TeV Gamma Rays Expected from Supernova Remnants in a Uniform Interstellar Medium

E. G. Berezhko¹ and H. J. Völk²

¹Institute of Cosmophysical Research and Aeronomy, 31 Lenin Ave., 677891 Yakutsk, Russia
²Max-Planck-Institut für Kernphysik, Heidelberg, D-69029, Germany

Abstract

Kinetic nonlinear calculations of TeV gamma rays, produced by shock accelerated cosmic rays (CRs) in nuclear collisions, from supernova remnants (SNRs) evolving in a uniform interstellar medium (ISM), are presented. The sensitivity of \( \gamma \)-ray production to a physical parameter set is studied. The critical value of the CR diffusion coefficient is determined, which is roughly ten times the Bohm diffusion coefficient. For lower CR diffusion coefficients, SNRs expanding in a so-called warm (or denser) ISM can be visible in TeV gamma rays within a distance of several kpc. When the CR diffusion coefficient essentially exceeds the critical value, then the maximum CR energy becomes lower than 10 TeV and the expected TeV gamma-ray flux drops below the detectable level.

1 Introduction:

Direct evidence that the observed nuclear Galactic CRs are indeed produced through diffusive shock acceleration in SNRs, at least up to the energy \( 10^{14} \) eV, will hopefully come from observations of SNRs in \( \gamma \)-rays. Theoretical estimates of the \( \pi^0 \)-decay \( \gamma \)-ray luminosity of SNRs have led to the conclusion that the expected TeV \( \gamma \)-ray flux from nearby SNRs should be detectable (Dorfi, 1991; Drury et al., 1994).

The spectra of \( \pi^0 \)-decay \( \gamma \)-rays, produced by shock accelerated CRs in SNRs evolving in a uniform interstellar medium (ISM), were studied in detail in a kinetic approach (Berezhko & Völk, 1997). Here we present further calculations in order to demonstrate how the expected TeV \( \gamma \)-ray flux from SNRs depends upon the ISM density, ejected mass and CR diffusion coefficient.

2 Results and Discussion:

We use here the kinetic model for particle acceleration in SNRs which selfconsistently describes diffusive shock acceleration of CRs including the nonlinear CR backreaction on the structure and dynamics of the expanding spherical SNR shock (Berezhko et al., 1996; Berezhko & Völk, 1997). CRs naturally arise from a suprathermal postshock gas particle population at the shock front, where after shock heating some fraction \( \eta \ll 1 \) of gas particles are injected into the acceleration process. Since for a wide range of possible injection rates the CR acceleration efficiency is very high and almost constant (Berezhko et al., 1996), we use here the particular value of the injection parameter \( \eta = 10^{-4} \). This value of \( \eta \) gives an injection rate which is more than an order of magnitude lower compared with results from collisionless shock plasma simulations (Trattner & Scholer, 1991; Giacalone et al., 1993) for a purely parallel shock. Our lower value of \( \eta \) effectively takes into account the influence of the shock obliquity: according to Ellison et al. (1995) and Malkov & Völk (1995) already at angles \( \theta \approx 45^\circ \) between the upstream magnetic field and the shock normal the injection rate is about an order of magnitude smaller than in the purely parallel shock case.

In order to illustrate the sensitivity of \( \gamma \)-ray production upon relevant physical parameters, we present in Fig.1 calculations of the expected integral \( \gamma \)-ray flux \( F_\gamma(1 \text{ TeV}) \) normalized to the distance \( d = 1 \text{ kpc} \) for eight different parameter sets. Four of them are typical for type Ia SN: a SN explosion energy \( E_{\text{sn}} = 10^{51} \) erg, ejecta mass \( M_{\text{ej}} = 1.4M_\odot \), and a value \( k = 7 \) of the parameter \( k \) that describes the ejecta velocity distribution; four others correspond to the SN Ib /SN II case: \( E_{\text{sn}} = 10^{51} \) erg, \( M_{\text{ej}} = 10M_\odot \), \( k = 10 \).

Note that an essential part of the Galactic volume is occupied by a rarefied, so-called hot ISM phase with a density that is about two orders of magnitude lower compared with another representative warm ISM phase,
Figure 1: Integral TeV $\gamma$-ray flux normalized to the distance $d = 1$ kpc (a) and CR maximum momentum (b) as a function of time.

whose hydrogen number density is $N_H = 0.3$ cm$^{-3}$. According to kinetic theory CRs are also efficiently produced in SNRs in the hot ISM. But as far as the expected $\gamma$-ray flux is concerned, even simple estimates show that this flux is far below the detection threshold due to the extremely low density. Therefore, in both cases calculations were performed for ISM hydrogen number densities $N_H = 0.3$ cm$^{-3}$, and 30 cm$^{-3}$. The last case can model a SNR inside a cloud.

According to calculations (Berezhko & Ksenofontov, 1999) performed for the case of the shell-type SNR SN 1006, whose TeV-energy $\gamma$-ray emission was detected recently (Tanimori et al., 1998), CR diffusion is less effective compared with the usually used Bohm-type diffusion. Therefore we consider two different CR diffusion coefficients. The first is the ordinary Bohm-type diffusion coefficient $\kappa = \kappa_B$; in the second case we use a value ten times higher, $\kappa = 10\kappa_B$. Note that different values of $\kappa$ almost only influence the value of CR maximum energy $\epsilon_{\text{max}}$, achieved during their acceleration (or maximum momentum $p_{\text{max}} = \epsilon_{\text{max}}/c$, where $c$ is the speed of light). In all cases we use the standard value of the ISM magnetic field $B_0 = 5 \mu$G.

Since the energy $\epsilon \approx 10 \epsilon_\gamma \approx 10$ TeV of CRs, which generate $\gamma$-rays with energy $\epsilon_\gamma = 1$ TeV, is close to the expected maximum CR energy, the actual value of CR diffusion coefficient influences the expected $\gamma$-ray flux $F_\gamma(1$ TeV) significantly: in the case $\epsilon_{\text{max}} < 10$ TeV it should be essentially lower than at $\epsilon_{\text{max}} \approx 10$ TeV. Therefore we present in Fig. 1b the calculated value of the maximum CR momentum $p_{\text{max}}$ as a function of time. By definition $p_{\text{max}}$ is that value of the CR momentum $p$ for which the overall spectrum $N(p, t)$ of CRs, accelerated up to time $t$, deviates from the dependence $p^{-2}$ by a factor of $e$: $N(p_{\text{max}}, t)/N(mc, t) = (p_{\text{max}}/mc^2)^2/e$.

For the sake of convenience in Fig.1 time is measured in units of $t_0$ which is related with the sweep up radius $R_0$ and the mean initial ejecta speed $V_0$: $t_0 = R_0/V_0$; $R_0 = (3M_{\text{ej}}/4\pi\rho_0)^{1/3}$; $V_0 = \sqrt{2E_{\text{sn}}/M_{\text{ej}}}$.

Here $\rho_0 = 1.4N_H m$ is the density of an ISM that contains 10% helium, and $m$ is the proton mass.
The maximum momentum of CRs accelerated by the expanding SN shock is determined by geometrical factors (Berezhko, 1996). The CR spectrum starts to decline significantly from the power law form at some momenta $p \sim p_{\text{max}}$, where the shock becomes too small and too slow to fill the upstream region by CRs with their number density, required for providing a power law spectrum. The shock size $R_s$ and its speed $V_s$ are the most relevant parameters, which influence the value of $p_{\text{max}}$. In the free expansion phase ($t < t_0$) $p_{\text{max}} \propto R_sV_s$ is an increasing function of time, whereas during the Sedov phase ($t \gtrsim t_0$) the value of $p_{\text{max}}$ is almost constant because the shock weakens at this stage, the value of $R_sV_s \propto t^{-1/5}$ decreases with time, and the shock produces CR particles only up to a momentum $p_m \propto R_sV_s$ which is a decreasing function of time. In this phase the value of $p_{\text{max}}$ is related with CRs accelerated at the end of the free expansion phase. At $t \gg t_0$ the influence of the shock on these highest energy CRs (so-called escaping particles) becomes almost negligible and their energy remains nearly constant. The calculated results (Fig. 1b) correspond very well to the expected dependence (Berezhko, 1996).

$$p_{\text{max}} \propto E_{\text{kin}}^{1/2} M_{ej}^{-1/6} N_H^{-1/3}/(\kappa_B/\kappa).$$  \hspace{1cm} (1)

At the same time this relation was derived in the test particle approximation and it does not take into account the nonlinear effects which also influence the value of CR maximum momentum. As one can see from Fig. 1b, the exact value of $p_{\text{max}}$ during the Sedov phase is slightly higher at larger ejected mass $M_{ej}$, contrary to eq. (1). The modified SN shock more effectively produces high energy CRs which leads to an increase of their maximum momentum by a factor of a few compared with the case of an unmodified shock. For example, in the case $\kappa = \kappa_B, N_H = 0.3 \text{ cm}^{-3}$ and at $M_{ej} = 10M_\odot$ the shock becomes essentially modified already at the end of the free expansion phase. The shock compression ratio is $\sigma = 5.5$ at $t = t_0$ and reaches a maximum value of 5.8 at $t = 2t_0$. Therefore the maximum CR momentum reaches its upper value $p_{\text{max}} \approx 2 \times 10^5mc$ soon after the beginning of the Sedov phase (Fig.1b). It is slightly larger than the estimate $1.6 \times 10^5mc$ from eq.1. Due to the higher mean ejecta speed $V_0$ in the case $M_{ej} = 1.4M_\odot$, the shock reaches the peak modification significantly later: it is only slightly modified ($\sigma = 4.5$) at $t = t_0$ and the maximum shock compression ratio $\sigma = 5.8$ is reached at $t = 10t_0$. At this stage, when the shock most effectively produces CRs, the production $R_sV_s$ has a lower value than at $t = 10t_0$. Therefore the maximum value $p_{\text{max}} = 1.8 \times 10^5mc$, reached at $t = 10t_0$, is slightly smaller than the test particle prediction $2.2 \times 10^5mc$.

As seen from Fig.2, at $N_H = 0.3 \text{ cm}^{-3}$ the maximum CR momentum remains above the critical level $p_{\text{max}}(t) \gtrsim 10^4mc$ at least for $t/t_0 \gtrsim 0.1$ even for a large diffusion coefficient $\kappa = 10\kappa_B$. Therefore the value of $\kappa$ does not influence much the expected flux $F_\gamma(1 \text{ TeV})$. In this case the predicted $\gamma$-ray flux $F_\gamma$ is roughly proportional to the ISM density in the whole SNR evolutionary range. The time profile $F_\gamma(t/t_0)$ is roughly the same in all four cases. Note that the essential part of the dependence of $F_\gamma(t/t_0)$ on $M_{ej}$ and $N_H$ is already included through the time $t_0$. The remaining part of the $F_\gamma(M_{ej})$- dependencies is the dependence of the time $t_m$, at which the peak value of the $\gamma$-ray flux is achieved, upon the value of ejecta mass $M_{ej}$. As one can see from Fig.1, $t_m/t_0 \approx 3$ at $M_{ej} = 10M_\odot$ and $t_m/t_0 \approx 10$ at $M_{ej} = 1.4M_\odot$. The reason is that the most relevant factor which determines the value of the $\gamma$-ray flux, is the CR energy content of the SNR. It reaches a peak value in the Sedov phase, when the shock speed $V_s$ drops to some critical value $V_m$ that is a function of ISM parameters (see Berezhko & Völk, 1997 for details). As far as during the Sedov phase $V_s \propto t^{-3/5}$, for the time when the shock speed has the same value $V_m$, we have the relation $t_m \propto M_{ej}^{-3/5}$ that satisfactory corresponds to results represented in Fig.1.

Only in the case of a dense ISM $N_H = 30 \text{ cm}^{-3}$ the increase of the CR diffusion coefficient leads to a drop of the CR maximum momentum below the critical level $p_{\text{max}} < 10^4mc$ that leads to an essential decrease of $F_\gamma(1 \text{ TeV})$ compared with the case $\kappa = \kappa_B$. Therefore the $\gamma$-ray emissivity drops exponentially with increasing energy $\epsilon_\gamma$. Note that this essential influence of the CR diffusion coefficient on the expected $\gamma$-ray flux takes place only in the energy range $\epsilon_\gamma \gtrsim 1 \text{ TeV}$. Due to the fact that at considerably lower energies $\epsilon_\gamma \ll 1 \text{ TeV}$ the $\gamma$-ray production is given by correspondingly lower-energy particles that belong to the power law part of CR spectrum, it remains almost insensitive to the value of the CR diffusion coefficient. As one can see from
Fig. 1, the increase of the ISM density from $N_H = 0.3 \text{ cm}^{-3}$ up to $30 \text{ cm}^{-3}$ in the case $\kappa = 10\kappa_B$ leaves the expected TeV $\gamma$-ray flux almost unchanged. It means that the effect of the $\kappa$-increase is about $10^{-2}$.

The highest energy CRs with momenta $p \gtrsim p_{max}$ become almost insensitive to the shock influence during the late Sedov phase $t \gg t_0$. They fill the volume almost uniformly and its size increases with time at this stage according to the diffusive law $R \propto \sqrt{\kappa t}$. Hence, we have the situation when CRs with number density $n_l(\epsilon) \propto R^{-3}$ interact with a progressively increasing amount of gas $\dot{M} \propto R^3$ which results in an almost constant TeV $\gamma$-ray flux $F_\gamma \propto M_n$ (see the curves which correspond to $\kappa = 10\kappa_B$ and $N_H = 30 \text{ cm}^{-3}$, in Fig. 1a).

Our calculations show that the efficiency of TeV $\gamma$-ray production by shock accelerated CRs in SNRs is characterized by a critical value for CR the diffusion coefficient, which can be represented in the form $\kappa_{crit} = K(B_0/5 \mu G)(N_H/0.3 \text{ cm}^{-3})^{-1/3} \kappa_B$, where the value of $K \approx 10$ is expected to be slightly dependent upon the injection rate. For CR diffusion coefficients essentially larger than $\kappa_{crit}$, one can expect maximum CR energies $\epsilon_{max} < 10 \text{ TeV}$ which would lead to a considerable decrease of the TeV $\gamma$-ray flux below a detectable level at all considered ISM densities considered.

3 Summary:

We have studied here how the background ISM density and the CR diffusion coefficient influence the expected TeV-energy $\pi^0$-decay $\gamma$-ray production in SNRs. Our calculations show that the peak TeV $\gamma$-ray flux $F_m$, normalized to a distance of $d = 1 \text{ kpc}$, of about $10^{-10}$ photons cm$^{-2}$ s$^{-1}$, is reached at $t_m$, which is about three sweep up times $t_0$ for an ISM number density $N_H = 0.3 \text{ cm}^{-3}$. In the Bohn limit for CR diffusion the flux $F_m$ scales proportional to the ISM density in the range $N_H = 0.3 \div 30 \text{ cm}^{-3}$. If the $\gamma$-ray flux is represented in the form $F_\gamma / t / t_0$, then the ejected mass $M_{ej}$ influences only the peak time $t_m$, which scales as $t_m \propto M_{ej}^{1/3}$. In the case of ten times less efficient CR scattering near the shock front, the maximum CR energy becomes lower than the critical value 10 TeV that provides a $\gamma$-ray production that is almost independent of the ISM density at a level which corresponds to $N_H = 0.3 \text{ cm}^{-3}$ in the Bohm limit. If the CR diffusion coefficient is significantly larger than $\kappa_{crit}$, then the expected TeV $\gamma$-ray flux drops considerably below the detectable level $10^{-12}$ photons cm$^{-2}$ s$^{-1}$. Note however that in this case SNRs would be hardly considered as the sources of the Galactic CRs.

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