of injected particles. We assume that the acceleration process starts at some distance \( R = r_0 = V t_0 \). As in the previous papers [1, 2] we assume high level of turbulence behind the shock, that provides \( \kappa_{||} (r < R) \ll \kappa_{||} (r > R) \).

The diffusion coefficients are determined by the relations

\[
\kappa_{||} = \frac{v^2 B^2}{32\pi^2 \omega_B E (k - \rho_B^{-1})}, \quad \kappa_{\perp} = \frac{\rho_B v^2}{3},
\]

where \( \rho_B = v/\omega_B \) is gyroradius, \( \omega_B = eB/mc \) is the gyrofrequency, \( m \) and \( e \) are mass and charge of proton, \( c \) is the speed of light, \( E(k) = d(\delta B^2/8\pi) / d \ln k \) is the energy density of Alfvén waves per logarithm of the wave number \( k \).

The background Alfvén wave spectrum \( E_0(r, k) = E(r, k, t = t_0) \) is modified due to the wave excitation by accelerated particle [4]

\[
\frac{\partial E}{\partial t} + w \frac{\partial E}{\partial r} = \Gamma E,
\]

where

\[
\Gamma(k) = -\frac{32\pi^3 c_a e^2 \kappa_{||}}{km c^2 v^2} \frac{(v = \omega_B/k)}{\int_{v_{\min}}^{\infty} dv v^3 \left( 1 - \frac{\omega_B^2}{k^2 v^2} \right) \frac{\partial f}{\partial r}}
\]

is the wave growing rate \((\partial f/\partial r < 0 \text{ at } r > R)\), \( v_{\min} = \max(v_{inj}, \omega_B/k) \), \( c_a \) is the Alfvén speed. Note that wave growth rate (4) is by a factor of 8/3 larger than that one derived by Lee [1]. In addition the particle diffusion coefficient \( \kappa_{||} (v) \) previously [1, 2] was under the integral.

### 3. Results and discussion

According to the assumption about the radially-directed IMF, its strength has a radial dependence \( B = B_e(r_e/r)^2 \), where \( r_e = 1 \) AU. The solar wind proton number density is described by the same kind of dependence \( N = N_e(r_e/r)^2 \). We take the background Alfvén wave energy density in the form

\[
E_0(r, k) = E_{0e}(k/k_0)^{-\beta} (r/r_e)^{-\delta},
\]

where \( k_0 = 5 \times 10^{-3} \) cm\(^{-1} \). We use the typical values of the solar wind parameters: \( w = 400 \) km/s, \( B_e = 7 \times 10^{-5} \) G, \( N_e = 5 \) cm\(^{-3} \), \( \beta = 1/2, \delta = 4 \), \( E_{0e} = 7 \times 10^{-14} \) erg/cm\(^3\)[5] and the injection parameters: \( \eta = 10^{-3} \). We also take into account the acceleration of \( \alpha \)-particles assuming that the solar wind plasma contains 5% of \( \alpha \)-particles relative to protons.

The differential intensity of accelerated protons \( J(\varepsilon) = (v^2/m) f(r, v, t) \) and the spectrum of Alfvén waves \( E(\nu) = E(k = 2\pi\nu/w) / \nu \) at the shock front \((r = R_s)\) calculated for the shock speed \( V_s = 650 \) km/s are shown for the several time moments in Fig.1 and Fig.2 respectively. Dashed curves correspond to the linear approximation, when selfconsistent generation of Alfvén waves by accelerated particles is not included. One
can see that the intensive wave generation (Fig.2) leads to the considerable increase of maximum (cutoff) energy of accelerated particles \( \epsilon_{\text{max}} = m u^2_{\text{max}} / 2 \), defined by the relation \( f_R(\nu_{\text{max}}) / f_0(\nu_{\text{max}}) = 1/e \), where 
\[
 f_0(\nu) = (N_{\text{inj}} q / 4\pi^2 \nu_{\text{inj}}^3)(\nu / \nu_{\text{inj}})^{-q}
\]
is the universal spectrum with \( q = 3\sigma / (\sigma - 1) \).

The maximum particle energy \( \epsilon_{\text{max}} \) at the initial period \( (R_s < 0.3 \text{ AU}) \) is mainly determined by the time factor, whereas near the Earth’s orbit \( (R_s > 1 \text{ AU}) \) geometrical factors become dominant and therefore at late phases \( R_s > 1 \text{ AU} \) maximum energy becomes almost time independent \([6, 2]\).

Efficient Alfvén wave generation (Fig.2) leads to the considerable decrease of particle diffusion coefficient \( \kappa \parallel \), that in turn provides more rapid particle acceleration. Therefore the maximum energy of accelerated protons considerably exceeds its value calculated in the linear approach (see Fig.1).

Low frequency cutoff in the selfgenerating Alfvén wave spectrum is situated at the frequency \( \nu_0 = \omega_B / (2\pi \nu_m) \), where \( \epsilon_m = m u^2_m / 2 \approx 0.5\epsilon_{\text{max}} \). At higher frequencies the wave spectrum goes like \( E(\nu) \propto \nu^{-2} \), including the region of nonresonant frequencies \( \nu > \nu_{\text{inj}} = \omega_B / (2\pi \nu_{\text{inj}}) \) in agreement with the steady state plane wave prediction \([4]\).

In Fig.3 we show the pressure of accelerated particles \( P_c = (4\pi m / 3) \int_{\nu_{\text{inj}}}^{\infty} d\nu \nu^4 f \) as function of the shock radius \( R_s \). The ratio of the pressure \( P_c \) to the shock ram pressure \( \rho_1 u^2_1 \) increases due to the increase of the particle maximum energy \( \epsilon_{\text{max}} \). Larger value of \( \epsilon_{\text{max}} \) in a quasilinear approach leads to higher particle pressure compared with the linear case (see Fig.3).

In Fig.4 we show the total energy density of selfgenerating Alfvén wave \( W = \int_0^{\infty} d\nu [E(\nu) - E_0(\nu)] \) at the shock front as a function of shock radius \( R_s \). According to the gyrodynamical approach, which treats the energetic particle population as a fluid, at a steady state condition energy density of selfgenerating Alfvén waves and energetic particle pressure are related by the expression
\[
 W / [B^2 / (8\pi)] = (u_1 / c_0) P_c / (\rho_1 u^2_1),
\]
where \( B^2 / (8\pi) \) is the energy density of IMF at the shock position \([7]\). It is seen from Fig.4 that numerical value of wave density goes in a rough agreement with the above relation.

As it is seen from Fig.4, the ratio of the energy density of selfexcited Alfvén waves to the IMF energy density for the shock radius

\[
 R_s = \begin{cases} 
 0.067 & \text{for} \ 1 \\
 0.193 & \text{for} \ 2 \\
 0.635 & \text{for} \ 3 \\
 2.503 & \text{for} \ 4 
\end{cases}
\]

\[
 J_{\text{max}} = \begin{cases} 
 1 & \text{for} \ 1 \\
 4 & \text{for} \ 2 \\
 10 & \text{for} \ 3 \\
 1000 & \text{for} \ 4 
\end{cases}
\]

\[
 E = \begin{cases} 
 1 & \text{for} \ 1 \\
 10 & \text{for} \ 2 \\
 100 & \text{for} \ 3 \\
 1000 & \text{for} \ 4 
\end{cases}
\]

\[
 n_{\text{inj}} = \begin{cases} 
 10^{-4} & \text{for} \ 1 \\
 10^{-2} & \text{for} \ 2 \\
 10^0 & \text{for} \ 3 \\
 10^2 & \text{for} \ 4 
\end{cases}
\]

\[
 R_s, \text{ AU} = \begin{cases} 
 1 & \text{for} \ 1 \\
 2 & \text{for} \ 2 \\
 3 & \text{for} \ 3 \\
 4 & \text{for} \ 4 
\end{cases}
\]

\[
 \nu_{\text{inj}} = \begin{cases} 
 1 & \text{for} \ 1 \\
 2 & \text{for} \ 2 \\
 3 & \text{for} \ 3 \\
 4 & \text{for} \ 4 
\end{cases}
\]

\[
 \nu_{\text{inj}} = \begin{cases} 
 1 & \text{for} \ 1 \\
 2 & \text{for} \ 2 \\
 3 & \text{for} \ 3 \\
 4 & \text{for} \ 4 
\end{cases}
\]

\[
 \nu_{\text{inj}} = \begin{cases} 
 1 & \text{for} \ 1 \\
 2 & \text{for} \ 2 \\
 3 & \text{for} \ 3 \\
 4 & \text{for} \ 4 
\end{cases}
\]

\[
 \nu_{\text{inj}} = \begin{cases} 
 1 & \text{for} \ 1 \\
 2 & \text{for} \ 2 \\
 3 & \text{for} \ 3 \\
 4 & \text{for} \ 4 
\end{cases}
\]

\[
 \nu_{\text{inj}} = \begin{cases} 
 1 & \text{for} \ 1 \\
 2 & \text{for} \ 2 \\
 3 & \text{for} \ 3 \\
 4 & \text{for} \ 4 
\end{cases}
\]
Figure 3. The pressure of accelerated particles as a function of shock radius. Solid and dashed lines correspond to quasilinear and linear approaches respectively.

Figure 4. Energy density of selfexcited Alfvén waves at the shock front as a function of shock radius (solid lines). Dashed lines represent the prediction given by Eq.(7).

grows with time and reaches about unity near the Earth’s orbit. Since $W \propto u_1 P_0 \propto \eta u_1^2$ this ratio is expected to be even larger for higher injection rate $\eta$ and/or larger shock speed $V_g$. In such cases the nonlinear interaction of Alfvén waves is expected to be relevant.

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References