Cosmic Ray Bounds on Violation of Lorentz Invariance

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Abstract

Recently Coleman and Glashow [Coleman and Glashow 1997] have developed a model which allows the introduction of a small violation of Lorentz invariance. Observational signatures arise because this interaction also violates flavor conservation and allows the radiative decay of the muon, $\mu \rightarrow e + \gamma$, whose branching ratio increases as $b \gamma^4$ where $\gamma$ is the Lorentz factor of the muon with respect to the reference frame in which the dipole anisotropy of the universal microwave radiation vanishes. In this paper we place a bound on the Lorentz invariance violating parameter, $b$, of $b<10^{-25}$ based on observations of horizontal air showers with $n_e \geq 5 \times 10^6$. Similar considerations of cosmic ray neutrinos in the atmosphere improve these bounds by twenty orders of magnitude.

To test by experiments the limits of validity of Lorentz invariance or indeed any of the fundamental principles of physics we need a theoretical model which assumes a specific form for the violation and makes predictions of physical phenomena which can be searched for by the experiments [Will 1993, Haugan and Will 1987, Fishbach et al 1985, Greene et al 1991]. The recent model of Coleman and Glashow incorporates tiny departures from Lorentz invariance which does not respect flavor conservation also [Coleman and Glashow 1997].

One of the signatures of such a flavor non conservation is the transition $\mu \rightarrow e + \gamma$ whose rate increases rapidly with the energy of the muon as measured in a preferred frame such as the one in which the 2.7K universal microwave background does not have any dipole anisotropy. Following their suggestion we calculate the possible contributions of such a process to the flux of “horizontal air showers” and $\mu$-less showers which provide useful estimates for the possible strength of such an interaction and also provide a good bound on such violations[Cowsik, Sreekantan 1999]. We now extend the model to neutrinos and calculate the bound on $b_{\nu_\gamma}/\tau_{\nu_\gamma}$ based on the decay scheme $\nu_\mu \rightarrow \nu_e + \gamma$.

The idea on which the bound on flavor violating interactions is derived becomes clear by noting that the primary cosmic rays consist mainly of nuclei which interact strongly when they are incident on the top of the earth’s atmosphere. The amount of shielding provided by the atmosphere in the vertical direction above the earth is about 1000 g cm$^{-2}$ and increases as the secant of the zenith angle $\theta$ up to $\sim 80^\circ$. The total grammage in the horizontal direction is about 36500 g cm$^{-2}$. The primary cosmic rays interact in the atmosphere and create a ‘nuclear active’ cascade. Since the atmosphere is tenuous with a scale height $h \approx 7 \times 10^5$ cm, pions and kaons in the cascade decay producing the cosmic-ray muonic and neutral component. Nuclear interactions of pions and kaons with the atmosphere compete with their decay and become increasingly dominant as the particle energy increases, so that the spectrum of the muonic and neutrino component at high energies is steeper than that of the nuclear active component by a factor E$^{-1}$. Also the muonic and neutrino components at high energies increases as $\sim \sec \theta$, as the scale height of the atmosphere also has this dependence. Since the interaction mean free path of the hadronic components is $\sim 70$ g cm$^{-2}$, after reaching their maximum development, they are absorbed with an absorption mean free path of $\sim 100$ g cm$^{-2}$. In contrast the muons and neutrinos suffer only fewer interactions and propagate with hardly any reduction in flux. Now note that as we move away from the vertical towards the horizontal direction, with increasing sec $\theta$ the nuclear active components get severely absorbed but the high energy penetrating component increases as $\sim \sec \theta$! Thus at large angles we have a nearly pure beam of high energy muons and neutrinos, traversing distances of the
order of few times the scale height $h_\theta \sim h_{sec}\theta$. Now should the muons and neutrino decay radiatively, the
decay products will induce an electromagnetic cascade which can easily be observed signalling the violation
of flavor conservation, as described in the model of Glashow and Coleman. Indeed as the energy of the
penetrating component increases the observability of the cascade increases as it penetrates deeper, spreads
wider and produces more observable electrons and photons. The electromagnetic cascade has a very broad
peak at about 500 g cm$^{-2}$ from the point of initiation for an electron or $\gamma$ of energy $E \sim 10^4$ GeV and the
depth of the maximum increases logarithmically with energy. The total number of electrons at the peak of
an electromagnetic cascade is approximately equal to the energy of the initiating electron or gamma ray in
GeV units. Thus any array of particle detectors deployed to detect extensive air showers will be able to detect
such showers generated by the radiative decay of the muon and neutrinos. There will be negligible amount of
nuclear active particles and muons in these showers. The background due to showers induced by the primary
cosmic ray nuclei become negligible as we go to large zenith angles. Thus ‘$\mu$-less’ showers appearing in near
horizontal directions constitute a signal of the new process described by Coleman and Glashow.

To quantify these ideas first consider the decay of muons into electrons and photons. We note that the
spectrum of muons at high energies near the earth may be parametrized as

$$\mu(E) = \frac{\kappa_1 \sec \Theta}{E^{\beta_1+1}} \cdot \text{cm}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1} \cdot \text{GeV}^{-1}$$

with

$$\kappa_1 = 10, \beta_1 = 2.7 \quad \text{for} \quad 10^3 \text{GeV} < E < 10^5 \text{ GeV}$$

and

$$\kappa_2 = 10^4, \beta_2 = 3.3 \quad \text{for} \quad 10^5 \text{GeV} < E < 3 \times 10^7 \text{ GeV}$$

Here $\beta_1$ and $\beta_2$ are the power law exponents of the primary cosmic ray spectrum at energies of 10 to 30
times the energy of the muon.

According to Coleman and Glashow[1] the total decay probability per unit time, $\Gamma$, of a muon of Lorentz
factor $\gamma$ is given by:

$$\Gamma = \Gamma_0 + \Gamma_r = \frac{1 + b \gamma^4}{\gamma \tau_0} = \frac{1}{\gamma \tau_0} + \frac{b \gamma^3}{\tau_0}$$

Here $\tau_0 \approx 2.2 \times 10^{-6}$ s is the life-time of the muon and $b$ is a very small parameter describing the violation
of Lorentz invariance and flavor conservation. For a muon to decay close to the earth, say at a distance $d$
of about 5 km ($\approx 700\text{g cm}^{-2}$) from the air shower array, it has to survive decay during its flight though the
atmosphere upto this point i.e. a distance of few times $h_\theta$, the scale height in that direction. Thus the number
of muons decaying in the 5 km stretch is given by

$$s(E) \approx \kappa_\sec \Theta E^{-\beta-1} \exp \{-j \cdot h_\theta E/c \} \cdot \Gamma d/c$$

where $j$ is a number of the order of 2 to 3. Noting that $\Gamma$ is a small number and that at high energies $\Gamma \sim \Gamma_r$, the exponential in eq. 5 may be set to unity and eq. 5 is rewritten as

$$s(E) \sim \kappa_\sec \Theta E^{-\beta-1} \cdot \Gamma_r d/c \approx \kappa_\sec \Theta \cdot \frac{c \tau_0}{b \gamma^3} E^{2-\beta}$$

where $\eta = d m_\mu^{-3} \sec \Theta / c \tau_0 \cdot \text{GeV}^{-3} \approx 5 \times 10^4 \cdot \text{GeV}^{-3}$, for $\langle \sec \Theta \rangle \approx 7$. The products of the radiative
decay of the muon generate an extensive air shower which contains a large number of electrons,$n_e$, near the
maximum, related to the muon energy through the simple relation

$$n_e \approx E/\epsilon$$
where \( \epsilon \approx 1 \text{ GeV} \) for an electromagnetic shower of primary energy in the range \( 10^4 \text{ GeV} - 10^6 \text{ GeV} \). The number spectrum of particles that will be seen by an air shower array is given by

\[
f(n_e) \approx \epsilon^{3-\beta} \cdot n_e^{2-\beta} \tag{8}\]

Or the number of showers \( F \) of size larger than \( n_e \) is given by

\[
F(n_e) = \int_{n_e}^{\infty} f(n_e')dn_e' \tag{9}
\]

\[
F_2(n_e) = \frac{\epsilon^{3-\beta} \cdot n_e^{3-\beta}}{\beta_2 - 3} \quad \text{for } n_e \geq 10^5 \tag{10}
\]

\[
F_1(n_e) = \frac{\epsilon^{3-\beta} \cdot n_e^{3-\beta}}{3 - \beta_1} \left[ 10^{5(3-\beta_1)} - n_e^{3-\beta_1} \right] + F_2(10^5) \quad \text{for } n_e < 10^5 \tag{11}
\]

We compare the integral number spectrum of horizontal air showers obtained by Nagano et al. [Nagano et al, 1986] with the Akeno array in Fig. 1 for \( b = 10^{-23} \) and \( b = 10^{-25} \). The theoretically derived spectrum is very flat, \( \sim n_e^{-0.3} \), in contrast with the observed spectrum of horizontal air showers which shows \( \sim n_e^{-2} \) behavior. Note that \( b \approx 3 \times 10^{-23} \) is excluded even by the lower energy data at \( n_e \approx 10^5 \) and that the bound \( b < 10^{-25} \) is obtained when we consider the fluxes of horizontal air showers quoted by Nagano et al for \( n_e \approx 5 \times 10^6 \). Clearly these bounds are considerably more stringent than those derived by looking at the depth intensity curves for muons and as such small values of branching ratio for radiative decay will not have any detrimental effects on the functioning of muon colliders (Coleman and Glashow, 1998).

It is interesting to note that in the Coleman Glashow model this limit translates to \( |1 - c| \leq 6 \times 10^{-21} \). Now, let us suppose that the radiative decay process for the muon suggested by Coleman and Glashow is also applicable to the neutrinos leading to the decay scheme:

\[
\nu_\mu \rightarrow \nu_e + \gamma,
\]

then the horizontal showers may be used to set a strict bound on this process as well. The calculations of the high-energy cosmic-ray neutrino fluxes in the near horizontal direction (Cowsik et al., 1966, Gaisser et al.) may be approximated to a single power law

\[
\nu_\mu(E) \approx \frac{\kappa_\nu \cdot \sec \theta}{E^{\beta_\nu + 1}} \cdot c m^2 \text{s}^{-1} \text{sr}^{-1} \text{GeV}^{-1} \tag{12}
\]

for \( 10^3 \text{GeV} < E_\nu < 10^6 \text{GeV} \), with \( \kappa_\nu \approx 10 \) and \( \beta_\nu = 3 \). As before its decay according to the equations (14) and (4), leads to a horizontal air shower flux,

\[
F_\nu(n_e) = \frac{\epsilon^{3-\beta_\nu} \cdot \kappa_\nu \cdot \sec \theta \cdot b_{\nu\gamma}}{3 - \beta_\nu} \cdot \tau_{\nu\gamma} \left( \frac{2m_2^2}{m_2^2 - m_1^2} \right)^{3-\beta_\nu} \cdot \frac{1}{m_2^2} \cdot n_e^{3-\beta_\nu} \tag{13}
\]

where \( m_2 \) is the mass of the muon neutrino, \( m_1 \) is the mass of the electron neutrino, \( b_{\nu\gamma} \) is the branching ratio and \( \tau_{\nu\gamma} \) the life time for the process given in equation (14). Notice that equation (16) is weakly sensitive to
\[ \Delta m^2 = m_2^2 - m_1^2 \] and highly sensitive to \( m_2 \). Comparison with the data on horizontal showers as in figure 1 yields the following strict bound for \( b_{\nu\gamma}/\tau_{\nu\gamma} \).

Table 1:
Bound on \( b_{\nu\gamma}/\tau_{\nu\gamma} \) for various values of \( \Delta m^2 \) and \( m_2 \) obtained by comparing the expected integral flux from Calomann-Glashow process with the integral spectrum given Nogano et al. as in Fig. 1.

<table>
<thead>
<tr>
<th>( \Delta m^2 ) (eV(^2))</th>
<th>( m_2 ) (eV)</th>
<th>bound on ( b_{\nu\gamma}/\tau_{\nu\gamma} ) (sec(^{-1}))</th>
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<td>( 10^{-2} )</td>
<td>( 1 \times 10^{-11} )</td>
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<td>( 10^{-1} )</td>
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If we assume \( b_{\nu\gamma} \) to be one, this will yield \( \tau_{\nu\gamma} \sim 10^{11} \) s!

**References**


