Extensive air shower studies with small arrays

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Abstract. We have studied Extensive Air shower (EAS) with two small arrays of 1 m² scintillation detectors in Tehran, 1200 m above sea level. The distribution of air showers in zenith and azimuth angles has been studied and a $\cos^n \theta$ distribution with $n = 7.2 \pm 0.2$ was obtained for zenith angle distribution. An asymmetry has been observed in the azimuthal distribution of EAS of cosmic rays because of magnetic field of the Earth. Amplitudes of the first and the second harmonics of observed distribution depend on zenith angle as $A_I \approx (0.02 + 0.34 \sin^2 \theta) \pm 0.02$, and $A_{II} \approx (0.027 + 0.5 \sin^4 \theta) \pm 0.026$. Meanwhile, the uncertainties arising from the instrument, transit location of shower particles in the scintillator and fluctuations in the shower front have been calculated.

1 Introduction

The ultra high energy (UHE) cosmic rays having energies greater than 100 TeV are usually observed by detection of their air showers. The shower itself is detected by a surface array of detectors which usually consist of several scintillator detectors and sometimes other types of particle detectors for improvement of accuracy. The arrival direction of an air shower can be determined from fast timing data of the detectors and hence the accuracy of the obtained direction depends on the accuracy of time measurements. The detectors can also determine the local density of shower particles. In this paper we are concerned only with scintillation detector arrays. Each detector is formed by an enclosure with reflective interior surface housing the plastic scintillator and one or more photomultiplier tubes (PMTs) viewing it. An air shower array with 1 m² scintillation detectors has been constructed in Tehran (35°43′N,51°20′E). The elevation of the site is 1200 m above sea level (890 gr cm⁻²). The purposes of the experiment are as follows:

1. Estimation of the effect of uncertainty in time of CR crossing on angular resolution; We estimated the uncertainty of the CR arrival time measurements due to the light enclosures.
2. Determination of zenith angle distribution of air showers at site level.
3. Investigation of the Earth’s magnetic field effect on the azimuthal distribution of EAS of cosmic rays. This experiment is the first step toward construction of an EAS array on Alborz mountain at an altitude of over 2500 m near Tehran.

2 Experimental arrangements

Three large area (100 × 100 × 2 cm³) plastic scintillators were used both to detect the air showers and to record the arrival time of the particles. Each scintillator was enclosed in a pyramidal light enclosure of 15 cm height with white painted walls (Bahmanabadi et al., 1998) and a 5 cm diameter PMT type EM9813KB at the vertex of the light enclosure. The triggering of the apparatus was made, when the three scintillators are fired. When triggering condition was confirmed by logic unit, the time lags between the output signals of scintillators 1 and 2, and 2 and 3 are read out by a computer. Two arrangements were used for this experiment. In arrangement I, the three scintillators lie on top of each other with a separation of 70 cm, so that most of the triple coincidences are due to travel of a single muon. In arrangement II, the three scintillators were layed on a horizontal line 510 cm apart, with the coincidences due to extensive air showers.

A second experiment was carried out with four scintillators as a square array. When the four scintillators were fired, the apparatus is triggered by logic unit, and time lags between the output signals of scintillators (1,4), (3,4), and (2,3) are read out by a computer.

3 Data analysis method

In our analysis, we assume the front surface of an air shower disk is approximately a plane perpendicular to the direction
of the primary cosmic ray. Let a unit vector \( \hat{n} \) represent the incident direction of the air shower. We define \( T_{21} \) to be the time lag between pulses 1 and 2 and \( T_{23} \) between pulses 2 and 3. If we measure \( T_{21} \) and \( T_{23} \) from three scintillators, we can determine the arrival direction, \( \hat{n} \). In order to interpret the experimental data we require expressions for \( T_{21} \) and \( T_{23} \) involving the geometrical properties of showers that we wish to measure, and take into account the thickness of the shower disks and the instrumental fluctuations:

\[
T_{21} = \frac{1}{c} d_1 \cdot \hat{n} + (T_2 - T_1) + (\tau_2 - \tau_1) + (t_2 - t_1) \\
T_{23} = \frac{1}{c} d_2 \cdot \hat{n} + (T_2 - T_3) + (\tau_2 - \tau_3) + (t_2 - t_3)
\]

(1)

The differences in the arrival times of the front at the various scintillators due to axis orientation are represented by terms involving \( d_1 \) and \( d_2 \), the position vectors of the scintillators 1 and 3 relative to scintillator 2. We assume all particles have velocity of light, \( c \). \( T_1, T_2, \) and \( T_3 \) are the fixed delays artificially introduced for display purposes. \( \tau_1, \tau_2 \) and \( \tau_3 \) represent the random instrumental errors, and \( t_1, t_2, \) and \( t_3 \) are the differences between the arrival times of the front on the three scintillators and the actual arrival times of the first particle at the scintillators. In the following discussion we use the statistical concept of dispersion of a variable \( x \) which is denoted by \( D(x) \). We assume in our analysis that the instrumental fluctuations are the same for all three pulses. As mentioned earlier, most of the coincidences with arrangement I are produced by downward moving single muons. Thus there is no term involving \( t \) for this arrangement and time dispersions will be:

\[
D_I(T_{21} + T_{23}) = 6D(r) \\
D_{II}(T_{21} + T_{23}) = 6D(r) + 6D(t)
\]

(2)

for the arrangements I and II respectively, and hence:

\[
D(t) = \frac{1}{6} [D_{II}(T_{21} + T_{23}) - D_I(T_{21} + T_{23})]
\]

(3)

Since \( D(t) \) is the dispersion of times of first electron arrival, averaged over shower sizes and over distances from shower axis, \( \sqrt{D(t)} \) is an indication of the mean thickness of the shower disks.

4 Experimental measurements and results

4.1 Determination of thickness of shower disk and instrumental fluctuations

In table 1 we have listed the values of \( [D(T_{21} + T_{23})]^{\frac{1}{2}} \) and the numbers of events used in the calculations. We have also listed the corresponding value of \( \sqrt{D(t)} \). The dispersion due to instrumental fluctuations is calculated by Eq. 2. This fluctuations arise partly from fluctuating delays in the light enclosure of scintillators and partly from electronic circuits. Assuming the dispersions of \( \tau_1, \tau_2 \) and \( \tau_3 \) to be equal, we find \( [D(r)]^{\frac{1}{2}} = 1.5 \) ns. The error in arrival time can be represented as \( [\sigma_{\text{elec.}}^2 + \sigma_{L.E.}^2]^{\frac{1}{2}} \), where \( \sigma_{\text{elec.}} \) and \( \sigma_{L.E.} \) are the errors in time due to the electronic circuits, and light enclosure of scintillators, respectively. The errors of the electronic circuits is 1 ns according to catalogues. Then the rest of errors is due to the light enclosure, that is 1.1 ns, which is consistent with our previous results (Bahmanabadi et al., 1998). Now, if the time of arrival of the first shower particles detected in two adjacent scintillators of an EAS array separated by a distance \( d \) are \( s_1 \) and \( s_2 \), neglecting the curvature of shower front, the direction of the two parallel shower particles, \( \hat{n} \), is obtained from simple relation: \( \sin \theta \sin \phi = \frac{2}{3}(s_2 - s_1) \). The incident cosmic rays direction has approximately an axial symmetry (Bertou et al., 2000). Assuming that the errors in \( \theta \) and \( \phi \) are equal, we can therefore write the errors in \( \theta \) as:

\[
\Delta \theta = \sqrt{2[c^2(\Delta s)^2 + \frac{1}{2}(\Delta d)^2 \sin^2 \theta]}^{\frac{1}{2}}
\]

(4)

where \( <\cos^2 \phi> = <\sin^2 \phi> = \frac{1}{2} \) has been used instead of \( \sin^2 \phi \) and \( \cos^2 \phi \), and \( \Delta s \) and \( \Delta d \) are the errors in determination of time of arrival and distance of separation of the two shower particles. The errors in arrival time can be written as \( \Delta t = [\sigma_{t}^2 + \sigma_{sh}^2]^{\frac{1}{2}} \), where \( \sigma_t = \sqrt{D(r)} \) is the inherent uncertainty in time measurement and \( \sigma_{sh} \) is the variance in time of arrival of shower particles of a given detector due to the thickness of the EAS disk. Our measurements show that \( \sigma_{sh} \) is greater than \( \sigma_t \) (table 1). From Eq. 1 of Linsley (1986) which parameterizes the shower thickness as a function of the distance of the detector from the shower core (\( r \)), we find the following expression for \( \sigma_{sh} \):

\[
\sigma_{sh} = (1.6 \text{ ns})(1 + r/30)^{1.65} / \sqrt{n(r, \theta)}
\]

(5)

where \( r \) is in meters, and \( n(r, \theta) \) is the number of shower particles crossing the detector located at a distance \( r \) from the core of a shower with zenith angle \( \theta \). For a 1 m\(^2\) detector \( n(r, \theta) \) is simply the shower particle density given by famous NKG formula. We have neglected the slight dependence of Eq. 5 on zenith angle (Linsley, 1986). Thus Eq. 4 is rewritten as

\[
\Delta \theta = \sqrt{2[c^2(\Delta s)^2 + 2.56(1 + r/30)^{3.3}/n(r)]} \\
+ \frac{1}{2}(\Delta d)^2 \sin^2 \theta)^{\frac{1}{2}}
\]

(6)

Eq. 6 shows that for large values of \( r \) the error due to shower thickness is the dominant term. However, near the shower core the inherent timing error (\( \sigma_t \)), which according to our estimation is 1.1 ns for light enclosure of white inside finish, and 1 ns for electronic circuits, is larger than the error due to shower thickness (\( \sigma_{sh} \)). Specifically, for 1 m\(^2\) detectors separated by 15 m, and a zenith angle of 20\(^o\), a typical shower may have a particle density of 50 at the core and thus we obtain \( \Delta \theta = 3.7^o \). For this example, the three error factors contributing to \( \Delta \theta \), i.e., inherent timing error, shower thickness, and location uncertainty, will be, 3.5\(^o\), 0.6\(^o\), and 1.3\(^o\) respectively. These uncertainties are taken to account in the analysis of the next experiment.
method three time lags, that is, \(T\) (Mitsui et al. 1990; Nishizawa et al. 1989). In the latter air shower can also be determined by least-square method \(T\) only two time lags i.e., present experiment at Tehran level. In this method we use 5 zenith angle in bins array (Ivanov et al., 1999). These distributions have been was also found for events above 5 in Fig. 2. It shows a north-south asymmetry. The asymmetry intervals used in the analysis has been given at the left side of the zenith angle computed from data. The differential zenith angle distribution can be represented by \(Z(\theta)\)\(d\theta \propto \sin \theta \cos^n \theta \ d\theta\). We find \(n = 7.6 \pm 0.2\) for this fit in the present experiment at Tehran level. In this method we use only two time lags i.e, \(T_{34}\) and \(T_{14}\). Arrival direction of an air shower can also be determined by least-square method (Mitsui et al. 1990; Nishizawa et al. 1989). In the latter method three time lags, that is, \(T_{14}, T_{34}, \) and \(T_{23}\) is used for finding direction of each shower axis. Accuracy of this method is more than the former. A set of about 14000 showers was used for this analysis. Fig. 1(a) shows the distribution of zenith angle, that has been obtained by this method. The exponent \(n\) is also calculated: \(n = 7.2 \pm 0.2\). The exponent \(n\) decreases as the shower size \(N\) or altitude increases, thus our result is consistent with previous result of Luorui & Winn (1984) who found \(n = 10.0\) at sea level for the range of relatively small showers (6 \(\times\) \(10^{14}\) \(\leq N \leq 5 \times 10^{16}\)).

Fig. 2 shows the azimuthal distribution of EAS events with zenith angle in bins 5° - 20°, 20° - 35°, 35° - 50°, and all zenith angles. The number of EAS events in the zenith angle intervals used in the analysis has been given at the left side in Fig. 2. It shows a north-south asymmetry. The asymmetry was also found for events above 5 \(\times\) \(10^{16}\) eV with the Yakutsk array (Ivanov et al., 1999). These distributions have been fitted to the following function:

\[
ASYM = 1 + A_I \cos(\phi - B) + A_{II} \cos(2\phi - C)
\]  

(7)

All fit parameters are shown in Table 2. The values for \(A_I\) are always greater than \(A_{II}\), that is the first harmonics are more important in the array region with the geomagnetic field zenith angle \(\theta_H = 38°\). For other arrays the situation may be different. For example, at the Tibet array (30.11°N, 90.53°E) where the field zenith angle is \(\theta_H = 45°\), both the first and the second harmonics are equally prevailing. At the Yakutsk array (62°N, 130°E; \(\theta_H = 14°\)) the first harmonic, and at the Chakaltuya array (16.35°S, 68.2°W; \(\theta_H = 88°\)) the second harmonic dominate (Ivanov et al., 1999). Using the amplitude of \((N_e - N_n)(N_{se} + N_{sn})\), where \(N_e(N_n)\) is the shower number from the south (north) half-space, as a function of zenith angle, we have compared our results (solid circles) with data of Yakutsk array (squares) in Fig. 3. As it is seen from this figure, the asymmetry amplitude increases with \(\theta\). Also in our case this amplitude is more than Yakutsk results, because the geomagnetic field zenith angle in our site is greater than the one in Yakutsk.

4.2 Square array

With arrangement of four scintillators as a square array, we measured the time lag between the detectors (1,4), (3,4), and (2,3) for each shower. The time lags are represented by \(T_{14}, T_{34}, \) and \(T_{23}\) respectively. Zenith and azimuth angle of each shower was obtained from Eq. 1. Fig. 1(a) is a histogram of the zenith angle computed from data. The differential zenith angle distribution can be represented by \(Z(\theta)\)\(d\theta \propto \sin \theta \cos^n \theta \ d\theta\). We find \(n = 7.6 \pm 0.2\) for this fit in the present experiment at Tehran level. In this method we use only two time lags i.e, \(T_{34}\) and \(T_{14}\). Arrival direction of an air shower can also be determined by least-square method (Mitsui et al. 1990; Nishizawa et al. 1989). In the latter method three time lags, that is, \(T_{14}, T_{34}, \) and \(T_{23}\) is used for finding direction of each shower axis. Accuracy of this method is more than the former. A set of about 14000 showers was used for this analysis. Fig. 1(b) shows the distribution of zenith angle, that has been obtained by this method. The exponent \(n\) is also calculated: \(n = 7.2 \pm 0.2\). The exponent \(n\) decreases as the shower size \(N\) or altitude increases, thus our result is consistent with previous result of Luorui & Winn (1984) who found \(n = 10.0\) at sea level for the range of relatively small showers (6 \(\times\) \(10^{14}\) \(\leq N \leq 5 \times 10^{16}\)).

Table 1. Specifications of experiments with arrangements I and II

<table>
<thead>
<tr>
<th>Arrangement</th>
<th>No. of events</th>
<th>Exposure time (sec)</th>
<th>(c\sqrt{D(t)}) (m)</th>
<th>(\sqrt{D(T_{21} + T_{23})}) (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>77000</td>
<td>15000</td>
<td>-</td>
<td>3.8 (\pm) 0.01</td>
</tr>
<tr>
<td>II</td>
<td>7530</td>
<td>72000</td>
<td>1.04 (\pm) 0.01</td>
<td>9.3 (\pm) 0.08</td>
</tr>
</tbody>
</table>

Fig. 1. Frequency of showers per solid angle vs. zenith angle, \(\theta\). Distributions were obtained by using two time lags (upper panel) and by least square method (lower panel)(Refer to text).

5 Conclusion

The arrival direction of air showers are usually estimated by fast timing shower detection with scintillator arrays. Each scintillator is usually enclosed by a light enclosure. We have estimated uncertainties in time of arrival of a shower particle. These uncertainties are due to the shower thickness, the unknown position of transit in the large area scintillator, and the equipment, i.e. electronics and light enclosure. The timing error principally originates from thickness of the
Table 2. Coefficients of the function ASYM which were fitted to the data.

<table>
<thead>
<tr>
<th></th>
<th>5° ≤ θ &lt; 20°</th>
<th>20° ≤ θ &lt; 35°</th>
<th>35° ≤ θ &lt; 50°</th>
<th>0° ≤ θ &lt; 90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_I</td>
<td>0.0356</td>
<td>0.090</td>
<td>0.174</td>
<td>0.081</td>
</tr>
<tr>
<td>A_{II}</td>
<td>0.0279</td>
<td>0.0488</td>
<td>0.1293</td>
<td>0.069</td>
</tr>
<tr>
<td>B</td>
<td>35°</td>
<td>78°</td>
<td>103°</td>
<td>95°</td>
</tr>
<tr>
<td>C</td>
<td>184°</td>
<td>234.5°</td>
<td>176.8°</td>
<td>−193°</td>
</tr>
</tbody>
</table>

Fig. 2. Relative numbers of EAS events in various zenith angle intervals (shown at each panel). Dot lines show the ASYM function (see the text).

Fig. 3. Relative difference of number of showers from the south and north half space as a function of zenith angle.

shower disk. However, near the shower core and for zenith angles less than 30° the inherent time error due to light enclosure is more important than other errors. It is therefore concluded that, in order to achieve better angular resolutions, EAS experiments must established along with the tracking techniques (Bernlohr et al., 1996). The zenith angle of the arrival direction of air showers, obeys a \( \cos^n \theta \) law with \( n = 7.2 \pm 0.2 \). On the other hand, when an air shower arrives at an angle to the earth’s magnetic field, the charged particles in the cascade can be deflected. Ivanov et al. (1999) have formulated the effect of geomagnetic field on EAS. For showers arriving from the north the shower particles have higher deflection than the southern showers of the equal energy with the equal zenith angle. Thus, it decreases the event rate as it is shown in Fig. 3. The amplitudes of the first two harmonics, can be fitted to \( A_I \approx (0.02 + 0.34 \sin^2 \theta) \pm 0.02 \), and \( A_{II} \approx (0.027 + 0.5 \sin^4 \theta) \pm 0.026 \).

Acknowledgements. This research has been partly supported by Grant No. NRCl 1853 of National Research Council of Islamic Republic of Iran.

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