Estimation of Muon Spectrum produced by Photons incident on Earth from the Direction of Markarian 421

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Abstract

The vertical integral energy spectrum of muons generated by the gamma rays incident on Earth from the direction of Markarian 421 has been derived. The spectrum of photons surveyed by McEnery et al. and the standard cascade equation for muons followed by recent photopion production cross-section obtained from DESY ep collider results have been adopted in this analytical calculation. The derived muon flux has been found to decrease drastically with energy.

1 Introduction:

TeV gamma ray detection technique has been developed by Compton Gamma Ray Observatory has been fairly surveyed by Weekes (1992). The photon spectra is also emitted from Crab Nebula, the Pulsar PSR 1706-44 and also from the Active Galaxy Markarian 421 have been detected by satellite borne active detector measurements at GeV to TeV energies. The TeV muons in underground initiated by high energy gamma rays under observation at AMANDA and Lake Baikal (Halzen and Staneev 1995). They pointed out that the majority of GeV gamma ray sources follow the power law energy spectrum ~ E^{-\gamma} with \gamma = 1 (Thompson et al. 1995). But extra galactic sources like from the Markarian 421 ntedout that the high energy photon may be absorbed in the source, or in the interstellar infrared and microwave background. Earlier Halzen et al. (1996) has pointed out that Markarian 421 is close enough to be detectable with currently available gamma-ray telescopes and TeV flux is also absorbed on the infrared light in the source. Sinitsynayna et al. (1997) have studied the sources of Markarian 421 using the mirror telescope with image system SHALON-1 and the data may be represented by the energy spectrum F(>E) ~ E^{-\gamma}. They have pointed out that the gamma rays from the Markarian 421 galaxy is a typical source of metagalactic cosmic rays. Markarian 421 of red shift z=0.031 is a X-ray selected BL Lacetae extragalactic source of \gamma-rays is the first detected for energies > 30 GeV (Punch et al. 1992). More recently McEnery et al. (1997) have investigated the TeV gamma ray spectrum of Markarian 421. The differential energy spectrum of Markarian 421 obtained from the observations taken on May 7, 1996 as expected from two different methods cited by Zweerink et al. (1997) and Rodgers et al. (1997) obey a power law fit spectrum with the energy spectral index of value \approx-2.56. In the present work we have used the gamma-ray spectrum (McEnery et al. 1997) reaching on earth from the arrival direction of Markarian 421 to estimate the energy spectrum of muons initiated by photopions emitted from such object [McEnery et al. (1997), Zweerink et al. (1997) and Rodgers et al. (1997)] by adopting the standard procedure modified earlier [Drees, Halzen and Hikasha (1989), Bhattacharyya (1997)].

2. Nuclear Physics and kinematics

The primary energy spectrum of \gamma-rays from the direction of Markarian 421 has been fairly surveyed by McEnery et al. (1997) which follows the power law fit of the form:

$$N_\gamma(E) = (2.24 \times 10^6) \ E^{-2.56} \ \text{photons (m}^2 \text{ s TeV)}^{-1}$$  \hspace{1cm} (1)
This can be considered as the source spectrum at the top of the atmosphere (t=0 g-cm$^{-2}$) for the calculation of muons from photoproduced charged pions initiated from gamma induced showers. The corresponding photon spectrum inside the atmosphere can be assumed to follow:

$$\Gamma_0(E) \equiv 8.75 \times 10^{-11} E^{-2.56} \text{cm}^2 \text{sTeV}^{-1}$$

(2)

We have adopted the limiting solution for the estimation of muons from photopions from Drees et al. (1989) from the muon atmospheric diffusion yields for TeV muons

$$\frac{dN_\mu}{dE} = \Gamma_0(E) \frac{\Lambda_\pi}{\lambda_\gamma N} \frac{L_\gamma}{\left[1 + \frac{L_\gamma}{H_\gamma}\right] E \cos \theta / \epsilon_\pi(\theta)}$$

(3)

where

$$L_\gamma = \left[1 - r_\pi(\gamma+1)^{-1}\right]_{\max}, \quad H_\gamma = \left[1 - r_\pi(\gamma+2)^{-1}\right]_{\max} + \frac{\ln \left(t_{\max} / \Lambda_\pi\right)}{(\gamma+2)(1-r_\pi)}$$

and

$$r_\pi = \frac{m_\mu^2}{m_\pi^2}$$

life time in air is the square of the ratio of muon to pion masses, $\epsilon_\pi = m_\pi c^2 H / (c \tau_\pi)$ represents the critical energy of pion decay for the isothermal atmospheric scale height $H=6.7$ km, the pion

$$\tau_\pi = 2.603 \times 10^{-9} \text{ sec}, \quad m_\pi = .139568 \text{GeV}$$

$$m_\mu = .10568 \text{GeV}$$

respectively. The calculated critical energies for meson decays

are $\epsilon_\pi(0^\circ) = 0.115 \text{ TeV}$ $\epsilon_\pi(45^\circ)$ for zenith angular incidence $0^\circ$ and $45^\circ$.

3 Results and discussions:

Taking $\tau_\pi = .57$, absorption length pions in air, $\Lambda_\pi = 120 \text{ g cm}^{-2}$ assuming maximum energy available in the system as $E_{\text{max}} = 100 \text{ TeV}$

and the photonuclear cross section $\sigma_{\gamma N}$ has been estimated from the fit to DESY ep collider results [Particle Data Group (1994)] that follows the relation

$$\sigma_{\gamma p} = [0.147 - 0.017 \ln(E_\gamma / \text{GeV}) + 0.0022 \ln^2(E_\gamma / \text{GeV})] \text{mb}$$

(4)

which yield a value 0.17705 mb at $E_{\text{max}} = 10 \text{ TeV}$ energy. The photonucleon mean free path $\lambda_{\gamma N}$ has been estimated from the expression

$$\lambda_{\gamma N} = \lambda_R \sigma_R / (A \sigma_{\gamma N})$$

(5)

which yield the value 7105 g/cm$^2$ for $\sigma_R = 480 \text{ mb}$ and $A = 14.55$. The simplified form of

$L_\gamma$ and $H_\gamma$ which follow:
\[ L_\gamma = 0.005775 \quad t_{\text{max}} \quad \text{and} \quad H_\gamma = 0.564946 \left[ 1 + \ln(0.00833 t_{\text{max}}) \right] \]

\[ t_{\text{max}} = \lambda_R \ln \left[ \frac{E_{\text{max}} < x \gamma \rightarrow \mu}}{E} \right] \]

Considering the radiation length in air as \( \lambda_R = 38 \text{g cm}^{-2} \), \( < x \gamma \rightarrow \mu = 0.25 \) and \( E_{\text{max}} = \text{TeV} \) the estimated values of \( t_{\text{max}} \) at different muon threshold energies \( E \) have been estimated and are displayed in the Table I along with the derived values of \( L_\gamma \) and \( H_\gamma \) respectively.

### Table I

<table>
<thead>
<tr>
<th>( E ) (TeV)</th>
<th>( t_{\text{max}} )</th>
<th>( L_\gamma )</th>
<th>( H_\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>148.66</td>
<td>0.8585</td>
<td>0.6858</td>
</tr>
<tr>
<td>1</td>
<td>122.32</td>
<td>0.7064</td>
<td>0.5756</td>
</tr>
<tr>
<td>2</td>
<td>95.98</td>
<td>0.5543</td>
<td>0.4387</td>
</tr>
<tr>
<td>5</td>
<td>61.16</td>
<td>0.3532</td>
<td>0.1841</td>
</tr>
<tr>
<td>7</td>
<td>48.37</td>
<td>0.2794</td>
<td>0.0516</td>
</tr>
</tbody>
</table>

Using the expression (3) and other parameters cited in the Table I the derived muon fluxes from the decay of photopions at zenith angles 0° and 45° have been estimated and are displayed in the Table II.

### Table II

<table>
<thead>
<tr>
<th>Muon Energy ( E ) (TeV)</th>
<th>( \frac{dN_\mu}{dE} ) at 0° ( \text{cm}^2 \text{s TeV}^{-1} )</th>
<th>( \frac{dN_\mu}{dE} ) at 45° ( \text{cm}^2 \text{s TeV}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>( 7.20 \times 10^{-13} )</td>
<td>( 1.44 \times 10^{-12} )</td>
</tr>
<tr>
<td>1.0</td>
<td>( 3.59 \times 10^{-14} )</td>
<td>( 7.19 \times 10^{-14} )</td>
</tr>
<tr>
<td>2.0</td>
<td>( 2.35 \times 10^{-15} )</td>
<td>( 4.70 \times 10^{-15} )</td>
</tr>
<tr>
<td>5.0</td>
<td>( 4.45 \times 10^{-17} )</td>
<td>( 9.37 \times 10^{-17} )</td>
</tr>
<tr>
<td>7.0</td>
<td>( 4.92 \times 10^{-18} )</td>
<td>( 9.84 \times 10^{-18} )</td>
</tr>
</tbody>
</table>
The calculated integral muon fluxes per \( \text{cm}^{-2} \cdot \text{s} \) from the differential flux data displayed in the Table II have been found to follow the power law fits at zenith angles 0° and 45°:

\[
N_{\mu}(>E) = 1.127 \times 10^{-14} E^{-3.40} \quad \text{fo} \theta = 0^0 \quad (7)
\]

\[
N_{\mu}(>E) = 2.269 \times 10^{-14} E^{-3.39} \quad \text{for} \quad \theta = 45^0 \quad (8)
\]

The above relations exhibit the fact that muon fluxes decrease drastically with energy and a flattening in the spectral shape is observed with the increase of zenith angular incidence.

**4 Conclusion:**

The integral muon energy spectra initiated by the Markarian 421 \( \gamma \)-ray produced photopion decay at zenith angles 0° and 45° have been calculated. The muon fluxes have been found to decrease sharply with the energy spectral index \( \Xi \approx -3.4 \).

**References**

Halzen F. and Stanev T., 1995, Univ. of Wisconsin, Madison preprint MADPH-95-901.