Energy Determination in Giant EAS

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Abstract

After a statistical treatment of extensive simulations carried out with CORSIKA (Le Gall, 1999) in the energy range 1-100 EeV, including LPM effect, combined with the fast calculations procedures, we have investigated the response of a giant array and the reconstruction of the primary spectrum. An interesting correlation between age parameter and $E_0/N_e$ has been pointed out as a complementary energy estimator. A minimization method on lateral densities has been also testified on the most energetic events registrated.

1 Introduction:

Those statistical Monte Carlo sampling are a first approach of the inverse problem, concentrated here on the problem of the primary energy determination; it has been adapted to the circumstance of giant arrays, like PAO, where a small number of detectors separated by a large grid distance (about 1.5Km) are hit by giant EAS. The options used in CORSIKA are QGSJET, HDPM and LPM (Capdevielle et al. 92, Heck et al. 97).

2 Longitudinal development and age parameter

2.1 New primary energy estimator

One correlation well known in cascade theory concerns the energy per shower particle observed $E_0/N_e$ and the longitudinal age parameter $s$. Such correlation was represented analytically by a simple quadratic form at lower energies (Capdevielle, Gabinski 90)

$$E_0/N_e = a(s-1)^2 + b$$

(1)

This relation is no more valid for the youngest electromagnetic showers distorted by the LPM effect, for primary energies between $10^8 GeV$ and $10^{11} GeV$ and cannot be used to optimize the evaluation of the primary energy $E_0$. Even, if we neglect the problems for shower size measurement, it appears difficult to estimate accurately the primary energy from electron size for young showers initiated by photons, without using fluorescent measurements. On the opposite, this correlation is still valid for protons or heavy nuclei primaries. As a preliminary rule of thumb, the relation (1) can be used in quality of primary energy estimator (in GeV) with co-efficients $a = 49.69$ and $b = 1.41$. An approach of longitudinal $s$, established from our simulations, is a global increase by 30% of the lateral NKG profile which can be adjusted to the electron density distribution registered between 500 and 1000m (as indicated further). From those elementary considerations, we can derive a preliminary EeV-meter for proton and nuclei initiated showers, summarized in the simple relation

$$E_0 \approx \{10.(s-1)^2 + 0.282\} \cdot \rho_{600}$$

(2)

This conversion of the charged particle density at 600m from axis, $\rho_{600}$, to the total energy $E_0$, expressed here in $EeV$, is a modulation by the lateral electron profile via $s \approx 1.3s_{ij}$ with $r_i = 600m$ and $r_j = 1000m$. 
2.2 Topological aspects and lateral distributions Even in the circumstances where the correlation of relation (1) remains valid, one major difficulty lies in the determination of \( s \) from the lateral profile observed far from shower axis and derived for a small number of densities. The topologic problems in the conversion of the lateral information on densities to \( s \) parameter were carried by introducing the local age parameter (lap) \( s_{ij} \) as the best fit of a portion of the lateral electron density distribution by an NKG function; with \( r_{ij} = \ln(r_i/r_j) \), \( \rho_{ij} = \ln(\rho_i/\rho_j) \), \( \rho_i \) and \( \rho_j \) being the densities at distances \( r_i \) and \( r_j \), \( R_{ij} = \ln\{(r_i + R_0)/(r_j + R_0)\} \), \( R_0 \) Molière radius), we have

\[
s_{ij} = \frac{(\rho_{ij} + 2r_{ij} + 4.5R_{ij})}{(r_{ij} + R_{ij})}
\]

(3)

Such formalism can be used when the mathematical conditions to solve diffusion equations are satisfied (limitations of Landau approximations, monotonic decrease of density with distances...) and needs some attention before being used for individual events. For instance, if one density happens to be larger than an other at lower distance, a larger bin must be taken containing the average density, according to the local configuration. It is also possible to use the average density distribution of several events. The schematic relation between the local profile at distance \( r \) characterized by \( s(r) \) and \( s \) was verified experimentally in Akeno (Nagano et al.83) for distances from 10 to 300m and suggests that near 500-1000m the l.a.p. is related to \( s \).

3 Primary spectrum reconstruction

3.1 Energy generator with ankle The preliminary stage involves the fast simulation codes (Capdevielle et al. 97) for the electromagnetic component and starts with a Monte Carlo generator of primary energy. This generator is characterized by the position of the ankle, \( E_{AK} \), and two different power laws in the differential primary energy spectrum, before and after the ankle. The ankle has been moved in the energy interval \([5EeV, 50EeV]\), starting from one threshold energy of 5 EeV, combined with two slopes choosen as \( m = -3.2 \) (below the ankle) and \( m = -2.2 \) (beyond the ankle).

The spectrum injected in the atmosphere, generating 10000 events (fig.2) with \( E_{AK} = 6.3EeV \), initiates the simulation of 10000 giant EAS on a shower array with a rectangular or hexagonal grid. The detector
spacing can be changed from 500 m up to 2 km and the shower axis is sampled randomly on the detectors contained in an area of 100 km². In order to appreciate the complex statistical effects of the primary spectrum, characterized by one ankle above 5.10¹⁸ eV and different slopes in its description by simple power laws, we have carried out special Monte Carlo simulations above 5 EeV. When trigger conditions are satisfied by an individual lateral distribution, ρ₀₀₀ is interpolated and the primary energy for each shower is derived.

In this first report, we present one part of our analysis, limited to the most simple combinations of primary parameters (near vertical showers between 0°, 30°).

![Primary spectrum reconstruction](image)

Figure 2: Injected and reconstructed spectra
- primary spectrum generated (dashed histogram)
- energy distribution of the events registrated (solid histogram)
- primary spectrum reconstructed (dotted histogram) (on horizontal axis, the logarithm of the energy in GeV and on vertical axis the number of events)

3.2 Trigger and axis determination A very elementary procedure is used here. The barycenter is supposed to be the core and the lateral charged particle distribution, similarly to AGASA (Yoshida et al.94) is taken as

\[ ρ = C x^{-α}(1 + x)^{-(η-α)} \left( 1 + x \frac{R_0}{2000} \right)^{-0.5} \]  

with \( α = 1.2 \), the last factor being added to take into account the steepness at very large distances. η is recalculated from each possible couple of densities ”measured” by the detectors and the density at 600 m is averaged for the different pairs. The primary energy \( E_3 \) is then reconstructed as in AGASA, converting \( ρ₀₀₀ \) following \( E_3 = 2.92.10^8 L_0^2 ρ₀₀₀ \)

The efficiency in the case of one grid spacing of 1.2 km is in fact reduced from 75.3% to 35.3% if we consider the reduction of the total area with the same number of detectors (fixed costs) for equivalent periods.
of exposure. It is interesting to note that the distance barycenter-core doesn’t depend very much neither from the configuration of the array nor from the ankle position.

\( \langle \rho_{600} \rangle \) is here the density at 600 m of the showers triggering the array. \( \langle E_1 \rangle \) is the primary energy averaged on the whole spectrum used for generation and entering in the atmosphere. \( \langle E_2 \rangle \) is the average primary energy of the shower triggering the array. The corresponding spectra are reproduced on fig.2 (respectively dashed and solid lines) for a square of 1.5 km and an ankle at 6.29 EeV. The spectrum reconstructed for \( E_3 \) is superimposed (dotted line) and it can be seen that the “bias” is extended a little above 20 EeV for grids near 1.5 km. The situation obtained for the hexagon is perfectly similar. The reduction of the grid to 1.2 km reduces the bias under about 10 EeV.

Inside, the showers of low energy composing this bias, it can be observed on fig.2 that \( E_3 \) is underestimated at very low energy. This corresponds to the triggering limit of the lowest energy showers where the axis has to fall approximately at equal distances from 3 detectors with minimal densities. In such circumstance, \( \eta \) cannot be determined with accuracy.

The small amount of energy overestimation corresponds also to densities larger at 2.3 Km for instance, than the value recorded at 1.7 Km, such situation in the absence of other signals separated by larger axis distance proving the most awkward for the determination of \( \eta \).

The energy reconstruction method has been improved by using a minimization procedure based on Minuit for axis location and \( \eta \) determination, coupled with a correlation on the sum of densities deposited. The energy of the most energetic event of AGASA was reduced by by this method to 170 EeV and the application to all the events recorded in Volcano Ranch and Yakutsk gave a substantial Chi-square reduction from the signals coming from all detectors.

4 Conclusion

In the case of GAS induced by nucleon or nuclei, the correlation (1) suggests a complementary estimator for primary energy from the couple (size, local age parameter), where it could be worthwhile to replace the size by the density \( \rho_{600} \).

At last, we cross the rather tricky problem of the reconstruction of the primary energy from a small number of densities recorded at large distance from the axis; in spite of the elementary procedure employed, we ascertained that a grid of 1.5 Km allows a correct estimation of the primary spectrum above 2 EeV.

References