Inelasticity Distribution of Hadron-Pb Collisions in the Energy Region exceeding $10^{14}$ eV
by Pamir Thick Lead Emulsion Chambers (Revised)

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1 Introduction:

The concept of the inelasticity is used widely in discussing the features of multiple particle production. In the energy region exceeding $10^{14}$ eV, however, there is no data of the inelasticity of hadron—hadron collisions by direct observation, because available high energy accelerators are all of collider-type, by which the surviving hadron and/or the particles produced in forward region are not easy to be observed. The data of hadron—nucleus collisions are available only at low energy region.

Hence, in the present paper, we try to estimate the inelasticity in the energy region exceeding $10^{14}$ eV by the data of cosmic-ray experiment. The data are from hadron-Pb collisions which are observed by thick emulsion chambers, exposed to cosmic rays at the Pamirs. The basic idea is that a hadron, incident upon the chamber, makes collisions successively in the chamber, and that the showers due to these collisions are detected individually. Hence one can estimate the energies of the individual collisions and the total energy sum of them which corresponds to the energy of the incident hadron.

1.1 Pamir thick lead chambers Pamir thick lead chambers are constructed at the Pamir Station (4,300 m, Tadjikistan) by Pamir Collaboration.$^{[1,2]}$ They have distinguished characteristics of large thickness (60 cm Pb or 3.2 mean free path of inelastic collision of nucleon) and uniform structure. (See Fig. 1 for the structure.) The former assures almost 100% collision probability of hadrons in the chamber, and consequently $\gamma$-rays$^1$ and hadrons, incident upon the chamber, are detected in the minimum-biased way. The latter permits a uniform detection of showers all over the depth of the chamber and the simplest way of energy determination.

1.2 Hadrons in Pamir thick lead chambers — Successive interactions The hadron, incident upon the chamber, causes a nuclear collision — multiple particle production — in the chamber. The surviving hadron and the produced hadrons undergo nuclear collisions again at various depths in the chamber, too. (See Fig. 2.) The process is repeated in the chamber until the hadrons, incident and produced, leave the chamber. Among the showers, produced by these collisions in the chamber, those which have the energies exceeding the threshold energy $E_{th}$ are detected by the emulsion chamber.

Consequently a single high energy hadron, incident upon the chamber, produces in the chamber $n$ showers ($n = 1, 2, \cdots$) which have appearance of aligning longitudinally and having the same direction. The event of $n \geq 2$ is called 'successive interactions', while the one of $n = 1$ is called 'single-shower'.

$^1$The electromagnetic component (electron and photon) are collectively called 'gamma-rays' in emulsion chamber experiment.
1.3 Distribution of $z$ by the experimental data Let us denote the energy of the first, the second, \ldots, shower in the event by $E_1^{(γ)}$, $E_2^{(γ)}$, \ldots, and that of the incident hadron by $E_0$ (Fig. 2). The parameter $z$ is defined as $z = E_1^{(γ)} / \sum E_i^{(γ)}$, which is a measure to estimate the inelasticity of the interaction of the first shower.

The distribution of $z$ is shown in Fig. 3 from 66 events which have $\sum E_i > 30$ TeV with $E_{th} = 4$ TeV. Among 66 events, 13 are single-shower events and the rest of 53 events are those of successive interactions.\(^2\) A set of 66 events consists of 32 hadrons in the families and 34 of single arrival.

Due to limited statistics of events, the histogram has large bins and detailed discussion on the inelasticity distribution is left for the future exposure of large scale.

1.4 Inelasticity $z$ distribution is calculated analytically in the following way. The energy spectrum of the produced charged particles in multiple particle production is assumed to be

$$
\varphi(E_0, E) dE = \left[ \frac{10}{3} \xi_1 \frac{(1 - x)^4}{x} + \frac{2}{3}(1 - \xi_1) \xi_2 \delta(x - \epsilon) \right] dx
$$

where $x = E/E_0$ and $\delta(x)$ is the delta-function. The two terms on the right-hand side describe the produced particles in the forward region and those in the central region, respectively, which correspond to the experimental observation that the energy spectrum of the produced particles in hadron-nucleus collision can be approximated by that of the hadron-hadron collision in the forward region with enhancement in the central region.

By integrating $\varphi(x)$, one obtains the total inelasticity $K$,

$$
K = \frac{3}{2} \int_0^1 x \varphi(x) dx = \xi_1 + (1 - \xi_1) \xi_2
$$

which shows that $\xi_1$ and $\xi_2$ are related to the inelasticity. That is, $\xi_1$ is the fraction of incident energy which is used to produce the particles in the forward region and $(1 - \xi_1)\xi_2$ is the fraction of the rest of the energy which is used to produced particles in the central region.

We assume that the parameters $\xi_1$ and $\xi_2$ are distributed;

$$
\varphi_1(\xi_1) d\xi_1 = N \left[ \alpha (1 - \xi_1)^{m_1-2} + \beta \xi_1^{m_2-2} \right]
$$

$$
\varphi_2(\xi_2) d\xi_2 = \frac{1}{2\delta} \theta(2\delta - \xi_2) d\xi_2
$$

\(^2\)In our previous report,[6] we made an analysis with 74 events of successive interactions. We found, however, that classification between single-shower and two-shower cannot be made easily because in most of the cases the second shower has low energy near the detection threshold. Hence, we make re-analysis in the present report including single-shower events.

The decrease of event number of successive interactions from 74 to 53 is due to the revised energy calibration.
where $\theta(x)$ is the step function. The parameters $\alpha, \beta, m_1, m_2$ and $\delta$ are adjusted to reproduce the $z$-distribution by the experiment.

1.5 $z$ distribution The nucleon of energy $E_0$, incident upon the chamber, makes the first interaction with inelasticity $K(\xi_1, \xi_2)$. Hence, after the collision, the surviving nucleon has the energy $(1 - K)E_0$ (with $K = \xi_1 + (1 - \xi_1)\xi_2$), and the charged pions, produced by the collision, have the energy spectrum $\varphi(E_0, E)dE$. The surviving nucleon and the charged pions undergo the nuclear cascade process. The total observed energy of the showers, produced by the process, is calculated in analytic way by solving the diffusion equation of the process, because a considerable number of particles are involved in the process which reduces the fluctuation reasonably. Hence one can obtain the $z$ distribution for the assumed $f_1(\xi_1)$ and $f_2(\xi_2)$ (or the inelasticity $K$). And we compare the distribution with the experimental data.

Fig. 1 presents the analytical calculation of $z$-distribution for $m_1 = 2.0, \ m_2 = 2.0, \ \alpha = 1.0, \ \beta = 1.0$ and $\delta = 0.0$ which is the case of the uniform distribution of the inelasticity, i.e. $K = 0.5$.

2 Summary and discussions

(i) We made an estimate of the inelasticity distribution of hadron-Pb collisions, using 66 events induced by cosmic-ray hadrons. Those are observed by Pamir thick lead chambers and have the total observed energies exceeding 30 TeV. It corresponds to $E_0 > 100$ TeV and $\langle E_0 \rangle = 2.3 \times 10^{12}$ TeV for the hadrons, incident upon the chamber, based on the energy spectrum of total observed energy (with the exponent of $-1.8$ in integral form) and the obtained value of inelasticity.

The average value of inelasticity is $\langle K \rangle = 0.60^{+0.02}_{-0.05}$ and the distribution of the best-fitting is

$$g(K)dK = \left[ \alpha(1 - K)^{m_1-1} + \beta K^{m_2-1} \right] dK$$

with $m_1 = 0.5, \ m_2 = 1.125, \ \alpha = 0.26, \ \beta = 0.55$, which is shown in Fig. 4.

The way of estimation is free from the absolute value of the shower energies.

(ii) The average value $\langle K \rangle = 0.60^{+0.02}_{-0.05}$ of hadron-Pb collision at $E_0 = 2.3 \times 10^{14}$ eV is similar to 0.63 (at $\sqrt{s} = 6.8 \times 10^2$ GeV) by Hama and Paiva [3], but smaller than 0.82 (at $E_0 = 100$ TeV) by Tamada[4]. The former calculation is made on the basis of the interacting gluon model, and the latter on the basis of the geometrical model for hadron–nucleus interactions and UA5 simulation code for particle production. The average inelasticity $\langle K \rangle$ in $p$-Pb collisions at $E = 100$ TeV is around 0.75 both in VENUS and QGSJET simulation codes.[5]

(iii) The distribution of the inelasticity is presented in Fig. 4. The inelasticity distributions of $m_1 = 0.5$ (the best fitting), 1.0, 1.5, 2.0 in the case of $\langle K \rangle = 0.6$ (with $K_1 = 0.6$ and $K_2 = 0$) have the value of $D$ within allowed region ($D < 0.01$), and they are similar another. The distribution, obtained by Hama and Paiva, is similar to these, too. One by Tamada is different reasonably because of the difference of the average inelasticity.

(iv) In the present analysis the inelasticity is assumed to consist of two parts, i.e. $\langle K \rangle = K_1 + K_2$. It corresponds to the experimental data of hadron–nucleus collision that the energy spectrum of produced particles is described by that of $p - p$ collision with enhancement in the central region. Present analysis shows that $K_2 < 0.07$ belongs to the allowed region keeping $\langle K \rangle = 0.60$. It follows that $K_1 = 0.53 \sim 0.6$, which is an approximate estimate of the inelasticity of hadron-nucleon collision for non-single-diffractive events.

References

Fig.3 Histogram of $z$, where $z$ is the ratio of the energy of the first shower to the energy sum of all the showers. (See the text.) The bold solid line is that of the experimental data (Errors are due to the number of events only.) and the chain line that of the best fitting, based on the assumed inelasticity distribution. The thin solid line is that for the uniform distribution of the inelasticity.

Fig.4 Inelasticity distribution of the best fitting (the thick solid line), where $m_1 = 0.5$, $K_1 = 0.6$, $K_2 = 0.0$. The thin solid lines ($a$, $b$, and $c$) corresponds to the cases of $m_1 = 1.0$, 1.5 and 2.0 (with $K_1 = 0.60$ and $K_2 = 0.0$), respectively. The chain line ($d$) corresponds to the case of $m_1 = 0.5$ with $K_1 = 0.55$ and $K_2 = 0.05$ (consequently $< K > = 0.6$). The dotted lines ($e$ and $f$) are the calculations, based on the theoretical models by Hama and Paiva and by Tamada, respectively.