Results of EAS characteristics calculations in the framework of the universal hadronic interaction model NEXUS

N. N. Kalmykov\textsuperscript{1}, S. S. Ostapchenko\textsuperscript{2,1}, and K. Werner\textsuperscript{3}

\textsuperscript{1}Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia
\textsuperscript{2}Institute für Kernphysik, Forschungszentrum Karlsruhe, Karlsruhe, Germany
\textsuperscript{3}SUBATECH, Université de Nantes IN2P3/CNRS Ecole des Mines, Nantes, France

Abstract. An extensive air shower (EAS) calculation scheme based on cascade equations and some EAS characteristics for energies \(10^{14} - 10^{17}\) eV are presented. The universal hadronic interaction model NEXUS is employed to provide the necessary data concerning hadron-air collisions. The influence of model assumptions on the longitudinal EAS development is discussed in the framework of the NEXUS and QGSJET models. Applied to EAS simulations, perspectives of combined Monte Carlo and numerical methods are considered.

1 Introduction

The simulation of the extensive air shower development and the reliability of model predictions are of prime importance in studies of super-high (\( > 10^{15}\) eV) energy cosmic rays. Indeed, the reconstruction of primary particle characteristics by measuring EAS characteristics implies the knowledge of the interaction model whereas the models used are phenomenological ones and their validity is open to question above the energy range attained by modern colliders (about \(10^{15}\) eV for equivalent fixed target energy). It should be noted that a considerable gap exists between this upper limit and the energy region \(10^{20} - 10^{21}\) eV which is presently the object of much attention (see AUGER Collaboration (1999)).

It would not be an overestimation to say that the most popular technique to provide necessary theoretical predictions of EAS characteristics is the Monte Carlo (MC) method which may be realized in two main variants. The first one (employed in the program CORSIKA (Heck et al., 1998)) uses the direct MC simulation down to the lowest particle energies under consideration. Such an approach produces results that can be easily compared with experimental data including not only average EAS characteristics but their fluctuations as well. But it proves to be very time-consuming and this serious drawback prevented to use the direct MC above \(10^{17}\) eV. Even at lower energies there are difficulties with simulation of sufficient number of events. The alternative is to use so-called “thinning” (Hillas, 1981) – multilevel sampling of secondary branches of the cascade where one ignores the majority of secondary particles and follows the fate of a few of them introducing proper weights. Reducing greatly the simulation time, this procedure distorts fluctuations and comes up with some other problems (Kobal et al., 1999).

But there exist effective methods to calculate EAS development using the combination of MC and numerical techniques. As all essential contributions to EAS fluctuations come from the initial part of the cascade process i.e. from the fluctuations due to the behaviour of the most energetic particles, it is sufficient to employ explicit MC simulations only for particles with energies above some cutoff \(E_{\text{thr}} = kE_0\) \((k \approx 10^{-2} - 10^{-3}, E_0\) is the primary energy). Contributions of secondary particle cascades of smaller energies may be accounted for in average using numerical solutions of corresponding cascade equations. This approach was, for example, successfully used in (Kalmykov et al., 1997) as well as by many other researchers in the last few decades.

Recently a new hadronic interaction model NEXUS has been proposed (Drescher, 2001) which has much more solid theoretical basis than presently used models such as VENUS (Werner, 1989) or QGSJET (Kalmykov et al., 1997). This new model enables one to obtain more reliable predictions at super-high energies but it is more complicated and therefore more time-consuming. So the problem of the EAS simulation strategy assumes a greater importance.

In this paper we consider the calculations of EAS characteristics in the framework of the NEXUS model and discuss the EAS simulation strategy.

2 Solving cascade equations in the framework of the NEXUS model

The NEXUS model treats cross-section and particle production calculations consistently considering energy conserva-
tion strictly in both cases. Hard processes are introduced in a natural way without any unphysical dependencies. The set of model parameters is adjusted so as to fit basic data in proton-proton and lepton-nucleon scattering as well as in electron-positron annihilation. All that ensures a much safer extrapolation to super-high energies when compared to other models but at the same time the necessity to accelerate EAS simulations becomes more pronounced.

Calculations of average EAS characteristics in the framework of the NEXUS model were carried out in (Bossard et al., 2001). Using the system of hadronic cascade equations (see Gaissier (1990)) one describes average hadronic cascades by the differential energy spectra $h_n(E, X)$ of hadrons of type $n$ with energy $E$ at depth $X$. The corresponding system of integro-differential equations for $h_n(E, X)$ may be reduced (after discretizing over energy) to the system of linear differential equations that can be solved by standard methods. Our approach is based on the same ideas as in (Denko, 1965; Hillas, 1965) but with some improvements (Kalmykov and Motova, 1986) which enable to avoid too small steps when integrating over the depth. The system used incorporates nucleons (and anti-nucleons), pions and kaons. The inclusive spectra of secondaries of type $n$ produced in interactions of primaries of type $m$ were calculated using the MC technique and a special smoothing procedure was applied to eliminate the influence of statistical fluctuations. Other EAS characteristics (electron and muon numbers) were computed as functionals from $h_n(E, X)$ (see Bossard et al. (2001)). The method employed enables to obtain average EAS characteristics within $\sim 1\%$ accuracy. As the number of discretized energies is proportional to $\ln E$ the computing time appears to be quite negligible when compared to the direct MC approach. It is worth noting that in some cases the knowledge of the average EAS behavior is quite sufficient to analyze experimental data or to compare predictions of different models. Thus calculated shower maximum depths were compared with experimental data to obtain the information on the primary mass composition near the knee (see Bossard et al. (2001)).

3 Comparison of NEXUS and QGSJET predictions

Some dependencies of EAS characteristics on the depth for a set of primary energies were presented in (Bossard et al., 2001). It is also of interest to compare the predictions of the NEXUS and QGSJET models as the latter one was frequently used in calculations at super-high energies. The results of this comparison are shown in Fig. 1 for electron and muon ($E_\mu > 1$ GeV) numbers at sea level and in Fig. 2 for shower maxima. Label 1 corresponds to the assumption that only inclusive spectra of hadrons are different whereas cross-sections are the same as in the QGSJET model (see Kalmykov et al. (1997)). Label 2 marks results obtained with different cross-sections. (The NEXUS model predicts higher values of cross-sections at energies above $10^{14}$ eV) As there is no essential discrepancy between QGSJET and NEXUS predictions at $10^{14} - 10^{17}$ eV it is hardly possible to expect any significant divergence in conclusions derived if one replaces one model by another. It follows from Fig. 1 that the exponent $\alpha_{\epsilon(\mu)}$ in the traditional fit

$$N_{\epsilon(\mu)} = K_{\epsilon(\mu)}E^{\alpha_{\epsilon(\mu)}}$$

does not differ more than by 0.02 for these two models. Calculations have shown that only for hadron numbers (in case of different cross-sections) expected deviations reach 0.03–0.04.

But variations discussed may increase as energy increases above $10^{17}$ eV.

$\frac{N(\text{NEXUS})}{N(\text{QGSJET})}$

**Fig. 1.** NEXUS to QGSJET ratio for electron and muon numbers at sea level vs. primary energy. 1—cross-sections are identical, 2—cross-sections are different.

$$X_{\text{max}}(\text{NEXUS}) - X_{\text{max}}(\text{QGSJET}), \text{g/cm}^2$$

**Fig. 2.** Difference of NEXUS and QGSJET shower maxima vs. primary energy. 1—cross-sections are identical, 2—cross-sections are different.

It may be also of interest to explore how the distribution of the projectile energy between hadrons and gammas influences on EAS characteristics. Fig. 3 demonstrates NEXUS model predictions for some EAS characteristics at sea level. The value of $K_\gamma$ incorporates results of $\pi^0$ and $\eta$ decays
and also some minor contributions. The influence of $K\gamma$ enhancement is rather well pronounced for hadrons ($E_h > 50$ GeV) and muons but may be neglected for electrons. It is essential to note that there are practically no variations of shower maxima due to $K\gamma$ enhancement.

$$N(K\gamma + 0.01)/N(K\gamma)$$

Fig. 3. Influence of $K\gamma$ enhancement on electron, muon, and hadron numbers at sea level.

4 Perspectives of simulation strategy

Although calculations of average EAS characteristics can provide valuable information it is highly desirable to have at hand a sufficiently fast procedure which accounts for EAS fluctuations properly. It is possible to split such a procedure into well separated blocks.

The first one is the solution of the cascade equations for different initial conditions. This block should produce spectra of shower particles at given observation levels as function of their types, energies, angles, transverse displacements and time delays. The results must be tabulated. It is essential to generalize cascade equations and their solutions from one-dimensional case described in (Bossard et al., 2001) to full three-dimensional cascades. In doing so one can employ the results of the standard ajoint equation approach (see Lagutin (1993)) to treat electron-photon cascades.

The second block is the explicit MC simulation of the high energy part of the cascade (for particles with energies $E_{thr} < E < E_0$) using the NE\textsc{x}US model. It is important that one may neglect scattering angles and employ one-dimensional procedure as $E_{thr}$ is sufficiently high. The calculation of EAS components for individual showers is realized by summing up all partial contributions. As a rule these contributions should be obtained by interpolation from the tables but, in principle, it is possible to solve cascade equations for random initial conditions representing individual showers.

It is also possible to employ pretabulated MC results for low energy cascades (below some value $E_{min} \ll E_{thr}$). These cascades should be simulated as in the CORS\textsc{i}KA program. The comparatively small time needed for simulations of low energy cascades could ensure necessary statistics of individual histories and thus achieve precise enough description of the distribution tails.

A number of additional blocks may be introduced to provide calculations of necessary EAS characteristics (e.g. fluorescence and cherenkov light).

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References

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