**MESON-NUCLEUS INELASTIC CROSS SECTIONS**

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**Abstract**

The known \(\pi\)- and K-proton inelastic and elastic cross sections (up to about 200 GeV) have been used to calculate the inelastic \(\pi\)- and K-nucleus cross sections with a method similar to that used for calculating the inelastic proton-nucleus cross section in a previous paper (Ann Arbor 1992). The calculated results show good agreement with measured \(\pi\)- and K-nucleus inelastic cross sections from about 2 GeV lab energy up to 200 GeV, and cosmic ray results at 8000 GeV for \(\pi\)-nucleus inelastic collisions. Assuming that this method can be used also at higher energies, the \(\pi\)- and K-air nucleus inelastic cross sections can be calculated, for instance, at 10^7 GeV. This gives for the inelastic \(\pi\)-air nucleus cross section: 406 mb, which can be compared to the calculated proton-air nucleus inelastic cross section, which is 471 mb at this energy.

1 **Introduction**

A method used to calculate the inelastic proton nucleus cross section [1] will here be used in a slightly different form to calculate the inelastic charged \(\pi\)- and K-meson nucleus cross sections. For low energies, where cross sections measured at accelerators are available, these experimental results can be used to help determine the parameters. At high energies, the assumption will be used that the differences of the square roots of different cross sections approach energy independence. This assumption was used for \(\sigma_{\text{inel}}(pp)\) and \(\sigma_{\text{el}}(pp)\) in ref. [1]:

\[
\frac{\sigma_{\text{inel}}(pp)}{\pi \cdot 10 \text{ mb}}^{1/2} - \frac{\sigma_{\text{el}}(pp)}{\pi \cdot 10 \text{ mb}}^{1/2} \rightarrow 0.603.
\]

2 **The proton nucleus inelastic cross section**

For atomic mass \(A > 9.5\) we find from accelerator measurements [2] that the inelastic proton nucleus cross section can be expressed by

\[
\sigma_{\text{inel}}(pA) = \pi \left\{1.193A^{1/3} - 0.045 + \Delta_{\text{step}} + (A^{1/3} + 1)\Delta_{11}\right\}^2 \cdot 10 \text{ mb} \tag{1a}
\]

and for \(A > 9.5\) we have according to ref. [1] from accelerator experiments, after adding a term \(\Delta_C\)

\[
\sigma_{\text{inel}}(pA) = \pi \left\{1.313A^{1/3} - 0.299 + \Delta_{\text{step}} + (A^{1/3} + 1)\Delta_{11} + \Delta_C\right\}^2 10 \text{ mb} \tag{1b}
\]

where \(\Delta_C = -0.03\) for carbon \((A = 12.011)\) and \(\Delta_C = 0\) for other nuclei. We have \(\Delta_{\text{step}} = 0.166\) generally for Serpukhov measurements, and \(\Delta_{\text{step}} = 0\) for Fermilab measurements. \(\Delta_{11}\) takes into account elastic scattering into the nucleus with successive inelastic collision, along with the energy dependence. Therefore, \(A = 1\), in eqs.(1) above does not give the inelastic pp cross section. \(\Delta_{11}\) is obtained from \(\sigma_{\text{inel}}(pp)\) and \(\sigma_{\text{el}}(pp)\) as given in ref. [1]. The normalization is such that \(\Delta_{11} = 0\) at about 200 GeV lab energy. That is:

\[
\sigma_{\text{inel}}(11) = \sigma_{\text{inel}}(pp) + \sigma_{\text{el}}(pp) - \sigma_{0}^\text{el} \sigma_{\text{inel}}(pp)/\sigma_{\text{tot}}(pp) \quad \text{with} \quad \sigma_{0}^\text{el} = 6.276 \text{ mb}, \quad \sigma_{\text{tot}} = \sigma_{\text{inel}} + \sigma_{\text{el}} \quad \text{and} \quad \Delta_{11} = 1/2\left((\sigma_{\text{inel}}(11)/(\pi \cdot 10 \text{ mb}))^{1/2} - 1.014\right).
\]

Numerical values for \(\sigma_{\text{inel}}(pp)\) and \(\sigma_{\text{el}}(pp)\) are given in ref.[1]. Eqs.(1) do not apply to energies near threshold for particle production in pp collisions, partly because the effect of Fermi motion is not included. At laboratory momenta as low as \(p = 1.8 \text{ GeV/c}\) and \(p = 1.52 \text{ GeV/c}\) [3] the agreement with measured inelastic pC, pAl, pTi and pCd cross sections is satisfactory.

3 **The antiproton nucleus inelastic cross section**

In addition to particle production in the same way as in \(\text{pp}\)-collisions, \(\bar{p}p\)-collisions ( \(\bar{p}\) is antiproton) also gives \(\bar{p}p\) annihilation. For the annihilation cross section in \(\bar{p}p\) collisions we obtain, by
comparison with the measured annihilation cross sections [2]:

$$\sigma^{\text{annihilation}}(\overline{p}p) = 36.47 \{ \ln(p + 0.5 \text{ GeV/c}) \}^{-1.148} \text{ mb}$$  \hspace{1cm} (2)

We use the measured antiproton-nucleus inelastic cross sections from $p = 1.8 \text{ GeV/c}$ up to $280 \text{ GeV/c}$ [2, 3] to find an expression for the antiproton inelastic cross section and extrapolate to higher energies:

$$\sigma^{\text{inel}}(\overline{p}A) = \{(\sigma^{\text{inel}}(\overline{p}p) - \sigma^{\text{annihilation}}(\overline{p}p))/\sigma^{\text{inel}}(pp)\} \sigma^{\text{inel}}(pA) + 0.904A^{2/3} \sigma^{\text{annihilation}}(\overline{p}p)$$  \hspace{1cm} (3)

$p$ is the lab momentum in GeV/c and $A$ the atomic mass.

This implies $(\sigma^{\text{inel}}(\overline{p}A) - \sigma^{\text{inel}}(pA)) \rightarrow 0$ for high energies. Some model, however, do not support $\sigma^{\text{tot}}(\overline{p}p) \geq \sigma^{\text{tot}}(pp)$ and $(\sigma^{\text{tot}}(\overline{p}p) - \sigma^{\text{tot}}(pp)) \rightarrow 0$ for high energies [4].

4 The $\pi$-meson nucleus inelastic cross section

We use the relation between $\sigma^{\text{inel}}(pA)$ and $\sigma^{\text{inel}}(\pi^+ A)$ given in ref. [1] at 200 GeV and introduce

$$\sigma^{\text{inel}}(\pi,11) = \sigma^{\text{inel}}(\pi p) + \{\sigma^{\text{el}}(\pi p) - \sigma^{\text{el}}(\pi A)\} \sigma^{\text{inel}}(\pi p)/\sigma^{\text{el}}(\pi A)$$

with $\sigma^{\text{el}}(\pi A) = 1/2\sigma^{\text{el}}(\pi p)$ and $\sigma^{\text{inel}}(\pi p) = 1/2\{\sigma^{\text{inel}}(\pi p) + \sigma^{\text{inel}}(\pi p)\}$ and $\sigma^{\text{el}}(\pi p) = 1/2\{\sigma^{\text{el}}(\pi p) + \sigma^{\text{el}}(\pi p)\}$ and $\Delta_{\pi,11} = 1/2\{\sigma^{\text{inel}}(\pi,11)/(\pi 10 mb)\}^{1/2} - 0.814$ and introduce, taking $\sigma^{\text{inel}}(pA) = \pi(1.313A^{1/3} - 0.299)^2 10 \text{ mb}$ as at 200 GeV in ref.[1], for the inelastic $\pi A$ cross section

$$\sigma^{\text{inel}}(\pi A) = \pi(1.313A^{1/3} - 0.071(A-1)^{1/3} - 0.299 - 0.200 + \Delta_{\pi,\text{step}}$$

$$+ [0.6 + 1.4(A^{1/3} - (0.071/1.313)(A-1)^{1/3})\Delta_{\pi,11}]A^{2/3} 10 \text{ mb}$$  \hspace{1cm} (4)

with $\Delta_{\pi,\text{step}} = 1/2\Delta_{\pi,\text{step}}$. This expression for $\sigma^{\text{inel}}(\pi A)$ agrees well with the measured inelastic cross sections from lab momentum $p = 2 \text{ GeV/c}$ [5] up to $280 \text{ GeV/c}$ [2] and can be compared with cosmic ray results at $800 \text{ GeV}$ [6]: $\sigma^{\text{inel}}(\pi C) = 216 \pm 45 \text{ mb}$, $\sigma^{\text{inel}}(\pi Fe) = 723 \pm 105 \text{ mb}$ and $\sigma^{\text{inel}}(\pi Pb) = 1728 \pm 140 \text{ mb}$. Our result gives $\sigma^{\text{inel}}(\pi C) = 210 \text{ mb}$, $\sigma^{\text{inel}}(\pi Fe) = 669 \text{ mb}$ and $\sigma^{\text{inel}}(\pi Pb) = 1717 \text{ mb}$. Beyond the energies of accelerator experiments, we have introduced the asymptotic relation:

$$(\sigma^{\text{inel}}(pp))^{1/2} - (1.3101 \text{ mb})^{1/2}$$

and $$(\sigma^{\text{el}}(pp))^{1/2} - (0.719 \text{ mb})^{1/2}$$

rather than that the ratios $\sigma^{\text{inel}}(pp)/\sigma^{\text{inel}}(pp)$ and $\sigma^{\text{el}}(pp)/\sigma^{\text{el}}(pp)$ should reach constant values at high energies.

5 The charged kaon nucleus inelastic cross sections

Here we use similar relations as for pions:

$$\sigma^{\text{inel}}(K^+ A) = \sigma^{\text{inel}}(K^+ p) + (\sigma^{\text{el}}(K^+ p) - 1/3\sigma^{\text{el}}(K^+ p)\sigma^{\text{tot}}(K^+ p))\Delta_{K^+,\text{step}} = 1/2\Delta_{K^+,\text{step}}$$

$$\Delta(K^+,11) = 1/2((\sigma^{\text{inel}}(K+1,11)/(\pi 10 mb))^{1/2} - 0.744)$$

$$\sigma^{\text{inel}}(K^+ A) = \pi(1.313A^{1/3} - 0.096(A-1)^{1/3} - 0.299 - 0.250 + 1/2\Delta_{K^+,\text{step}}$$

$$+ [0.6 + 1.4(A^{1/3} - (0.096/1.313)(A-1)^{1/3})\Delta(K^+,11)]A^{2/3} 10 \text{ mb}$$  \hspace{1cm} (5)

$$\sigma^{\text{inel}}(K^- A) = \pi(1.313A^{1/3} - 0.096(A-1)^{1/3} - 0.299 - 0.250 + 1/2\Delta_{K^+,\text{step}}$$

$$+ [0.6 + 1.4(A^{1/3} - (0.096/1.313)(A-1)^{1/3})\Delta(K^-,11)]A^{2/3} 10 \text{ mb}$$  \hspace{1cm} (6)

$\Delta(K^+,11) = 0$ at about $200 \text{ GeV}$. From $1.8 \text{ GeV/c}$ lab momentum and up to $280 \text{ GeV/c}$ there is good agreement between our calculated inelastic $K^\pm$ nucleus cross sections and measured inelastic cross sections [2, 3].
6 Discussion
We can now calculate the inelastic \( p \), \( p^- \), \( \pi^- \) and \( K^\pm \) air nucleus cross sections. Up to about \( 10^5 \) GeV the calculated \( \sigma^{\text{inel}}(pA_{\text{air}}) \) is about the same as given by other models, see J. Knapp [7] for a comparison with other models. Above \( 10^5 \) GeV our result for \( \sigma^{\text{inel}}(pA_{\text{air}}) \) is higher than predicted by other models [7], but in agreement with cosmic ray experimental results. The proton proton inelastic cross section is also higher in this model, than what are predicted by most other models above \( 10^5 \) GeV. The results from this model are given in mb in the table below for air. The atomic mass for air is: \( A_{\text{air}} = 14.50 \)

<table>
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<th>( p/(\text{GeV/c}) )</th>
<th>( \sigma^{\text{inel}}(pp) )</th>
<th>( \sigma^{\text{inel}}(pA_{\text{air}}) )</th>
<th>( \sigma^{\text{inel}}(pA_{\text{air}}) )</th>
<th>( \sigma^{\text{inel}}(\pi A_{\text{air}}) )</th>
<th>( \sigma^{\text{inel}}(K^- A_{\text{air}}) )</th>
<th>( \sigma^{\text{inel}}(K^+ A_{\text{air}}) )</th>
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References
2. Carroll, A. S. et al., 1979, Phys. Lett. 82B, 319
6. Avakian, V. V. et al., 1990, XXI ICRC 8, 145