Signatures of the Propagation of Primary and Secondary
Cosmic Ray Electrons and Positrons in the Galaxy

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Abstract

In this thesis, some of the consequences of the acceleration and production, and propagation, of high energy electrons and positrons in the Galaxy are examined. In particular, predictions are made of the diffuse photon signals arising from the interactions of electrons and positrons with gas, low energy photons, and the galactic magnetic field during their transport in the Galaxy.

Cosmic ray propagation is discussed in terms of a transport equation similar to that originally suggested by Ginzburg & Syrovatskii (1964). For realistic models of the Galaxy, it is often difficult to obtain analytical solutions for these propagation equations, and numerical methods must be employed. Chapter 2 introduces such a method based on Monte Carlo simulation techniques, and the accuracy and reliability of this method is shown by considering some simple cases of the transport equation.

The sources of cosmic ray electrons and positrons in the Galaxy are then discussed in Chapter 3. The primary and secondary source components are identified with particle acceleration in supernova remnants, and particle production in the interstellar medium respectively. For the primary component, particles are accelerated to cosmic ray energies through shock acceleration, and this is discussed in the context of an infinite planar shock with a parallel magnetic field. A simple model of a cosmic ray accelerator (the so called ‘leaky box’ model) is then used to show how simple, yet realistic, constraints on the geometry of the accelerator result in a natural cut-off in the spectrum of particles escaping from the accelerator. The source spectra and spatial distribution of primary sources used in this thesis is then constructed. For the secondary component, the production of positrons and electrons in inelastic collisions between cosmic ray nuclei and gas in the interstellar medium is described, and a comparison is made between the present calculations of the production spectrum and those of other authors. As for the primary component, the source spectra and spatial distribution of these secondary particles is then constructed.

The propagation models used in this thesis are based on the diffusion-convection model introduced in Chapter 2. In Chapter 4, I discuss why convection can be excluded as playing a significant role in the transport of cosmic rays in the Galaxy. Then, using a simple one-dimensional form of the diffusion model, and cosmic ray nuclear abundance data, I constrain the parameters of the propagation model, such as the diffusion coefficient and halo
size. These parameters are then used in a three-dimensional propagation calculation for cosmic ray electrons and positrons in Chapter 5. The distributions of electrons and positrons resulting from the propagation calculations are used to calculate the diffuse photon signals arising from interactions with the galactic magnetic field, matter distribution, and low energy photon populations; photons produced through hadronic processes in the interstellar medium are also considered, but in a parametric way. Comparisons are then made between the model predictions and non-thermal radio data, and diffuse γ-ray observations from keV to TeV energies and higher, and the results are discussed in terms of recent models for shock acceleration in supernova remnants.
Statement of Originality

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

Troy Porter
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‘To quote a lyric from a song I feel is appropriate: ‘I’m not in it for love, I’m outta here’.
Chapter 1

High Energy Astrophysics - An Overview

1.1 Introduction

High energy astrophysics encompasses several different fields of physics, including cosmic ray astrophysics, and γ-ray astrophysics. Of these, cosmic ray astrophysics is the most mature, having almost a ninety year history of experimental and theoretical advances that first began with a series of balloon flights made by Hess (Hess 1912, 1913). Gamma-ray astrophysics has a shorter history, but progress accelerated in the 1970s and 1980s with the launch of the SAS-2 and COS-B satellites (Fichtel et al. 1975; Mayer-Hasselwander et al. 1980, 1982). More recently, the launch of the Compton Gamma-Ray Observatory (CGRO) in 1991 has provided unprecedented coverage of the γ-ray sky. Together with the use of Cherenkov telescopes, and air shower arrays, the study of astrophysics of γ-rays is rapidly establishing itself as a mature discipline in its own right.

The connection between cosmic rays and diffuse γ-ray production in the Galaxy has been established for more than 30 years. Over this time, there has been steady theoretical progress. Initially, the majority of the diffuse γ-rays emanating from the Galaxy was thought to be due to the interaction of cosmic ray nuclei, and bremsstrahlung of cosmic ray electrons, with gas atoms in the interstellar medium (ISM) (e.g. Stecker 1971). Gradually, inverse Compton (IC) scattering of cosmic ray electrons on the ambient galactic photon fields due to emission by stars, and other processes, was also incorporated into the view of diffuse γ-ray production in the Galaxy. However, the contribution by the IC process was believed to be relatively
minor, with the majority of the \( \gamma \)-ray production being due to cosmic ray-matter interactions in the Galaxy. This view prevailed until relatively recently. However, the interpretation of the results of the COMPTEL (Schönfelder et al. 1993) and EGRET (Thompson et al. 1993) instruments on the CGRO has caused this picture to change. It is currently believed that the majority of diffuse \( \gamma \)-rays emitted by the Galaxy is produced through interactions of cosmic ray nuclei with gas in the ISM, and IC scattering (e.g. Porter & Protheroe 1997; Pohl & Esposito 1998; Strong, Moskalenko & Reimer 1998). In fact, the IC process is now considered to account for most of the diffuse \( \gamma \)-ray spectrum below about 100 MeV, and above 5 GeV.

In this thesis, some of the consequences of the acceleration, production, and propagation of cosmic ray electrons and positrons in the Galaxy are discussed. In particular, I discuss the relation to the production of diffuse photon fluxes through interactions with the galactic matter distribution, ambient photon populations, and magnetic field that can be directly compared with observational data.

The treatment of cosmic ray transport in the Galaxy requires the adoption of a propagation model, and a method for solving the propagation equations to obtain cosmic ray distributions throughout the Galaxy. In Chapter 2, the propagation model used in this thesis is given, and a particular method for solving the model using Monte Carlo simulation techniques is described. This method is particularly efficient for treating the propagation of cosmic rays that possess different source distributions, but are the same type from the point of view of the interactions they can undergo while propagating (e.g. electrons and positrons), since the Green’s function for the partial differential equation used to describe the cosmic ray transport is directly obtained.

The acceleration and production of cosmic ray electrons and positrons in the Galaxy are discussed in Chapter 3. Electrons are considered to be directly accelerated at SNR shock fronts, as well as being produced in inelastic collisions between cosmic ray nuclei and gas atoms at rest in the ISM. First order Fermi acceleration at a plane shock, along with a simple leaky box model for a cosmic ray accelerator is also discussed in Chapter 3 in relation to the acceleration of electrons in SNRs. Positrons, on the other hand, are considered to be purely secondary particles, produced in the same hadronic interactions in the interstellar medium that give rise to a fraction of the cosmic ray electron flux. The production of secondary positrons and electrons through these processes in the Galaxy is also described in Chapter 3.

Observations of cosmic ray nuclei incident at the top of the Earth’s atmosphere, and
in interplanetary space, can be used to deduce information about their propagation and confinement in the Galaxy. In Chapter 4, recent cosmic ray abundance data are used to constrain the parameters of the propagation model used to describe the electron and positron transport in Chapter 5.

The propagation of electrons and positrons in the Galaxy is considered in detail in Chapter 5. Propagation calculations are performed to calculate the distribution of electrons and positrons accelerated or produced in the Galaxy as described above. Then, constraints on the source spectrum of electrons and injection in SNRs are found using observations of the galactic non-thermal emission, and the galactic diffuse $\gamma$-ray spectrum. The implications of the predictions made using the modelling of the diffuse $\gamma$-ray spectrum for recent models of cosmic ray acceleration in SNRs, and the location of the sources of galactic cosmic rays with energies up to $\sim 10^6$ GeV are also discussed.

In the rest of this Chapter, a brief overview of cosmic ray and $\gamma$-ray astrophysics is given. Particular emphasis is placed on the diffuse galactic $\gamma$-ray spectrum, and what information it may be able to provide concerning the sources of cosmic rays in the Galaxy.

### 1.2 Cosmic Ray Astrophysics

Cosmic ray nuclei from outside the solar system arrive at Earth with energies from several hundred MeV/particle to $\sim 3 \times 10^{20}$ eV. At GeV energies, the spectrum is described by a power-law in energy close to $E^{-2.75}$, and at higher energies closer to $E^{-2.65}$ up to the ‘knee’, whereupon it steepens to about $E^{-3.1}$. It then flattens out again near the ‘ankle’. At low energies, the cosmic ray spectrum is cut-off due to transport effects in the heliosphere. The spectrum of cosmic rays observed at Earth, displaying these features, is shown in Figure 1.1.

The origin of the cosmic ray spectrum is still not completely understood. For energies below the ‘knee’, it is generally believed cosmic rays are predominantly due to the explosion of normal stars into the interstellar medium (ISM) (Lagage & Cesarsky 1983), and the explosion of massive stars into their former stellar winds (Völk & Biermann 1988). For energies intermediate between the ‘knee’ and the ‘ankle’ there is no real consensus: Jokipii & Morfill (1987) have argued that a galactic wind termination shock might be able to provide these particles, while other authors (e.g. Protheroe & Szabo 1992) propose an extragalactic origin. A recent proposal that claims to account for the particle energies, spectra, and chemical composition in a quantitative manner up to energies $\sim 3 \times 10^{18}$ eV has been
Figure 1.1: Compilation of measurements of the differential energy spectrum of cosmic rays (The Auger Collaboration, 1997). The dotted line shows an $E^{-3.0}$ power-law for comparison. Approximate integral fluxes (per steradian) are also shown.

proposed by Biermann and coauthors (Biermann 1997, and references therein). For energies higher than the ‘ankle’ an extragalactic origin is required because of the extremely large gyroradii of such particles.

In this thesis, I am interested in cosmic ray acceleration and production in the Galaxy. This directly pertains to the range $\sim 1$ MeV up to $\sim 10^6$ GeV of energies covered by the cosmic ray spectrum. As noted above, cosmic ray acceleration in this energy range is believed to be mainly due to stellar explosions, and the remnants of these explosions. The rest of this Section will consider the current observational evidence for cosmic ray acceleration in such objects.
1.2.1 Evidence for Cosmic Ray Acceleration in Supernova Remnants

Although it has generally been accepted since the late 1960s that supernova remnants (SNRs) are the most likely sources of the majority of galactic cosmic rays, no direct evidence existed until relatively recently for cosmic ray acceleration up to the energies required to explain the observed spectrum below the ‘knee’. While SNRs have been considered sources of relativistic electrons with energies up to a few tens of GeV for quite some time (e.g. Green 1995), the recent X-ray detections of several SNRs have provided evidence for electron acceleration up to TeV energies and beyond. The first such detection was made by ASCA of the SNR SN 1006 (G327.6 + 14.6) (Koyama et al. 1995). The spectral analysis of the emission from this object indicates the presence of a power-law continuum, which is attributed to non-thermal emission by electrons shock accelerated to \( \sim 100 \) TeV energies (Reynolds 1996; Mastichiadis 1996). Other detections of the SNRs IC 443 by ASCA and ROSAT (Keohane et al. 1997), RX J1713.7-3946 by ASCA (Koyama et al. 1997), and Cassiopeia A (Cas-A) by RXTE (Allen et al. 1997), have provided further evidence for electron acceleration up to energies \( \sim 10 - 40 \) TeV. However, there are different interpretations of these results (e.g. IC 443, Gaisser, Protheroe & Stanev 1997), so the detection of these remnants may not be incontrovertible evidence for the acceleration of high energy electrons.

It was noted by Pohl (1996), and Mastichiadis & de Jaeger (1996) that electrons of these energies would also interact via IC scattering with the cosmic microwave background radiation (CMBR), and other ambient photon fields around SN 1006, and produce TeV energy \( \gamma \)-rays. This appears to have been confirmed by recent observations of SN 1006 by the CANGAROO telescope (Tanimori et al. 1998). So, despite some of the uncertainty associated with other SNRs it appears that SN 1006 is indeed an object in which shock accelerated electrons attain energies approaching those just below the knee in the cosmic ray spectrum.

Despite the evidence for electron acceleration, no real indication that shock acceleration of cosmic ray nuclei takes place in SNRs presently exists. The EGRET instrument on the CGRO has detected \( \gamma \)-rays signals above 100 MeV from at least two SNRs – IC 443 and \( \gamma \)-Cygni (Exposito et al. 1996). The interpretation of these results is controversial (Allen et al. 1995; de Jaeger & Mastichiadis 1997; Gaisser, Protheroe & Stanev 1997; Sturmer et al. 1997) but may be due to \( \gamma \)-rays resulting from decaying neutral pions produced in interactions between shock accelerated cosmic ray nuclei and gas associated with the remnants. If this
is the case, then the first direct evidence for the shock acceleration of cosmic ray nuclei in such objects exists.

1.3 γ-ray Astrophysics

Unlike cosmic rays, γ-rays are not deflected by the galactic magnetic field. Therefore, they travel in straight lines from their places of origin, and may be identified with specific sources, or processes generating γ-radiation in the Galaxy. As noted in Section 1.2.1, γ-rays have been used to ascertain that shock acceleration of cosmic ray electrons, and maybe nuclei, occurs in SNRs. Gamma-ray production is also associated with diffuse process in the ISM. After their production, or initial acceleration, cosmic rays propagate in the Galaxy, losing energy through interactions with the gas distribution, ambient photon fields, and galactic magnetic field. This energy is channelled into the production of pions, kaons, and other particles, and high energy X-/γ-rays, the spatial distribution of which can be associated with the distributions of the cosmic rays, and the gas, etc., producing them.

The dominant feature of the high-energy γ-ray sky is the narrow band of intense emission along the galactic plane. This emission was first seen by the OSO-3 satellite (Kraushaar et al. 1972), and with a high-altitude balloon experiment (Fichtel et al. 1972). The SAS-2 and COS-B satellites (Fichtel et al. 1975; Mayer-Hasselwander 1980,1982) provided the first detailed views of the diffuse emission from the galactic plane. More recently, the OSSE (Johnson et al. 1993), COMPTEL (Schönfelder et al. 1993), and EGRET (Thompson et al. 1993) experiments on the CGRO have provided the most detailed picture to date of the γ-ray sky covering six decades in energy from 50 keV to 50 GeV. Additional coverage of the diffuse galactic emission, extending the observed spectrum into the X-ray, and TeV γ-ray regimes is provided by other satellite experiments (e.g. Ginga, Yamasaki et al. 1997 and references therein), and Cherenkov telescopes such as Whipple (Cawley et al. 1990), and air shower arrays.

Despite much effort, the origin of the diffuse galactic γ-ray continuum radiation is still subject to considerable uncertainties. While the main diffuse γ-ray production processes are considered to be synchrotron radiation, bremsstrahlung, IC scattering of cosmic ray electrons, and neutral pion decay production by cosmic ray nuclei, their individual contributions and relative importance in various energy bands are still the subject of some debate. Until recently, the general consensus was the low-energy emission is generated through interactions
of cosmic ray electrons with gas and low energy photons, and by an unresolvable point source component. At high energies, the prevailing view was the emission is generated by cosmic ray nuclei interacting with gas in the ISM. However, recent modelling of the galactic diffuse emission (e.g. Porter & Protheroe 1997; Pohl & Esposito 1998; Strong, Moskalenko & Reimer 1998), has indicated that the importance of the IC process may have been previously underestimated.

Because the Galaxy is essentially transparent to $\gamma$-rays up to energies $\sim 10^{5}$ GeV, and the distributions of gas, photon fields, and magnetic field, and photon production processes are reasonably well known, quantitative calculations of the diffuse $\gamma$-ray emission of the Galaxy can be made if certain assumptions about the sources and propagation of cosmic rays are made. Several recent models for the galactic diffuse $\gamma$-ray emission have been constructed (Bertsch et al. 1993; Hunter et al. 1997; Strong et al. 1996; Strong, Moskalenko & Reimer 1998). The models of Bertsch et al. (1993), and Hunter et al. (1997) assume coupling between cosmic rays and the galactic gas distribution over kpc scales, and are used as the galactic background model by the EGRET team. The model of Strong, Moskalenko & Reimer (1998) is based on a self-consistent approach to the problem of cosmic ray propagation in the Galaxy: cosmic ray nuclei, electrons and positrons, and the resulting $\gamma$-rays and synchrotron radiation through interactions with gas, photon fields and magnetic fields in the Galaxy are included.

In Chapter 5, an approach to the problem of the propagation of cosmic ray electrons and positrons in the Galaxy, and the generation of diffuse photon fluxes from the interactions of these particles during propagation, is described. The approach described is in some respects similar to that of Strong, Moskalenko & Reimer (1998), but differs in that the emphasis is placed on what information on the distribution of galactic electron and positron sources can be inferred from predictions of the diffuse $\gamma$-ray spectrum from keV to TeV energies and higher. Models for electrons accelerated in SNRs, and electrons and positrons produced in inelastic collisions between cosmic ray nuclei and gas in the ISM, are considered, and predictions are made for the resulting galactic electron and positron distributions after propagation. These distributions are then used to calculate diffuse fluxes of photons for the various production processes. Initially, possible source models for electrons accelerated in SNRs are found using measurements of the galactic non-thermal radio emission, since this a constraint that must satisfied by any plausible source model. Then, the source models so constrained are used to make predictions of the diffuse X-/$\gamma$-ray spectrum from keV to
TeV energies and beyond, and compared with relevant observations by satellites, Cherenkov telescopes, and air shower arrays. The conclusions drawn from such a comparison are then contrasted with recent models of shock acceleration in SNRs.
Chapter 2

Propagation Models and Numerical Methods

2.1 Introduction

The high degree of isotropy and relatively large number of secondary cosmic ray nuclei, when compared with the solar system abundance of such particles, indicate long propagation times and effective mixing for cosmic rays during their confinement in the Galaxy. Because cosmic ray nuclei, electrons and positrons are charged particles, a significant role is played by the galactic magnetic field in determining the nature of their propagation. However, the details of the magnetic field and the physical mechanism responsible for controlling the motion of the cosmic rays are uncertain (but see e.g. Berezinskii et al. 1990 for several possible mechanisms). In the absence of any definitive theory giving the exact details of the cosmic ray propagation, approximate semi-empirical models are used. These models allow experimental data to be used in conjunction with parameters of the propagation models to investigate the nature of the transport of cosmic rays in the Galaxy.

In this Chapter the equations used in this thesis to approximate the propagation of cosmic ray nuclei, and electrons and positrons, are discussed. These equations use the diffusion approximation (e.g. Ginzburg & Syrovatskii 1964), where the cosmic rays can be envisaged to scatter off magnetic turbulence in the ISM, to model the propagation of the cosmic rays, and other terms are included to take into account the various energy loss (and gain) mechanisms nuclei, and electrons and positrons, suffer while propagating (these depend on many details, including the spatial dependence of the galactic gas distribution, galactic
magnetic field strength etc.). In general, it is only possible to find closed form (i.e. analytical) solutions for these equations for trivial dependencies of the physical parameters causing the energy loss/gains of nuclei, and electrons and positrons. Therefore, for realistic physical parameters, numerical methods have to be employed to obtain solutions of the propagation equations, and a particular method for doing this is also discussed in this Chapter.

In the next Section the propagation equations for nuclei, and electrons and positrons, are given. Following this, a description of a numerical method for obtaining the solution of the propagation equations based on Monte Carlo simulation techniques is given. To demonstrate the reliability and accuracy of the numerical method, two simple approximations of the propagation equation suitable for electrons and positrons are considered. Analytical solutions are obtained for these two cases, and the Monte Carlo method is used to generate the corresponding numerical solutions. The interdependence of the parameters of the Monte Carlo method are then investigated to find the optimal choice of parameters to ensure sufficient agreement between the analytical and numerical results whilst minimising computing resources.

2.2 Propagation of Cosmic Rays in the Galaxy

The transport of cosmic ray nuclei in the Galaxy may be approximated using the following partial differential equation (PDE) (e.g. Berezinskiĭ et al. 1990)

\[
\frac{\partial N_i}{\partial t} - \nabla \cdot \left[ \hat{K} \cdot \nabla N_i - \hat{v} N_i \right] + \frac{\partial}{\partial E} \left[ \left( b - \frac{\nabla \cdot \hat{v}}{3} E \right) N_i \right] + \left[ \sum_s n_s \sigma_{i,s} \beta c + \frac{1}{\tau_{ri}} \right] N_i - Q_i = 0
\]

where \( N_i \equiv N_i(E, \vec{r}, t) \) (GeV\(^{-1}\) cm\(^{-3}\)) is the differential number density of cosmic ray species \( i \) in the energy and spatial intervals \( E \rightarrow E + dE \) and \( \vec{r} \rightarrow \vec{r} + d\vec{r} \) respectively at time \( t \), \( \hat{K} \equiv \hat{K}(E, \vec{r}, t) \) (cm\(^2\) s\(^{-1}\)) is the diffusion tensor, \( \hat{v} \equiv \hat{v}(\vec{r}) \) (cm s\(^{-1}\)) is the velocity of the bulk motion of the interstellar medium (ISM), \( b \equiv b(E, \vec{r}, t) \) (GeV s\(^{-1}\)) characterises the continuous energy loss (and gain) of the cosmic rays, \( n_s \equiv n_s(\vec{r}) \) (cm\(^{-3}\)) is the number density of gaseous particles of species \( s \) in the ISM, \( \sigma_{i,s} \equiv \sigma_{i,s}(E) \) (cm\(^2\)) is the total fragmentation cross-section of cosmic rays of species \( i \) on gas particles of species \( s \), \( \beta c \) (cm s\(^{-1}\)) is the velocity of the cosmic rays, \( \tau_{ri} \) (s) is the time dilated lifetime of cosmic rays of species \( i \) against radioactive decay, and \( Q_i \equiv Q_i(E, \vec{r}, t) \) (GeV\(^{-1}\) cm\(^{-3}\) s\(^{-1}\)) is the rate at which
cosmic rays are injected into the Galaxy. For cosmic ray nuclei injected from sources such as supernova remnants, the form of the source function is as given in Chapter 3; for cosmic ray nuclei that are the product of fragmentation by higher mass number nuclei in the ISM, the source function has the form

\[ Q_i(\vec{E}, \vec{r}, t) = \sum_{j>i} \left[ \sum_s n_s \sigma_{ij,s} \beta c + \frac{1}{\tau_{rij}} \right] N_j \]  

(2.2)

where the outer sum is over all cosmic ray species heavier than species \( i \), \( \sigma_{ij,s} \equiv \sigma_{ij,s}(E) \) (cm\(^2\)) is the fragmentation cross-section for species \( j \) into species \( i \) on gas particles of species \( s \), \( \tau_{rij} \) (s) is the time dilated lifetime of species \( j \) against radioactive decay into species \( i \), \( N_j \equiv N_j(E, \vec{r}, t) \) (GeV\(^{-1}\) cm\(^{-3}\)) is the differential number density of species \( j \), and all other terms have been defined earlier. Note that for secondary nuclei of species \( i \), a separate PDE for the number density \( N_j \) is required for each progenitor species \( j \), which results in a coupled set of PDEs to be solved to find the secondary species number density.

For cosmic ray electrons and positrons, not all terms in Equation 2.1 are required to describe the transport of these particles. Specifically, the terms involving fragmentation and radioactive decay do not apply in the case of electrons and positrons, and only one PDE is required. Therefore, the propagation of electrons and positrons in the Galaxy can be approximated by

\[ \frac{\partial N_i}{\partial t} - \nabla \cdot \left[ \hat{K} \cdot \nabla N_i - \vec{v} N_i \right] + \frac{\partial}{\partial E} \left[ \left( b - \frac{\vec{v} \cdot \vec{E}}{3} \right) N_i \right] - Q_i = 0 \]  

(2.3)

with the source function for these particles, \( Q_i \), having the form as described in Chapter 3 (here the subscript \( i \) refers to electrons or positrons).

In this thesis, I assume the diffusion tensor, \( \hat{K} \), has no off-diagonal elements, and is independent of spatial and temporal coordinates, i.e., \( \hat{K}(E, \vec{r}, t) = K(E) \hat{l} \) with \( \hat{l} \) the unit tensor. Also, the energy loss/gain rate, \( b \), and source functions, \( Q \), are assumed to be independent of time, i.e., \( b \equiv b(E, \vec{r}) \) and \( Q \equiv Q(E, \vec{r}) \). The assumption of isotropy for the diffusion tensor is made on the basis that although diffusion on local scales may be highly anisotropic, globally the large scale random magnetic field gives rise to random rotations of the local diffusion tensor that can make diffusion in the ISM isotropic (Berezinski et al. 1990). The assumption of the independence of \( K \) on spatial coordinates is made mainly for simplicity. Independence of \( K \), \( b \) and \( Q \) on \( t \) is assumed because I am generally only interested in steady state applications of Equations 2.1 and 2.3 (i.e. \( \partial N/\partial t = 0 \) in these
Equations).

2.3 The Numerical Method

In general, the solution of Equations 2.1 or 2.3 by analytical techniques is quite complicated and may only be done for certain limiting cases and boundary conditions. A realistic treatment of cosmic ray propagation in the Galaxy therefore requires a numerical method to be employed to treat the (often) non-trivial spatial and energy dependence of the physical parameters of the model. These numerical methods include finite-difference schemes to solve the PDEs (e.g. Strong & Moskalenko 1998), or Monte Carlo simulation (e.g. Owens & Jokipii 1977a,b; Porter & Protheroe 1997). In this Section, I describe a numerical method employing Monte Carlo simulation to determine the steady state Green’s function for the cosmic ray transport equation.

2.3.1 Description of the Method

Consider test particles with initial kinetic energy $E'$ released at $\vec{r}'$ to diffuse through the Galaxy and continuously lose, or gain, energy by interacting with the background radiation, magnetic field and gas distribution. Define the function $p(E, \vec{r}; E', \vec{r}')$ to be the probability density (cm$^{-3}$) of points in space at which the energy of the test particles was precisely $E$. Given the energy loss/gain rate

$$\frac{dE}{dt} = -b(E, \vec{r}) \text{ (GeV s$^{-1}$)}, \quad (2.4)$$

the average time spent in the energy interval $E \rightarrow E + dE$ is $dE/b(E, \vec{r})$ if a particle is located at or close to $\vec{r}$. Hence, the average time spent by a particle with energy in the range $E \rightarrow E + dE$ per unit volume at $\vec{r}$ is $p(E, \vec{r}; E', \vec{r}')dE/b(E, \vec{r})$. Thus, for some source distribution $Q(E, \vec{r})$ (GeV$^{-1}$ cm$^{-3}$ s$^{-1}$) the number density of particles $N(E, \vec{r})$ (GeV$^{-1}$ cm$^{-3}$) at $\vec{r}$ is

$$N(E, \vec{r}) = b(E, \vec{r})^{-1} \int dV' \int_{E}^{\infty} dE'Q(E', \vec{r}')p(E, \vec{r}; E', \vec{r}'). \quad (2.5)$$

The most time-consuming part of the calculation is working out $p(E, \vec{r}; E', \vec{r}')$ because this contains all the information about propagation, energy losses and interactions during the propagation.
In principle, this method is suitable for a general geometry of the source and particle propagation regions, and for particles undergoing continuous energy gains and losses. Modifications to the method must be made if the energy loss/gain is not the result of some continuous process, such as in the fragmentation of cosmic ray nuclei, but since the main focus of this thesis is the propagation of electrons and positrons it is sufficient to consider the method as given above.

To illustrate the accuracy and reliability of the numerical method I solve Equation 2.3 neglecting convection in the one-dimensional (1D) approximation in the next Section for two simple cases, and compare the analytical and numerical solutions obtained using the method described above.

### 2.3.2 Comparison of Analytical and Numerical Results

Consider the 1D approximation of the steady state diffusion equation neglecting adiabatic energy losses and convective transport:

$$
-K(E) \frac{\partial^2 N(E, z)}{\partial z^2} + \frac{\partial}{\partial E} [b(E)N(E, z)] = Q(E, z)
$$

(2.6)

where all terms have the same meaning as defined in Section 2.2 except that I assume here no spatial dependence for the energy loss rate (i.e. $b(E, z) \equiv b(E)$). I consider two cases for this approximation of the diffusion equation: (i) a constant energy loss rate $b(E) = B$, and a constant diffusion coefficient $K_0$, and (ii) an energy loss rate $b(E) = BE^2$ ($B \equiv$ constant), and an energy dependent diffusion coefficient $K(E) = K_0(E/E_0)^{\delta}$. The first case is of interest because it approximates the diffusion and energy losing processes of stable cosmic ray particles (such as electrons, positrons and protons) at low energies, while the second case approximates the diffusion and energy losing processes of cosmic ray electrons and positrons at high energies. In both cases considered, the Green’s function of Equation 2.6 is derived without boundaries for reasons of computational simplicity.

To derive the Green’s function, it is required to solve the equation

$$
-K(E) \frac{\partial^2 G}{\partial z^2} + \frac{\partial}{\partial E} [b(E)G] = Q_0 \delta(z - z_0) \delta(E - E_0)
$$

(2.7)

where $G \equiv G(E, z; E_0, z_0)$ is the Green’s function for particles released with initial energy and position $E_0$ and $z_0$ respectively, $Q_0$ ($\text{cm}^{-2} \text{s}^{-1}$) is a constant, and $\delta$ is the Dirac delta
distribution. I define a new function \( G' \) so that \( G = G'/b(E) \) and, following Ginzburg and Syrovatskii (1964), also define the function

\[
\lambda(E) = \int_{E}^{\infty} \frac{K(X)}{b(X)} dX
\]

so \( \partial \lambda/\partial E = K(E)/b(E) \). Equation 2.8 may be interpreted as being proportional to the square of the distance for a particle to cool down from infinity to energy \( E \). With these replacements, the diffusion equation becomes

\[
- \frac{\partial^2 G'}{\partial z^2} + \frac{\partial G'}{\partial \lambda} = Q_0 \delta(z - z_0) \delta(\lambda(E) - \lambda(E_0))
\]

where use has been made of the property of delta distributions \( \delta(x) = \delta(f(x))/|\partial f/\partial x| \). The procedure is now to invert the differential operator \( \partial^2/\partial z^2 + \partial/\partial \lambda \) to obtain \( G' \). Let \( \tilde{G}' \) be the Fourier transform of the function \( G' \) over \( \lambda \) and \( z \). Hence \( G' \) is

\[
G'(E, z; E_0, z_0) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk_z e^{-ik_z(z-z_0)} \int_{-\infty}^{\infty} dk_\lambda e^{-ik_\lambda(\lambda-\lambda_0)} \tilde{G}'(k_z, k_\lambda)
\]

where \( \lambda - \lambda_0 \equiv \lambda(E) - \lambda(E_0) \). Inserting Equation 2.10 into Equation 2.9 gives

\[
[\ldots] \left\{ - \frac{\partial^2}{\partial z^2} + \frac{\partial}{\partial \lambda} \right\} e^{-ik_z(z-z_0)} e^{-i\lambda(\lambda-\lambda_0)} \tilde{G}'(k_z, k_\lambda) = Q_0 [\ldots] e^{-ik_z(z-z_0)} e^{-i\lambda(\lambda-\lambda_0)}
\]

\[
[\ldots] e^{-ik_z(z-z_0)} e^{-i\lambda(\lambda-\lambda_0)} \left\{ -k_z^2 - ik_\lambda \right\} \tilde{G}'(k_z, k_\lambda) = Q_0 [\ldots] e^{-ik_z(z-z_0)} e^{-i\lambda(\lambda-\lambda_0)}
\]

\[
\left\{ -k_z^2 - ik_\lambda \right\} \tilde{G}'(k_z, k_\lambda) = Q_0
g \tilde{G}'(k_z, k_\lambda) = Q_0
\]

where \( [\ldots] \equiv 1/(2\pi)^2 \int_{-\infty}^{\infty} dk_z \int_{-\infty}^{\infty} dk_\lambda \). Transforming back into \( z - \lambda \) space gives the result

\[
G'(E, z; E_0, z_0) = \frac{Q_0}{(2\pi)^2} \int_{-\infty}^{\infty} dk_z e^{-ik_z(z-z_0)} \int_{-\infty}^{\infty} dk_\lambda \frac{e^{-ik_\lambda(\lambda-\lambda_0)}}{-k_z^2 - ik_\lambda}
\]

\[
= \frac{Q_0}{2\pi} \int_{-\infty}^{\infty} dk_z e^{-k_z^2(\lambda-\lambda_0)} e^{-ik_z(z-z_0)}
\]

\[
= \frac{Q_0}{[4\pi(\lambda - \lambda_0)]^{1/2}} e^{-\frac{(z-z_0)^2}{4(\lambda - \lambda_0)}}.
\]

The first integration has been performed by noticing that the integrand has a pole of order 1 at \( k_\lambda = -ik_z^2 \), and then applying the residue theorem with a suitably chosen contour in the
complex plane (see Figure 2.1). For the second integration I have used \( \int_{-\infty}^{\infty} e^{-p^2 x^2 + q x} dx = e^{q^2 / 4p} \pi^{1/2} / p \) (e.g. Gradshteyn and Ryzhik 1965). The Green’s function, \( G \), is then found as

\[
G(E, z; E', z') = \frac{1}{b(E)} \int_{-\infty}^{\infty} dz_0 \int_{0}^{\infty} dE_0 G'(E, z; E_0, z_0) \delta \left(z_0 - z'\right) \delta \left(E_0 - E'\right) \\
= \frac{1}{b(E)} \left(G'(E, z; E', z')\right) .
\] (2.13)

To find \( G \) for each of the cases considered, \( \lambda(E) - \lambda(E') \) must be specified. For case (i) \( \lambda(E) - \lambda(E') = K_0 |E - E'| / B \), and for case (ii) \( \lambda(E) - \lambda(E') = K_0 (E^{5-1} - E'^{6-1}) / BE_0^5 (1 - \delta) \).

Thus, the Green’s functions for case (i) and (ii) are

\[
G_{(i)}(E, z; E', z') = \frac{Q_0}{[4\pi K_0 B |E - E'|]^{1/2}} e^{-B \frac{(z - z')^2}{4K_0 |E - E'|}}
\] (2.14)

and

\[
G_{(ii)}(E, z; E', z') = \frac{Q_0}{[4\pi B E^4 K_0 (E^{5-1} - E'^{6-1}) / E_0^5 (1 - \delta)]^{1/2}} e^{-(1 - \delta) B E_0^5 \frac{(z - z')^2}{4K_0 (E^{5-1} - E'^{6-1})}}.
\] (2.15)

Substitution of both of these functions into the appropriate form of Equation 2.6 verifies that these are solutions of the propagation equation for a delta function source in energy and space, and therefore by definition Green’s functions.

I now describe the implementation of the Monte Carlo method, and directly compare the numerical results with the analytical solutions. First, consider case (i). The Green’s function given by Equation 2.14 was obtained for the source function \( Q(E, z) = Q_0 \delta(E - E') \delta(z - z') \), and I obtain the probability matrix \( p(E, z; E', z') \) for the same source function with \( z' = 0 \) below. I take 41 energy bins at intervals of \( \Delta \log E = 0.1 \) starting at a mid-bin energy \( 10^{-3} \) GeV. The Monte Carlo procedure is as follows. A particle is injected at \( z = 0 \) with initial kinetic energy \( E_j \). Based on random walk theory and its relation to diffusion (Chandrasekhar 1943), the diffusion in \( z \) is simulated by multiplying a randomly sampled normal deviate, \( \zeta \), by the scaling half-width

\[
\sigma_z = \min(\sqrt{2K \Delta t}, \sigma_{z,\text{max}})
\] (2.16)

where \( K \) is the diffusion coefficient, \( \Delta t = \Delta E / b(\sqrt{E_j E_{j+1}}, z) \) with \( \Delta E = (E_j - E_{j-1}) \) being the difference between the current mid-bin energy and the next lowest mid-bin energy,
Figure 2.1: Contour in the $k_\lambda$ plane used in the first integration to obtain Equation 2.12.

and $\sigma_{z_{\text{max}}}$ is the maximum scaling half-width of the distribution, and is chosen to minimise computing time while ensuring the Monte Carlo and analytical results agree. Typically, $\sigma_{z_{\text{max}}}$ is chosen to be small compared to the distance over which physical parameters of the model change significantly (but see Section 2.3.3). The energy used to compute the energy loss rate for $\Delta t$, $\sqrt{E_j E_{j-1}}$, corresponds to the energy half-way between two adjacent bins on a log scale. The new position is obtained by adding $\Delta z = \zeta \sigma_z$ to the current position.

If $\sigma_z < \sigma_{z_{\text{max}}}$ the particle’s energy is set to $E_{j-1}$ and, if the new position is in the “observing region”, i.e. the region for which it is required to obtain $N(E, z)$, the particle is recorded as having been observed with energy $E_{j-1}$. Otherwise, if $\sigma_z = \sigma_{z_{\text{max}}}$, the energy lost by the particle while diffusing is calculated using a time step of

$$\Delta t = \frac{\sigma_{z_{\text{max}}}^2}{K}$$

(2.17)

and subtracted from its current energy. If the particle’s energy falls below $E_{j-1}$, the approximate position where its energy became lower than $E_{j-1}$ is determined by interpolating the distance between the particle’s initial and final positions in which it took the particle to lose an amount $E - E_{j-1}$ of its energy while diffusing. If this position is within the observing
region, the particle is recorded as having been observed with energy \( E_{j-1} \). The energy bin counter \( (j-1) \) is then decremented by 1, and the above procedure is repeated until the particle reaches some large distance away from the galactic plane, taken to be 20 kpc in this instance; particles diffusing out to this distance would have lost an amount of energy large enough to place them below the lowest particle energy in the simulation, hence they are no longer of interest. The probability matrix is computed by injecting \( \mathcal{N} \) particles at each of the source energy bins and following the Monte Carlo procedure outlined above, the final result being divided by \( \mathcal{N} \) and the volume of the observing region.

As an example of the results that can be obtained using the Monte Carlo method, I show in Figures 2.2a and 2.2b, and Figures 2.3a and 2.3b, the distributions of particles in \( E \) and \( z \) for case (i) with source energies \( E'_I = 10^{-1.0} \) and \( E'_I = 10^{0.0} \) GeV, and case (ii) with source energies \( E'_I = 10^{4} \) GeV and \( E'_I = 10^{6} \) GeV respectively, calculated using the above method for \( \mathcal{N} = 10^5 \) particles and \( \sigma_{z_{\text{max}}} = 50 \) pc, and an observing region of half-width \( \Delta z_{\text{obs}} = 50 \) pc. The results for case (i) are obtained for \( Q_0 = 1 \) cm\(^{-2} \) s\(^{-1} \), \( K_0 = 2.5 \times 10^{28} \) cm\(^2\) s\(^{-1} \), and \( B = 5 \times 10^{-16} \) GeV s\(^{-1} \), while for case (ii) they are obtained for \( Q_0 = 1 \) cm\(^{-2} \) s\(^{-1} \), \( K_0 = 2.5 \times 10^{28} \) cm\(^2\) s\(^{-1} \), \( E_0 = 3 \) GeV, \( \delta = 0.6 \), and \( B = 9.12 \times 10^{-17} \) GeV\(^{-1}\) s\(^{-1} \). Also shown in the Figures are plots of Equation 2.14 and 2.15 for the same source energies and values for \( Q_0 \), \( K_0 \), and \( B \). Note that for case (ii), the probability matrix has been obtained by taking 61 energy bins for a mid-bin energy starting at \( 10^{0.0} \) GeV with spacing \( \Delta \log E = 0.1 \), and following the Monte Carlo procedure outlined above.

Both sets of Figures illustrate the effect on the particle distribution of the different energy loss rates. Below 1 GeV, a constant energy loss rate tends to steepen the distribution away from \( z = 0 \) as the injection energy is made progressively lower, even though the number of particles near \( z = 0 \) remains similar for all injection energies. The same effect on the particle distribution, but in the reverse sense, is seen above 1 GeV for an energy loss rate \( \propto E^2 \); as the injection energy is made progressively higher, the distribution near \( z = 0 \) is similar regardless of injection energy, but the distribution at larger \( z \) becomes significantly diminished.

In the Monte Carlo method, the free parameters are \( \sigma_{z_{\text{max}}} \) and \( \mathcal{N} \), and the size of the observing region, and these have to be adjusted to optimise the agreement between the analytical and numerical results while not producing unacceptably long run-times for the simulation code. For the particular choices of \( \sigma_{z_{\text{max}}} \), \( \mathcal{N} \), and observing region size used to generate Figures 2.2a and 2.2b, and Figures 2.3a and 2.3b, the agreement between the
Figure 2.2: Particle distributions in energy ($E$) and distance above the injection position ($z$) calculated using Equation 2.14 and the Monte Carlo method for source energies $E'$ (a) $10^{-0.5}$ GeV and (b) $10^{0.0}$ GeV. For each source energy the Monte Carlo results are plotted as solid histograms, and the analytical results as dashed lines. The upper curve shows the distribution at $z = 0$ kpc with lower curves showing the distribution at $z = 0.5, 1.0, 1.5$ and $2.0$ kpc. The lower curves have been multiplied by a factor $10^{-0.1}, 10^{-0.2}$ etc. to clarify the diagram.
Figure 2.3: Particle distributions in $E$ and $z$ calculated using Equation 2.15 and the Monte Carlo method for source energies $E'$: (a) $10^{10}$ GeV and (b) $10^{6.0}$ GeV. For each source energy, the Monte Carlo results are plotted as solid histograms, and the analytical results as dashed lines. The upper curve shows the distribution at $z = 0$ kpc with lower curves showing the distribution at $z = 0.5, 1.0, 1.5$ and $2.0$ kpc. The lower curves have been multiplied by a factor $10^{-0.2}, 10^{-0.4}$ etc. to clarify the diagram.
analytical and numerical results is excellent. However, these examples use a combination of parameters which were chosen through trial and error to give Monte Carlo results agreeing with the analytical solutions, and no particular emphasis was applied to the amount of computing time required to give the results shown in the Figures. It would therefore be purely serendipitous if the parameters used to generate the Figures were the optimal choice for $\sigma_{z_{\text{max}}}$, $N$, and $\Delta z_{\text{obs}}$ that minimise the overall computing time. In the next Section I examine the relationship between these parameters, and determine the optimal choice to be used in the propagation calculations presented in Chapter 5.

### 2.3.3 Parameter Selection for the Numerical Method

To examine the relationship between $\sigma_{z_{\text{max}}}$, $N$, and $\Delta z_{\text{obs}}$ I calculate particle distributions for case (i) in Section 2.3.2 with the same values for $Q_0$, $K_0$ and $B$ used to generate Figures 2.2a and 2.2b for a source energy of $E' = 10^{-1.0}$ GeV. These choices are made because case (i) is the quickest to calculate computationally (i.e. there is no energy dependence of the diffusion coefficient or energy loss rate), and the value of $E'$ gives a reasonably interesting particle distribution. Note that case (ii) from Section 2.3.2 could be used also, but this would tend to increase the computing resources required while not actually affecting the outcome of the following investigation.

I first consider the effect of varying $\sigma_{z_{\text{max}}}$ for a fixed value of $N$ and size of the observing region. For this I choose $N = 10^5$ and $\Delta z_{\text{obs}} = 50$ pc since these values give excellent agreement between the numerical and analytical results for $\sigma_{z_{\text{max}}} = 50$ pc at least, and generate the probability matrix for case (i) for particles injected at $z = 0$ for differing values of $\sigma_{z_{\text{max}}}$. Figures 2.4a-d show the particle distributions in $E$ and $z$ for a source energy of $E' = 10^{-1.0}$ GeV. For $\sigma_{z_{\text{max}}} = \infty$, corresponding to $\sigma_z$ in Equation 2.16 effectively being determined by the spacing between adjacent energy bins, the agreement is fair for the particle distributions with $z < 1.0$ kpc, but is poor for larger $z$. As $\sigma_{z_{\text{max}}}$ is reduced to 200 pc, the agreement of the Monte Carlo and analytical results improves. However, there is still some disagreement for the particle distributions at large $z$ and at low energies. Decreasing $\sigma_{z_{\text{max}}}$ further to 50 pc alleviates these discrepancies between the Monte Carlo and analytical results, and the overall agreement is excellent. A further decrease of $\sigma_{z_{\text{max}}}$ to 20 pc produces the same excellent agreement between the numerical and analytical results, but the computation time is practically doubled.

I now consider the effect of fixing both $\sigma_{z_{\text{max}}}$ and $\Delta z_{\text{obs}}$, and varying the number of
Figure 2.4: Particle distributions in \( E \) and \( z \) calculated using Equation 2.15 and the Monte Carlo method for a source energy of \( E' = 10^{-1.6} \text{ GeV} \). The particle distributions are calculated for \( N = 10^5 \) particles, \( \Delta z_{\text{obs}} = 50 \text{ pc} \), and for (a) \( \sigma_{z_{\text{max}}} = \infty \), (b) \( \sigma_{z_{\text{max}}} = 200 \text{ pc} \), (c) \( \sigma_{z_{\text{max}}} = 50 \text{ pc} \), and (d) \( \sigma_{z_{\text{max}}} = 20 \text{ pc} \). The histograms and curves have the same meanings as described in the key of these Figures 2.2a and 2.2b.
particles injected into an energy bin. In Figures 2.5a-d I show the particle distributions generated for \( \sigma_{z_{\text{max}}} = 50 \) pc and \( \Delta z_{\text{obs}} = 50 \) pc for different numbers of injected particles \( N \). For \( N = 10^3 \) particles the agreement is quite poor, especially for particle distributions for \( z > 1.0 \) kpc. As would be expected, as \( N \) is increased the agreement between the Monte Carlo and analytical results improves; for \( N = 10^4 \) the agreement is fairly good, with excellent agreement being found for \( N = 10^5 \) and \( N = 10^6 \) particles. However, for each of Figures 2.5a-d, the computing time increases by an order of magnitude as \( N \) is increased from \( 10^3 \) to \( 10^4 \) to \( 10^5 \) etc.

I now examine the effect of varying the size of the observing region, \( \Delta z_{\text{obs}} \), while keeping \( \sigma_{z_{\text{max}}} \) and \( N \) fixed. In Figures 2.6a-d I show the particle distributions generated for \( \sigma_{z_{\text{max}}} = 50 \) pc and \( N = 10^5 \) particles for different sizes of the observing region. The agreement between the analytical and numerical results is fairly good for \( \Delta z_{\text{obs}} = 10 \) pc, and is excellent for larger values of \( \Delta z_{\text{obs}} \). Ideally, the smallest value for \( \Delta z_{\text{obs}} \) would give the best agreement between the analytical and numerical results because it is closest to the size for which the analytical solution was calculated (formally, \( \Delta z_{\text{obs}} = 0 \) for the analytical results). However, the small size of the observing region in the numerical calculations means less particles are counted than if a larger value for \( \Delta z_{\text{obs}} \) was used. Hence, the agreement between the analytical and numerical results shown in Figure 2.6a is not as good as shown in the other Figures.

From examination of Figures 2.4 through 2.6 it can be seen that the choice for the particular set of values for \( \sigma_{z_{\text{max}}} \), \( N \) and \( \Delta z_{\text{obs}} \) that provides the best agreement between the analytical and numerical results is not obvious. Coupled with the additional constraint of requiring computing time to be reasonable, the task of finding the best set of parameters is beyond a simple ‘eyeball’ assessment of a few simulations such as are shown in the Figures. Therefore, to better quantify how well a set of parameters produce agreement between the analytical and Monte Carlo results for a particular simulation, I use a simple statistic that measures the average deviation between the Monte Carlo and analytical results per energy bin for a single simulation:

\[
\Sigma = \left[ \sum_{i=1}^{L} \left[ \sum_{j=1}^{M} \frac{(N_{i,j}^{MC} - N_{i,j}^{GFN})^2}{LM} \right] \right]^{1/2}
\]  

(2.18)

where the outer sum is over a representative number of particle distributions in \( z \), \( L \), the inner sum is over the number of energy bins, \( M \), calculated for each distribution, \( N_{i,j}^{MC} \) is the Monte Carlo generated number density for the \( i \)th particle distribution and \( j \)th energy
Figure 2.5: Particle distributions in $E$ and $z$ calculated using Equation 2.15 and the Monte Carlo method for a source energy of $E' = 10^{-1.6}$ GeV. The particle distributions are calculated for $\sigma_{z_{\text{max}}} = 50$ pc, $\Delta z_{\text{obs}} = 50$ pc, and for (a) $N = 10^3$ particles, (b) $N = 10^4$ particles, (c) $N = 10^5$ particles, and (d) $N = 10^6$ particles. The histograms and curves have the same meanings as described in the key of Figures 2.2a and 2.2b.
Figure 2.6: Particle distributions in $E$ and $z$ calculated using Equation 2.15 and the Monte Carlo method for a source energy of $E' = 10^{-1.6}$ GeV. The particle distributions are calculated for $\sigma_{z_{\text{max}}} = 50$ pc, $N = 10^5$ particles, and for (a) $\Delta z_{\text{obs}} = 10$ pc, (b) $\Delta z_{\text{obs}} = 50$ pc, (c) $\Delta z_{\text{obs}} = 100$ pc, and (d) $\Delta z_{\text{obs}} = 200$ pc. The histograms and curves have the same meanings as described in the key of Figures 2.2a and 2.2b.
bin, and $N_{ij}^{\text{GFN}}$ is the analytical number density for the $i$th particle distribution and $j$th energy bin. By way of example to illustrate how Equation 2.18 is to be calculated, $L = 5$ and $M = 21$ for Figures 2.6a-d.

For a given set of values for the parameters $\sigma_{z_{\text{max}}}$, $N$, and $\Delta z_{\text{obs}}$, the nature of the Monte Carlo simulation technique implies that there will be fluctuations in the value of $\Sigma$ calculated using Equation 2.18 between independent runs of the simulation code. Therefore, it is better to consider a value of $\Sigma$ for a given set of parameters averaged over a number of simulations, $S$. So, the procedure to generate a $\Sigma$ for a given set of $\sigma_{z_{\text{max}}}$, $N$ and $\Delta z_{\text{obs}}$ is to calculate $\Sigma$ for each of $S$ simulations for this set of parameters, sum up the individual $\Sigma$ values, and divide by $S$.

Table 2.1 shows the results obtained for Equation 2.18 for $S = 10$ simulations, for 4 different values of $\Delta z_{\text{obs}}$, and 4 different values also for $\sigma_{z_{\text{max}}}$ and $N$. Note that I also calculated Equation 2.18 for $S = 100$ simulations for the parameters $\sigma_{z_{\text{max}}} = 50$ pc, $N = 10^4$ and the 4 values of $\Delta z_{\text{obs}}$ used in Table 2.1, and found that the values of $\Sigma$ generated only differed in the second decimal place to the values of $\Sigma$ given for the same parameter set in the Table. Therefore, it should be sufficient to consider only $S = 10$ (this is also useful because it would be computationally prohibitive to calculate 100 simulations for large values of $N = 10^6$, or small values of $\sigma_{z_{\text{max}}}$).

For fixed values of $\sigma_{z_{\text{max}}}$ and $\Delta z_{\text{obs}}$, it would be expected that the values of $\Sigma$ calculated for $N = 10^6$ particles would be the lowest, which is simply a result of the Monte Carlo results converging to the analytical for increasingly larger values of $N$. This is the case, as given in the Table, for all values of $\sigma_{z_{\text{max}}}$ and $\Delta z_{\text{obs}}$. Therefore, by considering the values of $\Sigma$ obtained for different $\sigma_{z_{\text{max}}}$ values as a group for the $N = 10^6$ particles simulations, the best choice of $\Delta z_{\text{obs}}$ can be found. From the Table, the lowest values of $\Sigma$ for $\sigma_{z_{\text{max}}} = 20 - \infty$ pc for $N = 10^6$ particles are to found for $\Delta z_{\text{obs}} = 50$ and 100 pc respectively. Now, out of these two groups of $\Sigma$ for $N = 10^6$ particles, the lowest values of $\Sigma$ are obtained for $\sigma_{z_{\text{max}}} = 50$ pc, with that obtained for $\Delta z_{\text{obs}} = 100$ pc being only slightly lower than that obtained for $\Delta z_{\text{obs}} = 50$ pc. So, $\sigma_{z_{\text{max}}} = 50$ pc generally gives the best agreement with probably $\Delta z_{\text{obs}} = 100$ pc being the better choice out of the two sizes for the observing region.

At this stage no consideration has been given to the requirement that sufficient accuracy is ensured while the computing time is minimised. Obviously this requirement involves compromising between values of $N$ large enough to ensure a low value for $\Sigma$, but not making $N$ so large that the gains in accuracy are barely noticeable. Given that it has been determined
Table 2.1: The average value of Σ for $K = 10$ simulations per set of values for $\sigma_{z_{\text{max}}}$, $N$, and $\Delta z_{\text{obs}}$, calculated using Equation 2.18.

that $\sigma_{z_{\text{max}}}$ = 50 pc and $\Delta z_{\text{obs}}$ = 100 pc are probably the best choice of values for these parameters. $N$ can be chosen by considering how much $\Sigma$ decreases by as $N$ increases from $10^3 \rightarrow 10^6$. From the appropriate row in the Table, it can be seen that $\Sigma$ decreases by a factor $\sim 3$ for $N = 10^3 \rightarrow 10^4$ particles, a factor $\sim 3$ for $N = 10^4 \rightarrow 10^5$ particles, and a factor $\sim 2$ for $N = 10^5 \rightarrow 10^6$ particles. Recalling the results shown in Figures 2.5c-d, the practical difference between the Monte Carlo and analytical results for $N = 10^5$ and $N = 10^6$ particles is negligible, and this is borne out in the absolute values of $\Sigma$ for these two cases (obviously with a different value for $\Delta z_{\text{obs}}$ than used in Figures 2.5c-d) by the results given in the Table. Therefore, sufficient accuracy is obtained using $N = 10^5$ particles to not warrant the order of magnitude increase in computing time when $N = 10^6$ is used. The optimal choice of parameters is then: $\sigma_{z_{\text{max}}} = 50$ pc, $N = 10^5$ particles, and $\Delta z_{\text{obs}} = 100$ pc.
2.4 Summary

The equations used to describe the propagation of cosmic ray nuclei, and electrons and positrons, in the Galaxy in this thesis have been introduced, and discussed. A numerical method for solving the propagation equations for particles undergoing continuous energy losses and gains has been described. Two simple cases of the propagation equation for electrons and positrons have been used to illustrate the effectiveness of the numerical method; analytical expressions have been found for solutions of these two cases of the propagation equation, and numerical solutions for the two cases have been obtained using the Monte Carlo method. The agreement between the Monte Carlo and analytical results is, in general, excellent, and the numerical method has been demonstrated to reliably reproduce the analytical results. The interdependence between the adjustable parameters of the Monte Carlo method has been investigated to find the optimal set of parameters that produces the best agreement between the analytical and numerical results whilst minimising the computing resources required.
Chapter 3

Galactic Cosmic Ray Electron and Positron Sources

3.1 Introduction

Galactic cosmic ray electrons and positrons may be produced through a variety of mechanisms. Generally, it is thought that the majority of cosmic ray electrons are directly accelerated at shocks formed by supernova remnant (SNR) blast waves expanding into the interstellar medium (ISM) surrounding the precursor star (e.g. Blandford & Ostriker 1978). Electrons accelerated in this way comprise the bulk of the cosmic ray electron spectrum (Berezinskii et al, 1990) and are designated as the primary source component of the electron spectrum. On the other hand, it appears the bulk of cosmic ray positrons (and a small fraction of the electrons) are purely secondary particles, most likely produced in inelastic collisions between cosmic ray nuclei and gas in the ISM (e.g. Protheroe 1982; Barwick et al, 1997). Naturally, the spatial and energy distributions of the primary and secondary sources are not coincident. For example, the spatial distribution of the primary component depends upon the assumed distribution of SNRs in the Galaxy while that of the secondary component depends upon the distribution of cosmic ray nuclei and gas.

It is the purpose of this Chapter to describe the energy spectra and spatial distributions for the sources of primary and secondary electrons that will be used in propagation model calculations in subsequent Chapters. The acceleration of cosmic rays in SNRs is discussed in Section 3.2. A brief description is given of the mechanism by which cosmic rays are accelerated to relativistic energies, and the spatial distribution of primary sources is also
described. A primary electron source function for use in later Chapters is then constructed incorporating the energy spectra and spatial dependence of SNRs. In Section 3.3 the production spectrum of secondary electrons from inelastic collisions between cosmic ray nuclei and gas atoms in the ISM is described. A careful comparison of the present calculations for the production spectrum with the results obtained by other authors is given. The spatial distribution of secondary electron sources in the Galaxy is described, and as for primary electrons, the total secondary electron and positron source function used in later Chapters is then constructed.

3.2 Primary Electron Source Distribution

The primary source distribution for cosmic ray electrons in the Galaxy is constructed in this Section, assuming that SNRs are the acceleration sites.

The motivation for considering SNRs as the most likely acceleration sites for the majority of cosmic rays in the Galaxy dates back to the 1930s, when Baade & Zwicky (1934) advanced the hypothesis that cosmic rays are emitted by supernovae. Over time the view that SNRs, and not the actual supernova explosion, are responsible for the acceleration process (see, for example, the review by Axford 1991) has generally been adopted. In these objects, some form of diffusive acceleration at shock fronts operates to accelerate seed particles to relativistic energies. The shock fronts form when the ejected material from the supernova explosion moves supersonically through the ISM surrounding the precursor star, and the seed particles for the acceleration process come from the thermal particles in the ISM (see e.g. the models of Ellison, Drury & Meyer 1997 for a unified picture of the acceleration of interstellar and/or circumstellar gas and dust to relativistic energies).

In this thesis, I only consider electrons to be directly accelerated in primary sources. The possibility of positrons being similarly accelerated is not considered because observations of the local positron spectrum and fraction seem to indicate a purely secondary nature for cosmic ray positrons (e.g. Barwick et al., 1998).

The primary source function will depend on the spatial distribution of SNRs in the Galaxy, and the expected spectrum of particles emerging from a cosmic ray accelerator. For the source spectrum I consider cosmic rays to be accelerated at SNR shock fronts, and so a description of acceleration of cosmic rays at astrophysical shocks is given in the next Section. The acceleration rate of cosmic rays is derived assuming the first order Fermi process is
3.2. PRIMARY ELECTRON SOURCE DISTRIBUTION

responsible for particle acceleration in SNRs. This is then used in a simple model of a cosmic ray accelerator to obtain the source spectrum of cosmic rays at acceleration. The spatial distribution of primary cosmic ray sources in the Galaxy is then given, and the primary source distribution function is constructed by combining the spatial distribution and injection spectra.

3.2.1 Shock Acceleration of Cosmic Rays

The mechanism by which cosmic rays are accelerated is one of the most important unresolved questions in astrophysics. Most emphasis has been placed on the first order Fermi mechanism, mainly because it naturally reproduces a power-law particle spectrum as a result of the acceleration process. The subject of Fermi acceleration in astrophysical shocks has been reviewed in detail by Blandford & Eichler (1987), Berezko & Krymsky (1988), and Jones & Ellison (1991).

Shock acceleration is believed to be the primary process operating in SNRs to accelerate cosmic rays up to energies $\sim 10^5 - 10^6$ GeV (e.g., Berezinskii et al. 1990). In this thesis it is assumed that the first order Fermi process is the acceleration mechanism operating in the SNRs. It is possible that the second order Fermi process may operate in the ISM to accelerate low energy cosmic rays ('diffusive reacceleration'; see e.g., Seo & Ptuskin 1994), but this is not considered in this thesis.

In this Section, the acceleration rate of cosmic rays accelerated at a parallel shock is obtained.

3.2.1.1 Acceleration of Cosmic Rays at a Parallel Shock Front

Consider a plane shock in a plasma flow as shown in Figure 3.1. Assume the magnetic field is perpendicular to the shock front, and let the velocity of scattering centres in the unshocked ('upstream') and shocked ('downstream') regions be $u_1$ and $u_2$ respectively in the shock front's rest frame. Let the cosmic rays have diffusion coefficients $K_1(E)$ and $K_2(E)$ in the upstream and downstream regions. To calculate the acceleration rate of particles at the shock, the mean fractional energy gain per acceleration cycle $\langle \Delta E \rangle /E$ (upstream→downstream→upstream), and the time to complete one acceleration cycle $T_{\text{cycle}}$, are required. The derivation of these quantities following the method outlined by Gaisser (1990) is described below.

Consider a cosmic ray with energy $E_i$ in the upstream region to diffuse across the shock.
Let the angle the cosmic ray makes with the shock front normal be $\theta_i$, then in the rest frame of the downstream plasma flow the cosmic ray has energy

$$E'_i = \gamma E_i (1 - \beta \cos \theta_i)$$

(3.1)

where $\beta = V/c \equiv (u_2 - u_1)/c$ and $\gamma = (1 - \beta^2)^{1/2}$. Now, when viewed in the downstream rest frame, the energy of the cosmic ray remains constant during scattering, and therefore just before diffusing across the shock to the upstream region the cosmic ray energy is $E'_{i+1} = E'_i$. Transforming the cosmic ray energy to the upstream rest frame gives

$$E_{i+1} = \gamma E'_{i+1} (1 + \beta \cos \theta'_{i+1})$$

(3.2)

where $\theta'_{i+1}$ is the angle the cosmic ray makes with the shock front normal in the downstream rest frame. So the cosmic ray energy after an encounter with the shock characterised by the
angles $\theta_i$ and $\theta_i'_{i+1}$ is

$$E_{i+1} = \gamma^2 E_i (1 - \beta \cos \theta_i) (1 + \beta \cos \theta_i') .$$  \hspace{1cm} (3.3)

Noticing that $E_{i+1} = E_i + \Delta E$ gives the fractional energy gain for this encounter

$$\frac{\Delta E}{E_i} = \frac{(1 - \beta^2 \cos \theta_i \cos \theta_i'_{i+1} - \beta \cos \theta_i + \beta \cos \theta_i')}{1 - \beta^2} - 1.$$

(3.4)

To obtain the average fractional energy gain per acceleration cycle, the angular averages for $\theta_i$ and $\theta_i'_{i+1}$ have to be taken. This is performed by assuming an isotropic particle distribution and integrating over the probability of a cosmic ray crossing the shock with $\theta_i$ between $\theta_i$ and $\theta_i + d\theta_i$. The appropriate distributions are

$$\frac{dN}{d \cos \theta_i} = 2 \cos \theta_i, \quad -1 \leq \cos \theta_i \leq 0$$

(3.5)

$$\frac{dN}{d \cos \theta_i'} = 2 \cos \theta_i', \quad 0 \leq \cos \theta_i' \leq 1$$

(3.6)

and hence $\langle \cos \theta_i \rangle = -2/3$, and $\langle \cos \theta_i' \rangle = 2/3$. Therefore

$$\frac{\langle \Delta E \rangle}{E} = \frac{1 + \frac{4}{3} \beta + \frac{4}{9} \beta^2}{1 - \beta^2} - 1$$

$$= \left(1 - \beta^2 + \beta^4 - \ldots \right) \left(1 + \frac{4}{3} \beta + \frac{4}{9} \beta^2 \right) - 1$$

$$= \frac{4}{3} \beta - \frac{5}{9} \beta^2 - \frac{4}{3} \beta^3 + \ldots$$

$$\approx \frac{4}{3} \beta$$

(3.7)

where $\sum_{n=0}^{\infty} x^n = 1/(1-x)$ has been used and only the leading order term is retained because $\beta < 1$ always.

To calculate $T_{\text{cycle}}$, the propagation of cosmic rays in the upstream and downstream regions has to be considered. Consider first the upstream region. If only diffusion and convection of the cosmic rays are considered, then the transport of particles is governed by (see Equation 2.1)

$$\frac{dN}{dt} = \frac{\partial}{\partial z} \left[ K_1(E) \frac{\partial N}{\partial z} - u_1 N \right]$$

(3.8)

where $N \equiv N(E, z, t)$. In equilibrium $dN/dt = 0$ and therefore
\[ K_1(E) \frac{\partial N}{\partial z} = u_1 N. \] (3.9)

Thus, in the upstream region

\[ N(E, z) = N(E, 0)e^{\frac{zu_1}{K_1(E)}} \quad z < 0 \] (3.10)

where \( N(E, 0) \) is the number density at the shock. Therefore, the total number of cosmic rays per unit area of shock front is

\[ \int_{-\infty}^{0} N(E, 0)e^{\frac{zu_1}{K_1(E)}} dz = \frac{N(E, 0)u_1}{K_1(E)}. \] (3.11)

Now the rate per unit area at which cosmic rays cross the shock is found by assuming an isotropic particle distribution, and is (Gaisser 1990)

\[ \int_{0}^{1} \cos \theta_i N(E, 0)c \frac{d}{2} d(\cos \theta_i) = \frac{cN(E, 0)}{4}. \] (3.12)

By combining Equation 3.11 and 3.12 the mean residence time in the upstream region is found to be

\[ \frac{(N(E, 0)u_1)/K_1(E)}{(N(E, 0)c)/4} = \frac{4K_1(E)}{u_1 c}. \] (3.13)

An analysis of the downstream region gives a similar result (Drury 1983) and the time to complete one acceleration cycle is

\[ T_{\text{cycle}} = \frac{4}{c} \left( \frac{K_1(E)}{u_1} + \frac{K_2(E)}{u_2} \right). \] (3.14)

The acceleration rate is defined as

\[ \frac{1}{T_{\text{acc}}(E)} = \frac{dE}{E dt} = \frac{\xi}{T_{\text{cycle}}} \] (3.15)

where \( \xi = \langle \Delta E \rangle / E \). Inserting Equation 3.7 and 3.14 into Equation 3.15, the acceleration rate of cosmic rays at a parallel shock is

\[ \frac{1}{T_{\text{acc}}(E)} = \frac{4 \frac{u_2}{c} - \frac{u_1}{4} \left( \frac{K_1(E)}{u_1} + \frac{K_2(E)}{u_2} \right)^{-1}} \]

\[ = \frac{(R - 1) u_1}{3R} \left( \frac{K_1(E)}{u_1} + \frac{K_2(E)}{u_2} \right)^{-1}. \] (3.16)
with $R = u_1/u_2$ the compression ratio. If the energy dependence of the diffusion coefficient in the upstream and downstream regions is $\propto E^{\alpha}$, the acceleration rate is also energy dependent and $\propto E^{-\alpha}$.

The discussion given above has described a simplified view of shock acceleration in which the cosmic rays do not influence the plasma flow in the upstream or downstream regions (the 'test particle' approximation), the plasma flow velocities are non-relativistic, and the shock has planar geometry with parallel magnetic fields. This is obviously a highly idealised picture, and hence some or all of these conditions may not hold in reality. In particular, recent calculations by, for example, Reynolds & Ellison (1992) and Ellison, Baring & Jones (1996) illustrate how the shock profile is modified when the cosmic rays interact with the plasma flows, and the results when oblique shocks are considered.

The cosmic ray acceleration rate obtained for the idealised picture of shock acceleration given in this Section is used in the next, together with a simple model for a cosmic ray accelerator, to calculate the spectrum of cosmic rays emerging from a source.

### 3.2.1.2 Simple Picture of Shock Acceleration

Using the description of shock acceleration given in the previous Section, I obtain the source spectrum of cosmic rays emerging from an accelerator using a simple model for a cosmic ray source. Also, I consider the modification to the source spectrum when physical factors (e.g., the finite size of the acceleration region) act to limit the maximum energy to which the cosmic rays can be accelerated. The form of the source spectrum to be used to construct the total primary source function is then given.

If it is assumed that there is no diffusion or convection of particles within the cosmic ray source, and that no loss processes other than escape from the source region operate, then the source may be visualised as a leaky box accelerator (Szabo & Protheroe 1994; see Figure 3.2). In this model, the propagation equation for particles inside the source is

$$\frac{dN}{dt} = \frac{\partial}{\partial E} [b(E)N] + Q(E,t) - \frac{N}{T_{\text{esc}}(E)} \quad (3.17)$$

where $N \equiv N(E,t)$, $b(E) \equiv -dE/dt$ describes the continuous energy gains and losses of particles within the acceleration region, $Q(E,t)$ is the particle source function, and $T_{\text{esc}}(E)$ is the energy dependent escape time for particles from the accelerator.

The Green’s function for Equation 3.17, $G(E,t;E',t')$ for some injection energy $E'$ and time $t'$, is readily found (e.g., Stecker 1971) to be
Leaky Box Accelerator

\[ \text{Acceleration rate: } \alpha E^{-\alpha} \]
\[ \text{Escape rate: } \alpha E^{-\alpha} \text{ or } E^{-\alpha} + E_{\text{max}}^{-\alpha} \]

\[ \begin{array}{c|c}
E_0 & \frac{N(E)}{T_{\text{esc}}(E)} \\
\end{array} \]

Figure 3.2: Simplified picture of a cosmic ray source, cosmic rays are injected with energy \( E_0 \), and emerge with spectrum \( N(E)/T_{\text{esc}}(E) \). Acceleration inside the accelerator is at a rate \( \propto E^{-\alpha} \), and cosmic rays escape at a rate either \( \propto E^{-\alpha} \) or \( E^{-\alpha} + E_{\text{max}}^{-\alpha} \) with \( E_{\text{max}} \) defined in the text.

\[ G(E, t; E', t') = \frac{1}{|b(E)|} \delta \left( t - t' - \int_E^{E'} \frac{dX}{|b(X)|} \right) e^{-\int_E^{E'} \frac{dX}{|b(X)| T_{\text{esc}}(X)}} \] \hspace{1cm} (3.18)

from which the solution for \( N(E, t) \) can immediately be written as

\[ N(E, t) = \frac{1}{|b(E)|} \int_E^\infty dE' Q \left( E', t - \int_E^{E'} \frac{dX}{|b(X)|} \right) e^{-\int_E^{E'} \frac{dX}{|b(X)| T_{\text{esc}}(X)}}. \] \hspace{1cm} (3.19)

Now, consider \( N_0 \) particles of a single energy \( E_0 \) injected into the accelerator. The resulting equilibrium spectrum of particles inside the accelerator is obtained by integrating the steady state form of Equation 3.19 as

\[ N(E) = \frac{N_0}{|b(E)|} \int_E^\infty dE' \delta \left( E_0 - E' \right) e^{-\int_E^{E'} \frac{dX}{|b(X)| T_{\text{esc}}(X)}} = \frac{N_0}{|b(E)|} e^{-\int_E^{E_0} \frac{dX}{|b(X)| T_{\text{esc}}(X)}} \] \hspace{1cm} (3.20)

The discussion below follows that of Protheroe & Stanev (1998), although they consider the more complicated problem of cosmic ray energy gains and losses while in the acceleration...
region, If the cosmic rays only gain energy while in the accelerator, the continuous energy gain/loss term in Equation 3.17 can be specified by noting that in the previous Section it was found the energy gain rate of shock accelerated cosmic rays is inversely proportional to the energy dependence of the diffusion coefficient (assuming, of course, the same energy dependence for the diffusion coefficient in the upstream and downstream region of the shock). Therefore, assuming the acceleration rate of cosmic rays to have the form

\[ \frac{1}{T_{\text{acc}}(E)} = a E^{-\alpha} \]  

(3.21)

and using the definition of the acceleration rate (Equation 3.15), the continuous energy gain/loss term has the form

\[ \frac{dE}{dt} = a E^{(1-\alpha)}. \]  

(3.22)

Now, if an escape rate of the form

\[ \frac{1}{T_{\text{esc}}(E)} = c E^{-\alpha} \]  

(3.23)

is assumed, the spectrum of particles inside the accelerator for this simple model is

\[ N(E) = \frac{N_0}{a E^{(1-\alpha)}} e^{-\int E_0^{E} \frac{dX}{X}} = \frac{N_0}{a E^{(1-\alpha)}} \left( \frac{E}{E_0} \right)^{-c/a}. \]  

(3.24)

Of interest here is the spectrum of particles actually escaping the acceleration region, which is

\[
\frac{N(E)}{T_{\text{esc}}(E)} = N_0 \frac{c E^{-\alpha}}{a E^{(1-\alpha)}} \left( \frac{E}{E_0} \right)^{-c/a} = N_0 \frac{1}{a E_0} \left( \frac{E}{E_0} \right)^{-c/a-1} = N_0 (\gamma - 1) \frac{1}{E_0} \left( \frac{E}{E_0} \right)^{-\gamma}
\]  

(3.25)

where \( \gamma = c/a + 1 \). When just the energy gain rate by shock acceleration, and energy dependent escape from the acceleration region are considered, the emergent spectrum of particles is a power-law in energy. Note that if \( c = a \), \( \gamma = 2 \) and \( N(E) \propto E^{-2} \), which is the standard result for test particle acceleration at strong shocks.
Equation 3.25 describes the emergent spectrum of particles from an accelerator assuming no processes operate to limit the energy to which cosmic rays can be accelerated. In reality this is not correct because cosmic rays gain energy through the acceleration mechanism, and lose energy (e.g. through synchrotron and inverse Compton cooling for electrons) while in the accelerator. Furthermore, even in the absence of energy losses, acceleration will cease due to the finite size of the acceleration region (e.g. the gyroradius of the cosmic rays becomes comparable to the characteristic size of the shock).

To illustrate the modification to the spectrum of particles escaping the accelerator because of the finite size of the source region, consider an escape rate of the form

$$\frac{1}{T_{\text{esc}}(E)} = c E^{-\alpha} + c E_{\text{max}}^{-\alpha}$$  \hspace{1cm} (3.26)

where $E_{\text{max}}$ is termed the ‘maximum’ particle energy, even though some particles will be accelerated to energies higher than this. Note that the inclusion of energy loss processes will further modify the emergent spectrum, but it is sufficient to consider only geometrical constraints for the purposes of this Section (see Protheroe & Staney 1998 for calculations where energy losses are included).

The emergent spectrum of cosmic rays for the escape rate given by Equation 3.26 is

$$\frac{N(E)}{T_{\text{esc}}(E)} = N_0 \frac{c E^{-\alpha} + c E_{\text{max}}^{-\alpha}}{a E^{(1-\alpha)}} e^{-\int_{E_0}^{E_{\text{max}}} \frac{c E^{-\alpha} + c E_{\text{max}}^{-\alpha}}{a X^{1-\alpha}} dX}$$

$$= N_0 \frac{1}{a E} \left(1 + \left[\frac{E}{E_{\text{max}}}\right]^{\alpha}\right) e^{-\frac{E}{a}} \int_E^{E_{\text{max}}} \frac{dX}{X^{1-\alpha}}$$

$$= N_0 \frac{1}{E_0} \left(1 + \left[\frac{E}{E_{\text{max}}}\right]^{\alpha}\right) \left(\frac{E}{E_0}\right)^{-\gamma/a} e^{-\frac{E}{a}} E_{\text{max}}^{-\alpha} (E_{\text{max}} - E_0)$$

$$= N_0 \frac{\gamma - 1}{E_0} \left(1 + \left[\frac{E}{E_{\text{max}}}\right]^{\alpha}\right) \left(\frac{E}{E_0}\right)^{-\gamma} e^{-\frac{E}{\gamma a} \left[\left(\frac{E}{E_{\text{max}}}\right)^{\alpha} + \left(\frac{E_{\text{max}}}{E_0}\right)^{\alpha}\right]}.  \hspace{1cm} (3.27)$$

The modification to the escape term results in an emergent particle spectrum displaying a cut-off which is exponential in nature. Simply by considering a limitation on the maximum energy of particles from the accelerator imposed by purely geometrical considerations, it can be seen the spectrum is significantly modified from the pure power-law expected by test particle shock acceleration theories. In particular, the energy dependence of the diffusion coefficient plays a significant role in determining the spectrum.

While Equation 3.27 shows simple geometrical considerations on the escape rate of particles from an accelerator leads to an exponential cut-off in the source spectrum, it is not
especially useful for the propagation calculations performed in later Chapters. This is because \( N(E) \) is dependent on particular details within a source (e.g., diffusion coefficient energy dependence) which are subject to a high level of uncertainty, or are not even determined at all. Furthermore, Equation 3.27 would not apply to the case where energy losses in the acceleration region dominate over the constant escape term near \( E_{\text{max}} \). Therefore, I have adopted the following forms for the source spectrum of primary cosmic rays in the Galaxy

\[
q(E) = \begin{cases} 
N_0 E^{-\gamma} e^{-\frac{E}{E_{\text{max}}}} & \text{(a)} \\
N_0 E^{-\gamma} H \left[ E_{\text{max}} - E \right] & \text{(b)} 
\end{cases}
\]  

(3.28)

where \( N_0 \) is some normalisation constant, and \( H \) is the Heaviside function and is 1 for \( E < E_{\text{max}} \) and 0 otherwise. Equation 3.28a is an approximation of Equation 3.27, while Equation 3.28b has an abrupt cut-off. The form for Equation 3.28b was chosen to illustrate how predictions of high energy electron spectra, and the diffuse photon spectra resulting from the electrons interacting with the magnetic and diffuse photon fields in the Galaxy, depend on the assumed cut-off in the electron injection spectrum. This is a method of demonstrating ‘in principle’ how other modifications to the primary cosmic ray source spectrum caused by, e.g., the inclusion of energy loss processes for particles inside the accelerator, could possibly be detected by observations of the diffuse high energy photon spectra of the Galaxy (see Chapter 5).

### 3.2.2 Spatial Distribution of Primary Sources

For the spatial distribution of primary sources I take the following functional form

\[
F_{\text{SNR}}(R, z) = \left( \frac{R}{R_0} \right)^A e^{-B \left( \frac{R - R_0}{R_0} \right)} H \left[ z; -z_d, z_d \right]
\]  

(3.29)

where \( A \) and \( B \) are parameters, \( R_0 = 8.5 \text{ kpc} \) (Kerr & Lynden-Bell 1986) is the radial distance from the Sun to the Galactic centre, \( H \left[ x; a, b \right] \) is the Heaviside function and is equal to 1 if \( a \leq x \leq b \) and 0 otherwise, and \( z_d \) is the half-thickness of the primary source distribution. This form is motivated by statistical studies of the radial distribution of SNR in the Galaxy (e.g., Kodaira 1974; Stecker & Jones 1977; Case & Bhattacharya 1996) that fit the radial part of Equation 3.29 to the observed distribution for some values of the parameters \( A \) and \( B \). The \( z \)-variation of the distribution is taken simply to reflect the localisation of most of the primary sources of cosmic rays in a thin disk about the galactic plane.
For the radial part of Equation 3.29, the study of the SNR remnant catalogue of Green (1996) by Case & Bhattacharya (1998) gives the most recent values for $A$ and $B$: $A = 2.00 \pm 0.67$ and $B = 3.53 \pm 0.77$. Earlier studies (e.g., Ilovaisky & Lequeux 1972a,b; Stecker & Jones 1977) give values for $A$ and $B$ that produce a distribution more peaked for $R \sim 4-6$ kpc than the results of Case & Bhattacharya (1998). For example, Stecker & Jones (1977) give values for $A$ and $B$ in Equation 3.29 of $A = 1.2$ and $B = 3.22$ based on the SNR study by Kodaira (1974). However, these studies were probably dominated by selection effects and this tended to bias the derived distribution towards that observed within $\sim 3$ kpc of the Sun, and did not accurately reflect the true distribution of SNRs (Case & Bhattacharya 1998). For the source disk half-thickness I take the value $z_d = 0.2$ kpc. This value can be compared with the $z$ distances from the plane of radio selected SNRs in the catalogue of Ilovaisky & Lequeux (1972a) and Shaver (1982) where only $\sim 10\%$ of SNRs are located at $z > 0.2$ kpc (see also Medina Tanco 1993).

![Figure 3.3: The radial distribution of cosmic ray sources in the Galaxy. Solid line: distribution of SNRs given by Case & Bhattacharya (1998); Dashed line: distribution of SNRs given by Stecker & Jones (1977); Dotted line: distribution of sources obtained by Strong & Moskalenko (1998); Dash-dotted histogram: gradient of cosmic rays derived from $\gamma$-ray data above 100 MeV (Strong & Mattox 1996).]
The radial distribution of SNRs in the Galaxy for the parameters of Case & Bhattacharya (1998) and Stecker & Jones (1977) is shown in Figure 3.3. The Stecker & Jones (1977) distribution is only shown to demonstrate the greater peak around \( \sim 4 - 6 \) kpc generally obtained in earlier studies, and is not used any further in this thesis. Also shown in the Figure is the radial profile of cosmic ray sources derived by Strong & Moskalenko (1998), and the emissivity gradient of diffuse \( \gamma \)-rays with energies \( E > 100 \) MeV, normalised to the local value, derived from an analysis of EGRET data (Strong & Mattox 1996); this will be used later when deriving the spatial distribution of secondary electron and positron sources (see Section 3.3.2). The Strong & Moskalenko (1998) distribution is based on propagation model calculations of the distribution of 3 GeV protons in the Galaxy, and is the required form to obtain consistency with the \( \gamma \)-ray emissivity above 100 MeV which traces the distribution of these particles in the Galaxy. The parameters in Equation 3.29 required to give the Strong & Moskalenko (1998) distribution are \( A = 0.5 \) and \( B = 1.0 \).

The difference between the distribution of Case & Bhattacharya (1998), and that of Strong & Moskalenko (1998), is most dramatic for \( R \) outside the solar circle, where the SNR distribution shows a fairly steep fall off with increasing \( R \). Radial profiles for the distribution of other likely sources of cosmic rays (e.g., pulsars, Dogiel & Uryson 1988; Taylor, Manchester & Lyne 1993; Johnston 1994) display even steeper fall offs for \( R \) outside the solar circle than the Case & Bhattacharya (1998) distribution. The discrepancy between the Case & Bhattacharya (1998) and Strong & Moskalenko (1998) distributions is therefore unlikely to be remedied by including other cosmic ray sources, and it seems the derived SNR distribution is not completely reflective of the true distribution (Strong & Moskalenko 1998). Because the distribution of Strong & Moskalenko (1998) is obtained by requiring consistency with the observed \( \gamma \)-ray emissivity gradient, it is probably a better indicator of the true distribution of cosmic ray sources. Therefore, I use this for the radial profile of primary sources throughout the remainder of this thesis.

### 3.2.3 Total Primary Electron Source Function

The total primary source distribution is constructed by combining the the source spectrum with the spatial distribution. Therefore, the primary electron source distribution is

\[
q(E, R, z) = F_{\text{SNR}}(R, z) \begin{cases} 
N_0 E^{-\gamma} e^{-\frac{E}{E_{\text{max}}}} & \text{(a)} \\
N_0 E^{-\gamma} H[E_{\text{max}} - E] & \text{(b)}
\end{cases}
\]  

(3.30)
where all terms have previously been defined.

3.3 Secondary Electron Source Distribution

Unlike ‘primary’ electrons directly accelerated in SNRs to relativistic energies out of the thermal pool, secondary cosmic ray electrons and positrons are created in the Galaxy as a result of some interaction process between high energy cosmic rays and some target population of particles. In this Section, the production spectrum and spatial distribution of secondary electrons and positrons in the Galaxy, used for calculations in later Chapters, is derived.

There are a variety of ways in which electrons and positrons can be produced through some secondary process. The process generally believed to generate the majority of secondary electrons and positrons is the interaction between cosmic ray nuclei and gas atoms in the ISM (e.g. Stecker 1971) In this process, the cosmic rays interact with the gas atoms and lose energy in an inelastic collision. The energy is transformed into secondary mesons, which can subsequently decay to electrons, positrons, neutrinos or γ-rays. This is the production mechanism for secondary electrons and positrons that will be used to calculate the electron and positron production spectra in this Chapter. Other processes that could generate electrons and positrons are interactions between high energy γ-rays and diffuse galactic starlight (Aharonian & Atoyan 1990; Mastichiadis, Protheroe & Stephens 1991), and the decay of exotic particles in the galactic halo (e.g. Rudaz & Stecker 1988; Kamionkowski & Turner 1991). However, these other processes are of a more speculative nature than production in inelastic collisions (e.g. $\gamma + \gamma \rightarrow e^+ + e^-$: Aharonian & Atoyan 1990; Mastichiadis, Protheroe & Stephens 1991), and they are not considered further in this thesis.

The production of electrons and positrons by collisions between cosmic ray nuclei and gas atoms at rest is discussed in the next Section. Then, the spatial distribution of secondary electron and positron sources in the Galaxy is derived assuming separability of the spatial and energy dependence of the cosmic ray proton spectrum (this assumption will be justified in Section 3.3.2). The total electron and positron source function is then given in Section 3.3.3.

3.3.1 Electron and Positron Production in Inelastic Collisions

The interstellar spectrum of electrons and positrons produced by inelastic collisions between cosmic ray nuclei and gas atoms at rest is calculated in this Section. The results of the
present calculations are carefully compared with the predictions of other authors, and a
detailed description of each stage of the production process is given.

In an inelastic collision between a cosmic ray nucleus and a proton at rest, the cosmic
ray loses a portion of its energy, which is converted to mesons. These subsequently decay
through a variety of decay chains to produce electrons, positrons, and other particles. For
the production of electrons and positrons, the most important process is the production and
decay of charged pions via the following decay chain

\[
p + p \rightarrow \pi^\pm + X \\
\downarrow \\
\mu^\pm + \nu_\mu/\bar{\nu}_\mu \\
\downarrow \\
\nu_e/\bar{\nu}_e + \nu_\mu/\bar{\nu}_\mu
\]

where \( X \) are the other products of the interaction process (e.g. the target and initial nucleus, 
other particles produced in the interaction). The production of electrons and positrons 
through this process has been considered by various authors (e.g. Orth & Buffington 1976; 
Protheroe 1982; Wong & Ng 1986; Murphy, Dermer & Ramaty 1987; Moskalenko & Strong 
1998a).

The decay chain shown above is a three stage process, and involves the adoption of an
interaction model to treat the production of pions in the first stage, a treatment of the decay 
of pions to muons, and then of the subsequent decay of muons to electrons. Generally, the
treatment of the first and last stage of decay chain are the points at which the calculations 
of various authors differ; the treatment of the decay of pions to muons is universal by all authors. For example, for the pion production step, different interaction models have been 
used to describe the production of mesons (e.g. Protheroe 1982; Dermer 1986a), while various 
approximations have been used for the treatment of the decay of muons to electrons (e.g. 

The essential ingredients to obtain a prediction of the interstellar production rate of 
secondary electrons and positrons from the above process are the production cross-section 
for electrons or positrons of energy \( E_e \) by protons with energy \( E_p \), and the interstellar proton 
spectrum. These are convolved together to obtain the production spectrum of secondary 
electrons and positrons. In Appendix D, the calculation of the energy dependent production
cross-section for the process $pp \rightarrow e^{\pm} X$ is described. A hadronic interaction model based on the work summarised by Dermer (1986a,b) is used for the meson production, and the appropriate particle distributions for each step of the decay chain are given. However, in this Section I am not only interested in comparing the final electron and positron production spectra results with the predictions of other authors, but also in identifying why the present calculations may differ from other results. Therefore, the intermediary results, such as the pion production spectra, must also be examined.

As a point of reference for this Section, I have used the predictions of Protheroe (1982) against which to compare my calculations. This is because, in the literature, the Protheroe (1982) work is considered to be the ‘benchmark’ prediction against which all others are generally compared. In the following, the cosmic ray proton spectra used to calculate the spectra of pions, muons, and electrons and positrons are given, and then the pion production step of the calculation is examined: this essentially compares the hadronic interaction models used to describe the meson production. The decay of pions to muons, and then electrons is examined. Finally, the production spectrum of secondary electrons and positrons incorporating contributions by other processes (e.g. $K^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}/\overline{\nu}_{\mu}$; to be discussed below) is calculated.

### 3.3.1.1 Cosmic Ray Proton Spectrum

As noted above, an essential ingredient for the prediction of production spectra for secondary electrons and positrons from cosmic ray $pp$ collisions in the ISM is the spectrum of incident protons. The spectra of cosmic ray protons used in calculations of the production spectra of secondary electrons and positrons (and also $\gamma$-rays; see Chapter 5) are given in this Section.

For a proper comparison between my calculations and those of Protheroe (1982), the same incident proton spectrum should be used. For reference, the proton spectrum used by Protheroe (1982) is shown in Figure 3.4, along with experimental data for the locally observed proton spectrum up to $\sim 10^{3.5}$ GeV (Ryan 1972; Smith 1973; Webber et al. 1987; Seo et al. 1991). Also shown in the Figure are another parameterisation of the locally observed spectrum due to Mori (1997) (this is designated the ‘median’ proton spectrum), and a range of proton spectra possibly representative of the proton spectrum outside the heliosphere also due to Mori (1997) (these are designated the ‘best fit’ spectra). The median and best fit spectra were used in calculations by Mori (1997) to fit the observed galactic diffuse $\gamma$-ray spectrum above $\sim 1$ GeV.
3.3. SECONDARY ELECTRON SOURCE DISTRIBUTION

Figure 3.4: Local interstellar proton spectra as a function of proton kinetic energy used to obtain the secondary electron production function. The two solid lines show the spectrum used by Protheroe (1982) with the range of the spectrum at low energies being due to uncertainties associated with the demodulation of the observed spectrum; dash-dot line shows the ‘median’ spectrum described by Mori (1997); dashed region shows the range of ‘best fit’ spectra of Mori (1997) to EGRET data. Experimental data are taken from Ryan (1972) (open boxes), Smith (1973) (open circles), Webber et al. (1987) (solid boxes) and Seo et al. (1991) (solid circles).

For energies below $\sim 0.5 - 1$ GeV there are considerable differences between the spectra shown in the Figure. From the point of view of pion production these differences are not important because the threshold proton total energy for the production of a neutral pion in the galactic rest frame (the ‘laboratory system’ hereafter LS) is 1.21 GeV, with the thresholds for charged pion production being slightly higher. At higher energies both the Protheroe (1982) and Mori (1997) median proton spectra are $\propto E_p^{2.75}$ with $E_p$ the proton energy, and differ by a factor $\sim 1.4$. As can be seen, the data and both parameterisations are consistent. The best fit proton spectra are bounded by spectra $\propto E_p^{2.41}$ and $E_p^{2.55}$.

The Mori (1997) median spectrum shown in the Figure will be used in this Chapter
to illustrate how the production spectrum of charged pions, and hence that of secondary electrons and positrons, is affected by the choice of incident proton spectrum, while the Mori (1997) best fit spectra will be used later when investigating the limits imposed on the cosmic ray proton spectrum outside the heliosphere by the locally observed positron spectrum. The median proton spectrum will also be used in Chapter 5 for the calculation of the emissivity of γ-rays from neutral pion decay.

From the point of view of comparing predictions by various authors of secondary electron and positron production spectra, the adoption of different parent proton spectra is an important point to consider. This is because various authors (e.g. Wong & Ng 1986) do not consistently use the same proton spectrum as Protheroe (1982) when comparing their calculations with his, hence making it difficult to ascertain the cause of differences in the predictions.

### 3.3.1.2 Charged Pion Production Spectra for pp Interactions

Ideally, a comparison of the interaction models used to describe the pion production would examine the predicted distributions of pions in the lab system (LS) given by the model of Protheroe (1982) and the present calculations, and compare with available accelerator data, in much the same as done by Dermer (1986b) for neutral pion production. Unfortunately, Protheroe (1982) does not give the LS distribution of pions for his model, but does give the spectrum of positive charged pions produced for his adopted cosmic ray spectrum. Therefore, I have calculated the spectra of charged pions for the interaction model described in Appendix D (hereafter, the hybrid model) using the Protheroe (1982) proton spectrum.

The production spectrum of charged pions at a LS energy $E_\pi$ can be obtained by convolving the incident proton spectrum with the pion production cross-section as

$$Q_{pp\to\pi^\pm X}(E_\pi) = \int_{E_p^{th}}^{\infty} J_p(E_p) \left\langle \zeta_{pp\to\pi^\pm X}(E_p) \right\rangle \frac{dN(E_\pi, E_p)}{dE_\pi} dE_p$$

(3.31)

where $J_p(E_p)$ (GeV$^{-1}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$) is the incident proton spectrum, $\left\langle \zeta_{pp\to\pi^\pm X}(E_p) \right\rangle$ (cm$^2$) is the multiplicity weighted cross-section for the production of either positive or negative charged pions at an incident proton energy $E_p$, and $dN(E_\pi, E_p)/dE_\pi$ (GeV$^{-1}$) is the LS pion distribution function. Expressions for the pion distribution functions and inclusive cross-sections are given in Sections D.1 and D.3 respectively of Appendix D. The threshold proton energy for either process, $E_p^{th}$, is found by equating $s^{1/2} = [2m_p(E_p^{th} + m_p)]^{1/2} =$
\[ m_X^2 + E_X^2 - m_{\pi^\pm}^2 \right)^{1/2} + E_{\pi}^* \], where \( E_{\pi}^* = \gamma_c (E_{\pi} - \beta_c p_{\pi}) \). Here, \( s \) is the square of the centre-of-momentum system (CMS) energy, \( m_p \) is the proton rest mass, \( E_{\pi}^* \) is the pion energy in the CMS, \( m_{\pi^\pm} \) is the charged pion rest mass, \( p_{\pi} \) is the pion momentum in the LS, \( \gamma_c = s^{1/2}/2m_p \) and \( \beta_c \) are, respectively, the Lorentz factor and velocity of the CMS with respect to the LS, and the mass parameter \( m_X \) depends on the reaction channel, as described in Section D.1 of Appendix D. The constraint on the threshold proton energy comes about because there must be enough energy available in the CMS to create the pion.

Figure 3.5: Production spectra of charged pions per interstellar H-atom. Thick solid lines and hatched regions show the charged pion production spectra obtained using the hybrid interaction model and the demodulated proton spectrum given by Protheroe (1982). Note the uncertainty in the pion production rates below \( \sim 3 \) GeV are due to the uncertainties in the demodulation of the incident proton spectrum. Thin solid lines show the \( \pi^+ \) production spectrum calculated by Protheroe (1982). Dash-dot curves show the charged pion spectra calculated as for the solid lines, but using the Mori (1997) median proton spectrum.

The spectra of charged pions in interstellar space from \( pp \) collisions calculated using Equation 3.31 for the Protheroe (1982) and Mori (1997) median proton spectra are shown in Figure 3.5. Note that the pion production spectrum originally given by Protheroe (1982)
included the effect of heavier nuclei in the cosmic ray spectrum, and an ISM composition of 90\% hydrogen and 10\% helium by number. This results in an enhancement of the charged pion production rate over the case of pure \( pp \) collisions that has variously been estimated by different authors to be a constant factor \( e^M \sim 1.45 \sim 1.60 \) (e.g. Stephens \& Badhwar 1981; Dermer 1986b) for cosmic ray energies below \( \sim 100 \) GeV. Therefore, for comparison purposes, I have converted the Protheroe (1982) pion production rate to an effective production rate from \( pp \) interactions by dividing by a factor 1.5 to account for heavier cosmic ray nuclei, and multiplying by a factor 1.4 to account for the ISM composition assumed by Protheroe (1982), and this is the production spectrum shown in the Figure. Although this is not exact (the full details of the total cosmic ray spectrum used by Protheroe (1982) are not given) it should be sufficient for the comparison in this Section.

For the same incident proton spectrum, it can be seen that the hybrid interaction model results in an enhanced pion production rate when compared with the Protheroe (1982) spectrum below \( \sim 2 \) GeV. Up to \( \sim 10 \) GeV the production rates are comparable, and above \( \sim 10 \) GeV the Protheroe (1982) calculation predicts more pions to be produced. The enhanced production rate at low energies for the hybrid model can be ascribed to the description of the hadronic interaction process at low incident proton energies provided by the isobar model described in Section D.1 of Appendix D, deuterium production \( (pp \rightarrow \pi^+d) \) near threshold, and presumably to different parameterisations of the Lorentz invariant cross-sections used in the hybrid model and by Protheroe (1982).

To illustrate these points more clearly I have calculated the \( \pi^+ \) production spectrum using an interaction model based on the scaling representation employed to describe pion production above incident proton energies of 7 GeV in the hybrid model. I have simply extended the scaling model to describe pion production from the threshold proton energy for \( \pi^+ \) production up to 7 GeV, and discarded the other components of the hybrid model. The effect of the isobaric portion of the hybrid model is clearly seen by comparing the \( \pi^+ \) spectra below \( \sim 3 \sim 4 \) GeV in Figures 3.5 and 3.6. Comparing the two different parameterisations of the invariant cross-section, at low energies both predict essentially the same production spectrum, while at high energies the Protheroe (1982) model predicts a production spectrum somewhat flatter than the present calculation. Both calculations actually predict the pion spectrum to be harder than the incident proton spectrum, although the spectral flattening in the case of the present calculation is obviously significantly less than the Protheroe (1982) model. Note the high energy predictions of the Protheroe (1982) model are similar to those
Figure 3.6: Production spectra of positive pions per interstellar H-atom. Thick solid lines and hatched regions show the \( \pi^+ \) production spectra obtained using only the scaling representation of the hybrid interaction model and the demodulated proton spectrum given by Protheroe (1982). Thin solid lines show the \( \pi^+ \) production spectrum calculated by Protheroe (1982).

of Orth & Buffington (1976) and Wong & Ng (1986) who both give pion spectra slightly flatter than the incident proton spectra used for the production spectra calculations. The greatly reduced hardening of the pion spectrum in the present calculations is in agreement with the calculations of Badhwar, Stephens & Golden (1977) who found that the production of hadrons scales for energies greater than \( \sim 20 \) GeV for their invariant cross-section parameterisation.

Returning to Figure 3.5, the sort of differences to be expected in the pion production predictions for reasonable incident cosmic ray proton spectra can be seen. For example, the \( \pi^+ \) production spectrum corresponding to the Mori (1997) median proton spectrum is approximately 40% higher than the prediction for the Protheroe (1982) proton spectrum for pion energies > 5 GeV. This directly reflects the factor \( \sim 1.4 \) difference for proton...
energies \(>10\) GeV between the two incident proton spectra. This is important from the point of view of making direct comparisons of the interaction models used to predict the charged pion, and hence the electron/positron, production rates because a consistent use of incident proton spectra must be employed. Furthermore, some criticism can be levelled at work by other authors (e.g. Wong & Ng 1986; Moskalenko & Strong 1998) who directly compare their predictions with those of Protheroe (1982), yet do not maintain consistency by using his proton spectrum essentially making it very difficult to properly attribute the cause of differences between the predictions. It will actually be shown later that the adoption of different incident proton spectra complicates the interpretation of the locally observed positron spectrum.

3.3.1.3 Electron and Positron Production Spectra for \(pp\) Interactions

Having examined the differences between the interaction models used for the charged pion production process, the muon spectrum resulting from the decay of the pions, and the treatment of the muon decay to produce electrons and positrons is now considered. The decay of pions to muons is a two body process, and the appropriate LS particle distribution is given in Section D.2 of Appendix D. The treatment of this step of the calculation, which is the second step in the decay chain shown earlier, is identical between the present work and the other authors referenced in this Section. However, the treatment of the decay of muons to electrons and positrons needs to be examined because of the methods different authors have used for this step of the calculation. For example, Ort\(h\) & Buffington (1976) use the same decay distribution in the muon rest frame for both electrons and positrons \((\xi = -1\) in Equation D.10), while Wong & Ng (1986) use the energy distribution given by Zatsepin & Kuzmin (1962). Protheroe (1982) treats the muon decay to electrons and positrons in the same way as Ort\(h\) & Buffington (1976). Because the decay distributions of electrons and positrons given by Equation D.10 are quite different, it is important to examine how these different treatments affect the predictions of the electron and positron production spectra.

The charged muons created at the second stage of the decay chain are fully polarised, which leads to an asymmetry in the decay to electrons and positrons, and results in a difference in the production spectra of secondary electrons and positrons. The importance of the muon decay asymmetry has been noted by Dermer (1986)a, and the cross-sections for the production of secondary electrons and positrons from collisions of monoenergetic protons have been presented by Murphy, Dermer & Ramaty (1987), and Moskalenko & Strong (1998).
Figure 3.7: Distributions of positrons from decaying muons produced via the $\pi^+ \rightarrow \mu^+ + \nu_\mu$ process. Upper thick solid band and line shows the production spectrum of positive pions obtained for the incident proton spectrum of Protheroe (1982). Upper thin solid lines show the production spectrum of positive pions calculated by Protheroe (1982). Lower thick solid band and line, and thick dashed lines, show the positron spectrum calculated using the thick solid pion spectrum for the cases of $\xi = +1$ and $\xi = -1$ in Equation D,10 respectively. Lower thin solid lines show the positron spectrum calculated by Protheroe (1982).

To compare the difference between the Protheroe (1982) treatment for the muon decay, and the present calculation, the secondary positron spectrum for different positron distributions in the muon rest frame has been calculated, and this is described below.

Due to the identical treatments of the second step in the decay chain by the present work and that of Protheroe (1982), a comparison of the muon spectra has been bypassed because this would only show the same characteristics as the pion spectrum in Figure 3.5. Instead, the secondary positron spectrum has been directly calculated for three different positron distributions in the muon rest frame: $\xi = +1, 0$ and -1 in Equation D,10.

The spectrum of positrons is calculated in a similar way to the charged pion spectrum, by convolving the incident proton spectrum together with the production cross section of
secondary positrons

\[ Q_{pp \rightarrow e+X} (E_e) = \int_{E_{p}^{\text{th}}}^{\infty} J_p(E_p) \frac{d\sigma_{pp \rightarrow e+X}(E_e, E_p)}{dE_e} dE_p \]  

(3.32)

where \( d\sigma_{pp \rightarrow e+X}(E_e, E_p) / dE_e \) (GeV\(^{-1}\) cm\(^2\)) is given by Equation D.22 in Appendix D. The threshold proton energy, \( E_{p}^{\text{th}} \), is obtained by substituting \( E_\pi = m_\pi \) for \( E_e \leq E \equiv \frac{1}{2} m_\mu \gamma_\mu'(1 + \beta_\mu') \), and \( E_\pi = \frac{1}{2} m_\pi (E_{c}/E_{c} + E/E_{c}) \) for \( E_{c} > E \) with \( \gamma_\mu' = (m_{\pi}^2 + m_{\mu}^2) / 2m_{\pi}m_{\mu} \approx 1.039 \) and \( \beta_\mu' \approx 0.2712 \) the Lorentz factor and velocity of the muon in the pion rest frame respectively, into the expression for determining the threshold energy in Equation 3.31.

The positron production spectrum calculated using Equation 3.32 for the Protheroe (1982) proton spectrum is shown in Figure 3.7. Also shown is the positive pion spectrum from Figure 3.5, and the positive pion and positron spectra of Protheroe (1982). The correction factor applied to the Protheroe (1982) pion spectra in the previous Section has also been used here to remove the effect of heavier nuclei, and the contribution of kaon decay modes has been removed using Equation 3.35 to yield an effective positron production rate from \( pp \) collisions only; this is what is shown in the Figure. For clarity, only the cases \( \xi = +1 \) and \( \xi = -1 \) for the positron distribution in the muon rest frame (Equation D.13) are shown in the Figure.

Although the integration over the positron distribution tends to smear out features, the relationship between the \( \xi = -1 \) curve and the Protheroe (1982) positron spectrum is reasonably similar to that between the pion spectra shown in the Figure. Above \( \sim 0.5 \) GeV, the difference between the \( \xi = +1 \) and \( \xi = -1 \) curves is a factor \( \sim 1.5 \). For an isotropic distribution of positrons in the muon rest frame (not shown), the positron spectrum is approximately half way between the \( \xi = +1 \) and \( \xi = -1 \) curves with a slight bias towards the \( \xi = +1 \) curve for energies higher than \( \sim 2 \) GeV.

The inclusion of the correct kinematics generally yields a higher positron production rate at high energies, but actually results in a lower spectrum below \( \sim 60 \) MeV than for the \( \xi = -1 \) case. This can be traced to the increased cross-section for the \( \xi = -1 \) versus \( \xi = +1 \) case below \( \sim 600 \) MeV (see Figure 3 of Moskalenko & Strong 1998 for positron production cross-sections for the three values of \( \xi \)), and to the fact that the threshold proton energy for positron production becomes constant for positrons with energies less than half the charged pion rest mass. The gradual change to a higher production rate for the correct kinematics is because the proton spectrum integration smears out the effect of the increased cross-section.
for the $\xi = -1$ case. The actual cause of the difference between the cross-sections for $\xi = +1$ and $\xi = -1$ is difficult to ascertain but is certainly related to the decay distribution functions given by Moskalenko & Strong (1998).

### 3.3.1.4 Total Secondary Electron and Positron Production Spectra

Having compared the present calculations of the secondary electron and positron cross-section (and intermediate cross-sections) with the predictions by other authors, I now calculate the electron and positron production spectra that will be used in subsequent Chapters.

The calculations given above are purely for pion production in $pp$ collisions. However, there are other nuclei present in cosmic rays and the ISM is not entirely composed of hydrogen. Additionally, processes other than pion production can contribute in a non-negligible way to the total electron and positron production rate. I briefly describe how these contributions are accounted for, and then the total electron and positron production spectra, including these effects, are given.

**The Effect of Heavier Nuclei**

The effect of nuclei other than protons in the cosmic ray spectrum and the ISM has been estimated by several authors to amount to a constant $\sim 1.45 - 1.7$ factor enhancement over the case of simple $pp$ interactions (e.g. Badhwar & Stephens 1981 or Dermer 1986b; see discussion in Section 3.3.1.2). This is sufficient for the production of electrons and positrons (and $\gamma$-rays; see Chapter 5) with GeV energies because the proton to helium ratio appears to be constant in the range 10–100 GeV. Recent observations indicate, however, that protons and heavier nuclei have differing spectral indices at high energies (e.g. Biermann, Gaisser & Stanev 1994; Wiebel-Sooth, Bierman & Meyer 1995). Following Mori (1997) I have obtained an energy dependence on the nuclear enhancement factor by assuming different spectral indices for protons, and other nuclei. I calculate the energy dependence of $e^M$ firstly by assuming that all nuclei heavier than protons can be described with a single spectral index, and then without this simplification by using the individual spectra for each spectral index, species from the summary given by Wiebel-Sooth, Biermann & Meyer (1998). Slightly pre-empting the results, it will be shown that both methods produce the same results, but it is useful to illustrate that the simplified method is just as good as a more complicated treatment.

Following Gaisser & Schaefer (1992) the nuclear enhancement factor can be written


\[ e^M(E_p) = \sum_i \left( \frac{J_i}{J_p}(E_p) \right) \frac{1}{2} \left[ w_{ip} \frac{\sigma_{ip}}{\sigma_{pp}} + w_{i\alpha} \frac{\sigma_{i\alpha}}{\sigma_{pp}} \left( \frac{n_{i\alpha}}{n_p} \right) \right] \]  

(3.33)

where a factor relating to propagation (only necessary for the case of anti-protons) has been dropped, and

\[ \left( \frac{J_i}{J_p}(E_p) \right) = \left( \frac{J_i}{J_p}(\text{GeV}) \right) \times \begin{cases} 
1 & i = p \text{ or } E_p \leq 100 \text{ GeV} \\
\left( \frac{E_p}{100 \text{ GeV}} \right)^{\gamma_p/\gamma_i} & \text{(otherwise)}
\end{cases} \]  

(3.34)

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<td>2.70</td>
</tr>
<tr>
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<td>111.0</td>
<td>2.64</td>
</tr>
<tr>
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<td>34.0</td>
<td>118.0</td>
<td>2.61</td>
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<tr>
<td>V</td>
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<td>36.0</td>
<td>124.0</td>
<td>2.63</td>
</tr>
<tr>
<td>Cr</td>
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<td>37.0</td>
<td>126.0</td>
<td>2.67</td>
</tr>
<tr>
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<td>133.0</td>
<td>2.46</td>
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<tr>
<td>Fe</td>
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<td>40.0</td>
<td>135.0</td>
<td>2.60</td>
</tr>
</tbody>
</table>

Table 3.1: Nuclear parameters for \( e^M \). Column 1 gives the nuclear species, with Column 2 the relative abundance of nuclei in cosmic rays, and Column 3 and 4 giving the multiplicity of pions in collisions between a nucleus \( i \) and protons (3) and helium (4). Column 5 gives the individual spectral indices, \( \gamma_i \), for each nuclear species.

where \( J_i/J_p(\text{GeV}) \) are the relative cosmic ray abundances in the GeV range and are given in Table 3.1 which is a reproduction of the relevant data given by Gaisser & Schaefer (1992),
3.3. SECONDARY ELECTRON SOURCE DISTRIBUTION

$\gamma_p$ and $\gamma_i$ are the spectral indices of protons and nuclei of type $i$ (also given in Table 3.1), $w_{ip}$ and $w_{i\alpha}$ are the total numbers of wounded nucleons in a collision between a nucleus $i$ and a proton or alpha particle, $\sigma_{pp}$, $\sigma_{ip}$ and $\sigma_{i\alpha}$ are the total inelastic cross-sections for a collision between two protons, a nucleus $i$ and a proton and a nucleus $i$ and an alpha particle respectively, and $n_{He}/n_H$ is the ratio of helium to hydrogen in the ISM.

Throughout this thesis the helium fraction in the ISM is assumed to be 90% hydrogen and 10% helium by number (e.g. Protheroe 1982; Strong & Moskalenko 1998). This can be compared with the ISM composition assumed by Gaisser & Schaefer (1992), and Mori (1997) of 93% hydrogen and 7% helium by number, recent photospheric determinations giving a helium fraction $0.10 \pm 0.008$ (Grevesse, Noels & Savuval 1996), and helioseismological methods which give a helium abundance $0.242 \pm 0.003$ by mass (Hernandez & Christensen-Dalsgaard 1994) corresponding to $0.075 \pm 0.001$ by number. Obviously there is some uncertainty in this ratio, but it seems reasonable to take the values I have because of the uncertainties associated with, e.g., the cosmic ray proton spectrum. This will have a greater bearing on the secondary electron and positron production rate than small differences in the assumed ISM composition.

The energy dependence of the nuclear enhancement factor calculated using Equation 3.33 is shown in Figure 3.8. Below 100 GeV $e^M$ takes the value 1.62, which differs from the result of Gaisser & Schaefer (1992) and Mori (1997). This is because I have assumed a slightly higher helium fraction in the ISM than by Gaisser & Schaefer (1992), and Mori (1997). Above 100 GeV $e^M$ rises to a value of 1.97 at 100 TeV, which is 7% higher than the result of Mori (1997). I calculated the energy dependence of $e^M$ for the same ISM composition as used by Mori (1997) and found that my results were the same as his. So, the difference in $e^M$ at all energies is entirely due to the different helium fraction used for my calculation. Hence, the approximation by Mori (1997) of the adoption of a constant spectral index for all nuclei heavier than protons gives the same results as using the different $\gamma_i$ for each nuclear species as given in Table 3.1.

Since it makes no difference which treatment is used, I employ the simplified method for the energy dependence of $e^M$.

**Inclusion of Kaon Decay Modes**

Processes other than the decay of charged pions also contribute to the total production rate of secondary electrons and positrons in the ISM. For example, charged and neutral
kaons (both short and long) contribute at a non-negligible level to the total production spectra of electrons and positrons. To estimate the contribution by the processes $K^\pm \to \mu^\pm \nu, \pi^\pm \pi^0, \pi^\pm \pi^\mp, \pi^0 e^\pm \nu, \mu^\pm \pi^0 \nu$, and $K^0_S \to \pi^+ \pi^-$, and $K^0_L \to \pi^\pm \mu^\mp \nu, \pi^\pm e^\mp \nu, \pi^+ \pi^- \pi^0$, I have used the formula given by Orth & Buffington (1976) based on a fit made by them to the results of their Monte Carlo model:

$$A_{K^\pm \to \pi^\pm} (E_e) = 1.10 + 0.05 \log (E_e/\text{GeV}).$$  \hfill (3.35)

where $A_{K^\pm \to \pi^\pm}$ is an energy dependent factor that multiplies the production function for electron/positron production from pion decays. This method for including the contribution by kaons was also used by Protheroe (1982) and Wong & Ng (1986).

Figure 3.9 compares the positron production spectra obtained by Moskalenko & Strong (1998) with and without a kaon decay mode contribution, and for the present calculation. The positron spectrum of Moskalenko & Strong (1998) that includes a kaon decay correction only includes the process $K^+ \to \mu^+ + \nu_{\mu}$. In the Figure, I have calculated the positron spectrum using the hybrid model for the Mori (1997) incident proton spectrum. I have used
the multiplicity weighted cross-sections for the process $pp \rightarrow \pi^+ X$ given by Dermer (1986a) to obtain the positron spectra shown in the Figure. Note that this is the same procedure as used by Moskalenko & Strong (1998), but is not the same as used for the other calculations given in this Section\(^1\).

As can be seen from the Figure, the kaon decay treatment of Moskalenko & Strong (1998) underestimates the positron production rate by 5–12% in the range 10–100 GeV when compared with the present calculation’s treatment of the kaon contribution. This

\(^1\)Formally, the production spectra at high energies should be obtained using the weighted cross-sections calculated for the scaling representation portion of the hybrid model.
is because neutral kaons contribute at approximately the same level as the charged kaon decay channels (Orth & Buffington 1976). It is therefore quite important to take the other processes into account when including the kaon decay contribution to the total production rate of secondary electrons and positrons.

**Total Production Spectra**

![Production spectra graph](image)

Figure 3.10: Production spectra of secondary electrons and positrons in interstellar space for the Protheroe (1982) proton spectrum. Thick solid lines show the positron (upper) and electron (lower) production spectra including kaon decay modes, and including the effect of heavier cosmic ray nuclei. Thin solid lines show the positron (upper) and electron (lower) production spectra for pp interactions only. Dashed lines show the positron production spectrum calculated by Protheroe (1982) including the effect of heavier cosmic ray nuclei.

The total production spectra of secondary electrons and positrons from inelastic collisions is now given. The total production spectra for both electrons and positrons are calculated using

$$Q_{\pm}(E_e) = A_{\pi^-\pi^0X}(E_e) \int_{E_p^h}^\infty J_p(E_p)c^M(E_p) \frac{d\sigma_{pp\rightarrow e^\pm}(E_e, E_p)}{dE_e} dE_p$$

(3.36)
Figure 3.11: Production spectra of secondary electrons and positrons in interstellar space calculated using the hybrid interaction model. Solid bands and lines are for the Protheroe (1982) proton spectrum, and include the effect of heavier nuclei and the kaon decay contribution. Dashed lines are for the Mori (1997) median proton spectrum, and include heavier nuclei and kaon decay contributions.

where all the terms have been defined before. Figure 3.10 shows the production spectra of secondary electrons and positrons calculated using Equation 3.36 for the Protheroe (1982) proton spectrum. Also shown are the production spectra for electrons and positrons for $pp$ collisions only, and for $pp$ collisions including the kaon decay contribution, and the production spectra calculated by Protheroe (1982).

The total positron production spectrum I have calculated, when compared with that of Protheroe (1982), is larger at all energies. This is because of the higher pion production rate predicted at low energies by the hybrid interaction model, the use of the proper muon decay distribution, and the treatment of the heavier nuclei contribution to the total production rate.

Figure 3.11 shows the total electron and positron production spectra calculated using
the hybrid interaction model for both the Protheroe (1982) and Mori (1997) median proton spectra. These production spectra will be used in the following Chapters when making propagation model predictions of the electron and positron spectra in the Galaxy.

3.3.2 Spatial Distribution of Secondary Electron and Positron Sources

In this Section, I construct the spatial dependence of the secondary electron and positron source distribution. This will also be used in Chapter 5 for the spatial dependence of γ-rays from the decay of neutral pions produced in inelastic collisions in the ISM.

The spatial distribution of the source function depends upon the spatial distribution of cosmic ray nucleons in the Galaxy, and the distribution of the target gas. For the spatial distribution of cosmic ray nucleons, instead of performing a propagation calculation to obtain the (interrelated) spatial and energy distributions, I simply assume that the spatial and energy dependences are separable. This assumption essentially means I do not consider that protons (and heavier nuclei) lose energy when propagating in the Galaxy through ionisation and Coulomb interactions, or that catastrophic energy losses (e.g., fragmentation) for heavier nuclei occur. These are obviously not realistic assumptions because, for example, the fragmentation of cosmic ray nuclei is a well established fact. However, they are reasonable approximations to make for the purposes of this thesis because only cosmic ray protons with energies higher than ~ 1 GeV contribute to the production of secondary electrons and positrons (and γ-rays) in the Galaxy. Therefore, any features in the proton (and heavier nuclei) spectrum at low energies due to ionisation and Coulomb interactions will not significantly contribute to the production of electrons and positrons. The problem of the effect of catastrophic losses for heavier nuclei does not need to be considered because I use Equation 3.33 to treat the contribution by these cosmic rays to the production rate of secondary particles.

The distribution of cosmic ray protons in the Galaxy may therefore be written

\[ J_p(E_p, R, z) = J_p(E_p)G(R, z) \]  \hspace{1cm} (3.37)

where \( G(R, z) \) is a dimensionless function describing the spatial variation of the proton density throughout the Galaxy. For \( G \), I use

\[ G(R, z) = F_{E > 100 \text{ MeV}}(R) \left( 1 - \frac{|z|}{z_h} \right) \]  \hspace{1cm} (3.38)
with $F_{E>100 \text{ MeV}}(R)$ given by the dash-dotted histogram in Figure 3.3, and $z_h$ is the height of the cosmic ray ‘halo’ beyond which it is assumed the cosmic ray flux is negligible. This form of the radial distribution of cosmic ray nucleons is taken because the integral galactic $\gamma$-ray flux above $\sim 100$ MeV is dominated by photons from neutral pion decays produced in inelastic collisions between cosmic ray nuclei and gas in the ISM. Hence the radial distribution shown in Figure 3.3, which is derived from an analysis of galactic diffuse $\gamma$-rays observed by EGRET (Strong & Mattox 1996), traces the distribution of cosmic ray nucleons in the Galaxy. The $z$ distribution is motivated by the results of propagation calculations by Bloemen et al. (1993) who give a linear decrease to zero for the cosmic ray flux perpendicular to the galactic plane for particles, such as high energy protons, unaffected by significant radiative energy losses.

The total distribution function describing the spatial extent of the secondary electron and positron sources is obtained by combining the spatial distribution of cosmic ray nucleons (Equation 3.38) with the gas distribution as

$$n_{e,\pm}(R, z) = G(R, z) \left[ n_{\text{HI}}(R, z) + n_{\text{H}_2}(R, z) + n_{\text{HII}}(R, z) \right]$$

(3.39)

where $n_{\text{HI}}(R, z) \text{ (cm}^{-3})$, $n_{\text{H}_2}(R, z) \text{ (cm}^{-3})$, and $n_{\text{HII}}(R, z) \text{ (cm}^{-3})$ are the spatial distributions of neutral, molecular and ionised hydrogen respectively in the Galaxy. The models for all three gas distributions are given in Section A.1 of Appendix A.

The combined spatial distribution of secondary electron and positron sources in the Galaxy given by Equation 3.39 is shown in Figure 3.12. The rather ‘jagged’ variation with $R$ of the spatial distribution function is due to the histogram nature of the gas and nucleon distribution functions.

### 3.3.3 Total Secondary Electron and Positron Source Function

Because of the assumed separability in energy and spatial coordinates of the cosmic ray nucleon spectrum, the total secondary electron and positron source function is obtained by combining the production spectra calculated in Section 3.3.1.4 with the spatial distribution given in Section 3.3.2. Thus,

$$Q_{e,\pm}(E_e, R, z) = Q_{e,\pm}(E_e) F_{E>100 \text{ MeV}}(R) \left( 1 - \frac{|z|}{z_h} \right) \left[ n_{\text{HI}}(R, z) + n_{\text{H}_2}(R, z) + n_{\text{HII}}(R, z) \right]$$

(3.40)
Figure 3.12: Spatial source distribution of electron and positrons produced by cosmic ray interactions with gas in the ISM. The source distribution is obtained by convolving the assumed distribution of cosmic ray nuclei with the gas distribution model given in Section A.1 of Appendix A.

where $Q_{\pm}(E_e)$ is given by Equation 3.36, and all the terms relating to the spatial distribution have been defined in Section 3.3.2.

### 3.4 Summary

The source functions for primary cosmic ray electrons, and secondary electrons and positrons, have been derived in this Chapter.

The primary source function has been obtained by combining the distribution of SNRs, with power-law source spectra. The SNR distribution used is based on propagation model studies of the cosmic ray proton distribution, and not the actual observed distribution which may be affected by observational biases, and other affects. The source spectra have been calculated for the first order Fermi process operating at a parallel shock in a simple model
of a cosmic ray accelerator. By introducing simple geometrical limitations on the maximum attainable energy of particles in the accelerator, it has been shown how the cosmic ray spectrum emerging from the accelerator has a cut-off at high energies. The actual source spectra used to construct the primary source function are approximations of the emerging particle spectrum from the accelerator. This is because the actual details inside the accelerator are not well determined.

The secondary source function has been obtained by assuming separability between the energy and spatial dependence of the cosmic ray proton spectrum. Secondary electrons and positrons are assumed to be produced by inelastic collisions between cosmic ray nuclei and gas in the ISM. I have compared in detail the aspects of my calculation with those of other authors to ascertain how differences arise between predictions of the secondary production spectra. I have found that the adoption of different incident proton spectra significantly affects the predicted production spectra at high energies. The correct decay distributions of intermediate particles in the decay chain also result in significant differences, when compared with the results of earlier authors. The spatial distribution of secondary sources was then constructed by convolving the distribution of cosmic ray protons, given by the observed diffuse γ-ray emissivity gradient for $E_\gamma > 100$ MeV, with the model of the galactic gas distribution.
Chapter 4

Propagation of Galactic Cosmic Ray Nuclei

4.1 Introduction

Observations of cosmic ray nuclei incident at the top of the Earth’s atmosphere, and in interplanetary space, can be used to deduce information about their propagation and confinement in the Galaxy. Studies of stable cosmic ray isotopic ratios, e.g. B/C and sub-iron/Fe, provide information on the total amount of matter (the ‘grammage’) traversed by cosmic rays during their confinement. On the other hand the abundances of radioactive cosmic ray species produced from the fragmentation of heavier cosmic ray species in the ISM, for example $^{10}$Be, $^{26}$Al, or $^{36}$Cl, are related to the mean age of the parent cosmic rays. The interpretation of these abundances is dependent on the assumed model for describing the cosmic ray propagation. For example, the homogeneous model, which is a model of uniform production and propagation of cosmic rays over the galactic confinement volume and is the conventional model used to interpret these cosmic ray abundances, leads to a different physical picture than that provided by halo models (Ginzburg, Khazan, & Ptuskin 1980). The former class of models generally show a lower average matter density sampled by cosmic rays during their confinement than the local galactic hydrogen density. This can be interpreted as implying cosmic rays propagate throughout an extended halo region (e.g. Lukasiak et al. 1994a), but homogeneous models do not actually provide meaningful estimates of the actual size for such a confinement volume (e.g. Strong & Moskalenko 1998). Generally, more complicated
models incorporating cosmic rays diffusing in some extended halo region offer a more realistic picture of the transport of cosmic rays in the Galaxy (e.g. Berezinskii et al. 1990) than is afforded by homogeneous models, and I consider the limits imposed on such models by observations of cosmic ray nuclei in this Chapter.

In this Chapter, I use a diffusion-convection model to describe the propagation of cosmic ray nuclei in the Galaxy. My primary goal is to constrain the parameters of this propagation model using information deduced from experimental cosmic ray abundance data. The model parameters found in this way will then be used in the next Chapter in propagation calculations for cosmic ray electrons and positrons.

I use analytical methods to treat the propagation of cosmic ray nuclei. Therefore, for simplicity, I use the one-dimensional (1D) form of the diffusion-convection model described in Chapter 2 instead of a fully three-dimensional (3D) model. This is a sufficient approximation to make for cosmic rays (such as nuclei) that are not significantly affected by severe radiative energy losses provided the size of the halo is significantly smaller than the maximum radial dimension of the Galaxy. In the next Chapter, I use a cylindrically symmetric 3D model with maximum radius $R_{\text{max}} = 20$ kpc for the electron and positron propagation calculations. So, slightly anticipating the limits on the halo size to be obtained later in this Chapter, the ratio of the halo size to the maximum radius is always less than $\sim 0.3$. Therefore, it is reasonable to use the 1D approximation to treat the propagation of cosmic ray nuclei.

The 1D diffusion-convection model is described in the next Section. The propagation equation for cosmic ray nuclei in this model is given and the importance of convection as a transport mechanism is discussed. The parameters of the model are then constrained using experimental cosmic ray abundance data.

4.2 The 1D Diffusion-Convection Model

cosmic ray nuclei within this class of models; the papers of other authors deal with cosmic ray electrons. Several useful results have come from these studies, and some of these will be used below.

The propagation equation for stable cosmic ray nuclei in the 1D diffusion-convection model is given next, along with a basic description of the propagation geometry. The importance of the convection for cosmic ray transport in the Galaxy is then discussed, and it is shown that convection is (at best) only a very weak influence on the cosmic ray propagation. Then, the parameters appropriate for the diffusion limit of the model, such as the halo size and diffusion coefficient, are constrained using cosmic ray abundance data.

### 4.2.1 Propagation Equation

The propagation of cosmic ray nuclei in the 1D diffusion-convection model may be described by (see Equation 2.1)

\[
-\frac{\partial}{\partial z} \left( K(E) \frac{\partial}{\partial z} - V(z) \right) N(E, z) + \frac{\partial}{\partial E} \left[ b(E, z) + \frac{1}{3} \frac{dV(z)}{dz} E \right] N(E, z)
\]

\[
- \left[ \sum_{s=\text{H,He}} n_s(z) \sigma_s(E) \beta c + \frac{1}{\tau_r} \right] N(E, z) + Q(E, z) = 0
\]

where \( K(E) \) (cm² s⁻¹) is the diffusion coefficient, \( N(E, z) \) (GeV⁻¹ cm⁻³) is the cosmic ray number density, \( V(z) \) (cm s⁻¹) is the velocity of the bulk motion of the ISM, \( b(E, z) \) (GeV s⁻¹) is the energy loss rate and the second term in square brackets describes adiabatic energy losses due to the convective outflow, \( n_s(z) \) (cm⁻³) and \( \sigma_s(E) \) (cm²) are the number densities and interaction cross-sections of cosmic rays with a gaseous species \( s \) in the ISM, \( \tau_r \) is the time-dilated lifetime for cosmic rays against radioactive decay (for stable cosmic ray nuclei \( \tau_r \to \infty \)), and \( Q(E, z) \) (GeV⁻¹ cm⁻³ s⁻¹) is the source function whose form for primary and secondary cosmic ray nuclei was discussed in Chapter 2.

The geometry of the basic model is illustrated in Figure 4.1. In this model, the cosmic ray sources are located in a thin region of half-thickness \( z_d \) symmetric about the galactic plane. This is termed the ‘disk’ region. It is assumed that the gas is also concentrated in the disk region, thus both primary and secondary cosmic rays are produced in the disk. After being injected in the disk region, the cosmic rays propagate within an extended confinement region of half-height \( z_h \) symmetric about the plane (the ‘halo’). In the disk, cosmic rays diffuse freely and hence their transport in this region is solely governed by the diffusion coefficient.
Outside the disk cosmic rays diffuse, and a convective outflow operates to transport particles away from the disk. Thus, in the halo the cosmic ray transport is governed by the diffusion coefficient and convection velocity. The cosmic rays are free to propagate throughout the disk and halo region until they reach ±zh where they escape to intergalactic space. Note that in the Figure the source function, gas density, diffusion coefficient, and convection velocity are the same in the halo region above and below the galactic plane.

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</table>

Figure 4.1: Geometry of the 1D diffusion-convection model described in the text.

### 4.2.2 The Importance of Convection

The possible role of convection was pointed out by Jokipii (1976), and early calculations for cosmic ray nuclei and electrons incorporating convective transport were performed by Owens & Jokipii (1977a,b). Later analyses of the non-thermal radio emission of the Galaxy using diffusion-convection models to determine the electron distribution (Dogiel et al. 1980,1981; Lerche & Schlickeiser 1980,1981) arrived at differing conclusions: Dogiel et al. (1980,1981) found that diffusion was the dominant transport mechanism, while Lerche & Schlickeiser (1980,1981) found the opposite, and obtained the convection velocity in the halo to be \( V \approx 8 \) km s\(^{-1}\). Later work by Webber, Lee & Gupta (1992), and Bloemen et al. (1993) focussed
on cosmic ray nuclei. The general consensus of these papers was that convection could only be an important transport mechanism for halo sizes \(< 2\) kpc. A key point of both of these papers was that the size of the halo and the strength of convection in the halo are interrelated, such that, as the halo size increases, the convection velocity diminishes. Specifically, for the Webber, Lee & Gupta (1992) calculations as \(z_h \rightarrow 4\) kpc, \(V \rightarrow 0\) km s\(^{-1}\). Given that the respective analyses in these papers of cosmic ray abundance ratios tended to favour halo sizes \(~ 3 – 4\) kpc, both sets of authors concluded that convection would only play a weak role transporting cosmic rays in the Galaxy. However, most of these calculations focussed on an analytical treatment of the nuclear (or electron) propagation equation, and none explicitly treated the energy losses by cosmic ray nuclei during propagation. Recently, Strong & Moskalenko (1998) have also reconsidered the question of the importance of convection from the point of view of cosmic ray nuclei using a detailed three-dimensional (3D) diffusion-convection (and diffusion-reacceleration) model incorporating a realistic gas distribution, and nucleon energy losses. In a detailed analysis of cosmic ray secondary-to-primary ratios, and the abundance of the cosmic ray radioactive isotope \(^{10}\)Be, they find the halo is limited to be \(4 \leq z_h \leq 12\) kpc for a diffusion model. For the convective case, Strong & Moskalenko (1998) prefer to impose limits on the adiabatic energy loss term in Equation 4.1. For a constant linear increase of \(V\) with \(z\) they find \(0 \leq dV/dz \leq 7\) km s\(^{-1}\) kpc\(^{-1}\), with a non-zero wind velocity only allowed for \(z_h > 4\) kpc.

The model calculations of Webber, Lee & Gupta (1992), Bloemen et al. (1993), and Strong & Moskalenko (1998) overlap for halo sizes \(~ 4\) kpc. For halo sizes \(~ 4\) kpc, the predictions of each set of authors favours convection velocities \(V \rightarrow 0\). Even though the treatments of the propagation of nuclei differ between the respective papers, it seems the implication is that convection does not play an important role in the transport of cosmic ray nuclei in the Galaxy. Although there appears to be a general consensus amongst the predictions for cosmic ray nuclei, it would be useful if some independent method could be used to constrain the halo size and hence determine the importance of convection. For this, I turn to the scale height of the non-thermal emission of cosmic ray electrons in the galactic magnetic field. From observations of the galactic non-thermal emission, Beuermann, Kanbach & Berkhuijsen (1985) find the scale height of the galactic electron distribution is \(3.6 \pm 0.4\) kpc, which corresponds well with a halo size \(~ 4\) kpc as found from the nucleon propagation studies above. Therefore, it is reasonable to conclude that convection is unimportant as a cosmic ray transport mechanism. On this basis I have not considered convection in the propagation.
model calculations presented later in this thesis.

4.2.3 Setting K and $z_h$ in the Diffusion Model

The model parameters for the diffusion model are constrained in this Section. For this, I use the analytical results of Freedman et al. (1980) who solved Equation 4.1 to obtain expressions for the grammage and surviving fraction of radionuclei (see below), together with cosmic ray abundance data to impose limits on the range of allowable halo sizes and diffusion coefficients. Note that in the following summary of formulae I have retained the most general case, as derived by Freedman et al. (1980), where convection is included. However, for the calculations of the model parameters I take the diffusion limit of Equation 4.1 to obtain parameters for use in propagation calculations in the next Chapter. Also, Freedman et al. (1980) considered the possibility of differing diffusion coefficients in the disk and halo regions; this is not considered here, and the diffusion coefficient is assumed to be the same in the disk and halo regions of the Galaxy.

In the diffusion-convection model, the grammage is given by the expression

$$X(E, \epsilon) = \frac{\rho z_d^2 \beta c}{K(E)} F(\epsilon)$$

(4.2)

where $\rho = (n_H m_H + n_{He} m_{He})$ (g cm$^{-3}$) is the matter density with $n_H$ and $n_{He}$, and $m_H$ and $m_{He}$, the number densities and masses of hydrogen and helium respectively, $z_d$ (cm) is the disk half-thickness, $\beta c$ (cm s$^{-1}$) is the velocity of the cosmic rays, $K(E)$ (cm$^2$ s$^{-1}$) is the diffusion coefficient, and $F(\epsilon)$ is the function

$$F(\epsilon) = \frac{1}{\epsilon} + \frac{1}{3} + \frac{\epsilon}{24 + 12\epsilon}$$

(4.3)

where the parameter $\epsilon$ is defined below. The relationship between convection and diffusion in this model comes about through the evaluation of $\epsilon$, which is given as

$$\epsilon = \frac{Q}{\beta Y} \left( \left[1 - e^{-[n \cdot \frac{2}{3}]^\frac{1}{\gamma}} \right]^{-1} + \frac{\gamma - 1}{3} \right)$$

(4.4)

where $Q = \frac{z_h V}{K}$, $Y = \frac{z_h}{z_d}$ and $\gamma$ is the spectral index of the cosmic ray sources. Conventionally, the parameter $Q$ is used to describe the propagation model as either diffusion dominated, $Q \ll 1$, or convection dominated, $Q \gg 1$. In a diffusion dominated model, $Q \ll 1$, and $V \to 0$. So using
\[
\frac{e^x}{e^x - 1} = \frac{1}{x + x^2/2 + \ldots} + \frac{1}{1 + x/2 + \ldots} + \frac{x/2}{1 + x/2 + \ldots}
\] (4.5)

Equation 4.4 reduces to

\[
e = \frac{z_d}{z_h - z_d},
\] (4.6)

and therefore the grammage (Equation 4.2) is

\[
X(E) = \frac{\rho z_d z_h \beta c}{K(E)} \left(1 + \frac{z_d}{z_h} \left[\frac{z_d}{24 z_h} - \frac{2}{3}\right]\right).
\] (4.7)

Note that for \(z_d \ll z_h\), which is usually the case, Equation 4.7 gives the same result as derived by Bloemen et al. (1993).

The other quantity of interest that can be derived is the surviving fraction of radioactive nuclei, \(f_s\). This is the ratio of the observed amount of a cosmic ray radioactive isotope to that expected if no decay occurs. In this model the surviving fraction of a radioactive cosmic ray species is given by the expression

\[
f_s = \frac{X_r}{X_r + X_i} \left[1 - \frac{\cosh(A_1) + Y \frac{A_1}{A_3} \sinh(A_1)}{1 - \left[\cosh(A_3) + \frac{A_1}{A_3} \sinh(A_3)\right]}\right]^{-1}
\] (4.8)

where

\[
A_1 = \frac{X}{F(\epsilon)} \left(\frac{1}{X_i} + \frac{1}{X_r}\right)
\] (4.9)

\[
A_2 = \left(\frac{1 + 2\gamma}{6}\right) \frac{Q}{\beta} + Y \sqrt{W} \coth \left\{(Y - 1)\sqrt{W}\right\}
\] (4.10)

\[
A_3 = \frac{X}{F(\epsilon) X_i}
\] (4.11)

with \(W = X/F X_r + (Q/2\beta Y)^2\), \(X\) given by Equation 4.2, \(F \equiv F(\epsilon)\), and \(Q\) and \(Y\) are defined above. The parameters \(X_i\) and \(X_r = \rho \beta c \tau_r\) are the interaction mean free path of the radioactive nuclei and the column density of gas traversed at velocity \(\beta c\) by a radioactive nucleus during its time dilated lifetime \(\tau_r = \gamma \tau_{1/2}/\ln 2\) with \(\tau_{1/2}\) the half-life against radioactive decay, respectively.

As can be seen from examination of Equation 4.7, the grammage alone only gives the ratio \(z_h/K\); the surviving fraction, \(f_s\), must be used to uniquely fix \(z_h\) and hence \(K\). Therefore, I now use formulae for \(X\) and \(f_s\) given above to constrain the diffusion coefficient and halo size
for the diffusion dominated limit of Equation 4.1. The procedure is to first fix the grammage, and then find values for the diffusion coefficient and halo size that correspond to the observed surviving fractions of radionuclei; for this purpose I use the measured values of $f_s$ for $^{10}$Be and $^{26}$Al given by Lukasiak et al. (1994a,b).

I assume a grammage of the form

$$X(E)/(g \text{ cm}^{-2}) = \begin{cases} 
12.5\beta & (r \leq 4.7 \text{ GV}) \\
12.5\beta(r/4.7 \text{ GV})^{-0.60} & (r > 4.7 \text{ GV})
\end{cases} \quad (4.12)$$

which is based on an analysis of cosmic ray secondary-to-primary ratios for an ISM composition similar to that assumed in this thesis (Webber et al. 1996). Here $r$ is the particle rigidity, $r = pc/Ze$, with $p = [E^2 - m^2c^4]^{1/2}/c$ the particle momentum and $Ze$ the particle charge, and $\beta = v/c$. The diffusion coefficient then has the form

$$K(E) = \begin{cases} 
K_0 & (r \leq 4.7 \text{ GV}) \\
K_0(r/4.7 \text{ GV})^{-0.60} & (r > 4.7 \text{ GV})
\end{cases} \quad (4.13)$$

where $K_0$ (cm$^2$ s$^{-1}$) is a normalisation constant to be determined. To fix the average matter density and disk half-height in Equation 4.2, I note that the product of the two is equivalent to half of the gas column density perpendicular to the galactic plane (recall that the gas is assumed to be concentrated wholly within the disk in this model). For the gas model described in Appendix A, the column density perpendicular to the galactic plane is $\sigma_{HI} = 1.25 \times 10^{21}$ H-atoms cm$^{-2}$. Thus, the average density of hydrogen atoms in this model is $n_H = 1.01$ cm$^{-3}$ where I have used $z_d = 200$ pc, the half-thickness of the primary source distribution from Section 3.2.

To determine $f_s$ for the cosmic ray radionuclei considered here, the value of $X_i$ must be specified for each. This is found by considering the interaction rate of a cosmic ray species, $M$, with gas in the ISM

$$\frac{1}{T_i} = (n_H \sigma_{HI-M} + n_{He} \sigma_{He-M}) \beta c \quad (4.14)$$

where $n_H$ (cm$^{-3}$) and $n_{He}$ (cm$^{-3}$) are the number densities of hydrogen and helium, $\sigma_{HI-M}$ (cm$^2$) and $\sigma_{He-M}$ (cm$^2$) are the fragmentation cross-sections for species $M$ on hydrogen and helium targets, and $\beta c$ (cm s$^{-1}$) is the velocity of the cosmic rays. The interaction mean free path is then simply
\[ X_i = \left( n_H m_H + n_{He} m_{He} \right) \beta c T_i \]
\[
\simeq \frac{1.44 m_H}{\sigma_{iH-M} \left( 1 + 0.11 \frac{\sigma_{iHe-M}}{\sigma_{iH-M}} \right)} \tag{4.15}
\]

for an ISM composed of 90\% hydrogen and 10\% helium by number, as assumed in this thesis.

For the interaction cross-sections on hydrogen targets I use the values given by Webber, Lee & Gupta (1992), based on the measured and parametric cross-sections of Webber, Kish & Schrier (1990a,b,c), for \(^{10}\text{Be}\) and \(^{26}\text{Al}\): \(\sigma_{iH-^{10}\text{Be}} = 212 \text{ mb} \) and \(\sigma_{iH-^{26}\text{Al}} = 434 \text{ mb} \). The interaction cross-sections on helium targets are obtained using the empirical formula given by Ferrando et al. (1988)

\[
\frac{\sigma_{iHe-M}}{\sigma_{iH-M}} = a A^{-\eta} \tag{4.16}
\]

where \(A\) is the mass number of the species \(M\), and \(a = 2.10 \pm 0.10\) and \(\eta = 0.055 \pm 0.013\) (Ferrando et al. 1988). For the assumed ISM composition, the interaction mean free paths for \(^{10}\text{Be}\) and \(^{26}\text{Al}\) are \(X_i \simeq 9.44 \text{ (g cm}^2\text{)}\) and \(X_i \simeq 4.65 \text{ (g cm}^2\text{)}\) respectively. I tested the sensitivity of these parameters to the values of \(a\) and \(\eta\) in Equation 4.15. As expected, it was found that the values of \(X_i\) differed only slightly (less than \(\sim 0.1 \text{ g cm}^2\)) for all combinations of \(a\) and \(\eta\) allowed within the error bars of each parameter.

Using the values for \(X_i\) obtained above, the half-lives against radioactive decay for \(^{10}\text{Be}\) (\(\tau_{1/2} = 1.6 \times 10^6 \text{ yr}\)) and \(^{26}\text{Al}\) (\(\tau_{1/2} = 8.7 \times 10^5 \text{ yr}\)) (e.g. Simpson & Connell 1998; Connell 1998), and Equations 4.7 and 4.8, the diffusion coefficient normalisation and halo size determined for a surviving fraction of \(^{10}\text{Be}\) of 0.185\(^{+0.071}_{-0.069}\) (Lukasiak et al. 1994a) are \(K_0 = 7.7^{+13.4}_{-1.5} \times 10^{28} \text{ cm}^2 \text{ s}^{-1}\) and \(z_h = 7.6^{+12.0}_{-4.0} \text{ kpc}\) respectively. For a surviving fraction of \(^{26}\text{Al}\) of 0.32 \pm 0.09 (Lukasiak et al. 1994b), I obtain the values \(K_0 = 2.1^{+3.3}_{-1.4} \times 10^{28} \text{ cm}^2 \text{ s}^{-1}\), and \(z_h = 2.0^{+3.0}_{-1.2} \text{ kpc}\). Unfortunately, the results from the Ulysses spacecraft with their better statistics (Simpson & Connell 1998; Connell 1998) do not have surviving fractions for either \(^{10}\text{Be}\) or \(^{26}\text{Al}\) reported, nor is it possible to deduce a value of \(f_s\) for either species from the information given in the respective papers.

The actual numerical results for \(f_s\) for both \(^{10}\text{Be}\) and \(^{26}\text{Al}\), calculated for various \(z_h\) and \(K_0\), are given in Table 4.1. In the Table, the combinations of \(z_h\) and \(K_0\) used were chosen to give the required grammage. Note that the actual measurements for \(^{10}\text{Be}\) and \(^{26}\text{Al}\) were obtained at energies \(\sim 100\) and 140 MeV/nucleon, and these were corrected for
solar modulation effects in the respective papers (Lukasiak et al. 1994a,b) to give interstellar energies ~ 300 MeV/nucleon and ~ 400 MeV/nucleon respectively. The interstellar energies for each cosmic ray species were used when calculating the results presented in Table 4.1.

<table>
<thead>
<tr>
<th>$z_h$ (kpc)</th>
<th>$K_0$ ($10^{28}$ cm$^2$ s$^{-1}$)</th>
<th>$f_s(^{10}\text{Be})$</th>
<th>$f_s(^{26}\text{Al})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.41</td>
<td>0.431</td>
<td>0.445</td>
</tr>
<tr>
<td>1.5</td>
<td>1.52</td>
<td>0.320</td>
<td>0.350</td>
</tr>
<tr>
<td>2.5</td>
<td>2.64</td>
<td>0.272</td>
<td>0.302</td>
</tr>
<tr>
<td>3.5</td>
<td>3.76</td>
<td>0.242</td>
<td>0.271</td>
</tr>
<tr>
<td>4.5</td>
<td>4.88</td>
<td>0.220</td>
<td>0.249</td>
</tr>
<tr>
<td>5.5</td>
<td>5.99</td>
<td>0.204</td>
<td>0.232</td>
</tr>
<tr>
<td>6.5</td>
<td>7.11</td>
<td>0.191</td>
<td>0.218</td>
</tr>
<tr>
<td>7.5</td>
<td>8.23</td>
<td>0.180</td>
<td>0.207</td>
</tr>
<tr>
<td>8.5</td>
<td>9.34</td>
<td>0.171</td>
<td>0.197</td>
</tr>
<tr>
<td>9.5</td>
<td>10.46</td>
<td>0.163</td>
<td>0.189</td>
</tr>
<tr>
<td>10.5</td>
<td>11.56</td>
<td>0.157</td>
<td>0.181</td>
</tr>
<tr>
<td>11.5</td>
<td>12.69</td>
<td>0.151</td>
<td>0.175</td>
</tr>
<tr>
<td>12.5</td>
<td>13.81</td>
<td>0.145</td>
<td>0.169</td>
</tr>
<tr>
<td>13.5</td>
<td>14.93</td>
<td>0.141</td>
<td>0.164</td>
</tr>
<tr>
<td>14.5</td>
<td>16.04</td>
<td>0.136</td>
<td>0.159</td>
</tr>
<tr>
<td>15.5</td>
<td>17.16</td>
<td>0.132</td>
<td>0.155</td>
</tr>
<tr>
<td>16.5</td>
<td>18.28</td>
<td>0.129</td>
<td>0.151</td>
</tr>
<tr>
<td>17.5</td>
<td>19.39</td>
<td>0.126</td>
<td>0.148</td>
</tr>
<tr>
<td>18.5</td>
<td>20.51</td>
<td>0.123</td>
<td>0.144</td>
</tr>
<tr>
<td>19.5</td>
<td>21.63</td>
<td>0.120</td>
<td>0.141</td>
</tr>
</tbody>
</table>

Table 4.1: Halo sizes and diffusion coefficient normalisations corresponding to a particular value of $f_s$ for $^{10}\text{Be}$ (column 3) and $^{26}\text{Al}$ (column 4) which is calculated using Equation 4.8, as described in the text.

The results obtained for $z_h$ from the surviving fractions for $^{10}\text{Be}$ and $^{26}\text{Al}$ given in Table 4.1 can be compared with those of other studies: $z_h \geq 7.8$ kpc (Freedman et al. 1980), 1.1 kpc $\leq z_h \leq 3.8$ kpc (Webber, Lee & Gupta 1992), $z_h \leq 3$ kpc (Bloemen et al. 1993), 1.9 kpc $\leq z_h \leq 4.0$ kpc (Lukasiak et al. 1994a), and 4 kpc $\leq z_h \leq 12$ kpc (Strong & Moskalenko 1998) for $^{10}\text{Be}$, and 0.8 kpc $\leq z_h \leq 16.4$ kpc (Webber, Lee & Gupta 1992) and 0.9 kpc $\leq z_h \leq 3.3$ kpc (Lukasiak et al. 1994b) for $^{26}\text{Al}$, and 2.0 kpc $\leq z_h \leq 4.0$ kpc (Webber & Soutoul 1998) using both $^{10}\text{Be}$ and $^{26}\text{Al}$. Generally, the results reported here for both cosmic ray species are similar to those obtained by other authors, often with more complicated propagation models (e.g. Strong & Moskalenko 1998). So the 1D approximation with analytical treatment of the cosmic ray propagation equation used in the present Chapter is a useful method for constraining $z_h$ and $K_0$. 
The model parameters obtained for $^{10}$Be and $^{26}$Al are not inconsistent with each other. Generally, the $^{10}$Be measurements tend to favour larger halo sizes and values for $K_0$, while $^{26}$Al favours smaller halo sizes and values for $K_0$. However, most of the lower range of allowed values for $K_0$ and $z_h$ by $^{10}$Be corresponds with the upper range of allowed values for $K_0$ and $z_h$ by $^{26}$Al. Numerically, the range of $K_0$ and $z_h$ simultaneously consistent with $f_s$ for both $^{10}$Be and $^{26}$Al is $3.0 \text{ kpc} \leq z_h \leq 5.0 \text{ kpc}$, and $3.2 \times 10^{28} \text{ cm}^2 \text{ s}^{-1} \leq K_0 \leq 5.3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$. These represent tighter constraints on the allowable ranges for $z_h$ and $K_0$ than would be obtained by simply considering only one radioactive species. Also, the range of $z_h$ allowed for both $^{10}$Be and $^{26}$Al is consistent with that obtained for the scale height of the galactic electron distribution obtained by Beuermann, Kanbach & Berkhuijsen (1985) from non-thermal radio studies.

### 4.3 Summary

The propagation of cosmic rays within the context of a 1D diffusion-convection model has been considered in this Chapter. It has been shown how convection can be neglected as playing a significant role in the transport of cosmic rays in the Galaxy. A summary of formulae suitable for obtaining estimates of the grammage and surviving fraction of cosmic ray radionuclei in the 1D diffusion-convection model has been given. In the diffusion dominated limit of the propagation model, these formulae have been applied to obtain limits on the range of allowable halo sizes and diffusion coefficients consistent with recent experimental data from spacecraft experiments. These model parameters will be used in the next Chapter when considering the propagation of cosmic ray electrons in the Galaxy.
Chapter 5

Diffuse $\gamma$-Rays and the Cosmic Ray Electron Spectrum

5.1 Introduction

Cosmic rays with energies up to $\sim 100$ TeV are thought to be accelerated by the first-order Fermi mechanism at supernova shocks (see Jones & Ellison 1991 for a recent review; also Section 3.2.1), and recently the EGRET instrument on the Compton Gamma-Ray Observatory has detected $\gamma$-ray signals above 100 MeV from at least two SNRs – IC 443 and $\gamma$-Cyg (Esposito et al. 1996). Further evidence for particle acceleration comes from recent ASCA and ROSAT detections of X-ray emission from the limbs of the type Ia SNR SN 1006 (Koyama et al. 1995; Willingale et al. 1996), and X-ray detections by ASCA of the SNRs IC 443 (Keohane et al. 1997) and RXJ1713.7-3946 (Koyama et al. 1997), and by RXTE of Cassiopeia A (Cas-A) (Allen et al. 1997). Reynolds (1996) and Mastichiadis (1996) interpret the X-ray observations of SN1006 as synchrotron emission by electrons accelerated in the remnant up to energies as high as $\sim 100$ TeV, while Keohane et al. (1997) and Allen et al. (1997) argue that the X-ray detections of IC 443 and Cas-A are evidence for electrons shock accelerated to $\sim 10$ TeV and $\sim 40$ TeV energies respectively. The non-detection of IC 443 in TeV $\gamma$-rays (Buckley et al. 1998) is, however, puzzling, and the observed X-ray emission of this remnant may have other explanations (e.g. Buckley et al. 1998). However, observations of SN 1006 in TeV $\gamma$-rays (Tanimori et al. 1998) appear to confirm the interpretation of the X-ray emission of this object as being due to electrons accelerated to $\sim 100$ TeV. So there probably exists at least one acceleration site of high energy electrons in the Galaxy,
and presumably more, as Pohl (1996) has argued that electron acceleration up to \( \sim 100 \text{ TeV} \) energies is probably not unique to SN 1006. Hence, the acceleration of electrons to TeV energies and beyond in cosmic ray sources should be relatively commonplace in the Galaxy.

Electrons accelerated to 100 TeV energies would eventually escape their acceleration sites and diffuse in the Galaxy, cooling through synchrotron radiation and inverse Compton (IC) scattering on the galactic magnetic and radiation fields respectively. For synchrotron cooling, electrons of these energies in a magnetic field strength of \( \sim 6 \mu \text{G} \) would give a diffuse flux of radiation in the X-ray regime, while IC scattering of 100 TeV energy electrons on the cosmic microwave background (CMBR) would give a diffuse flux of \( \gamma \)-rays at TeV energies. The spectrum of these processes is dependent on the high energy interstellar electron spectrum, which in turn is dependent on the initial source spectrum, distribution of sources, and propagation. If this radiation is detectable, it would provide a means of estimating the average interstellar electron spectrum, and hence the average spectrum of electrons at acceleration.

Protheroe & Wolfendale (1980), in an approximate calculation, considered the dual role of ultrarelativistic electrons in producing the diffuse galactic radiation from hard X-rays to TeV energies and above, and an analysis of Uhuru data by Protheroe et al. (1980) has indicated a considerable contribution by synchrotron emission in the soft X-ray band. More recently, Porter & Protheroe (1997) reconsidered this problem with a detailed propagation model, and found that the peculiar spectrum observed by OSSE below \( \sim 1 \text{ MeV} \) (e.g. Purcell et al. 1996) could be explained in terms of synchrotron emission by shock accelerated electrons with \( \sim 100 \text{ TeV} \) energies. However, other calculations (e.g. Strong et al. 1996; Hunter et al. 1997) of the low energy diffuse \( \gamma \)-ray spectrum have generally neglected synchrotron radiation as a significant production process, and have invoked other explanations than synchrotron emission for the observed spectrum below \( \sim 1 \text{ MeV} \) (e.g. Skibo et al. 1995; Schlickeiser 1997; Kinzer et al. 1997). Inverse Compton scattering on the ambient galactic radiation fields, on the other hand, is generally recognised as an important component of the diffuse \( \gamma \)-ray spectrum at MeV to GeV energies. In particular, detailed predictions of IC \( \gamma \)-rays above 70-100 MeV have been made by Bloemen (1985), Chi et al. (1989), Pohl & Esposito (1998), and Strong, Moskalenko & Reimer (1998). An analysis of EGRET data by Giller et al. (1995) suggested a contribution by the IC process of \( \sim 30\% \) at medium latitudes, and up to \( \sim 45\% \) of the total towards the galactic pole, in the energy range 30 MeV to 4 GeV. Recent modelling of the diffuse galactic emission (Strong, Moskalenko & Reimer 1998) indicates that the contribution by IC \( \gamma \)-rays may be even more important, and that the
contribution by bremsstrahlung only comprises at most \( \sim 10\% \) of the total emission below \( \sim 1 \text{ GeV} \). At high energies, the contribution by IC \( \gamma \)-rays is less clear but they may actually be the major \( \gamma \)-ray production process, dominating even over neutral pion decay (Porter & Protheroe 1997). This is a view that was initially greeted with some skepticism, however several models of the high energy spectrum observed by EGRET (Pohl & Esposito 1998; Strong, Moskalenko & Reimer 1998) now support a dominant contribution by IC \( \gamma \)-rays at high energies.

In this Chapter, I consider possible injection spectra of primary cosmic ray electrons, and the resulting diffuse \( \gamma \)-ray spectrum of the Galaxy. Also investigated is the possibility that the cosmic ray nucleus spectrum throughout the Galaxy differs from the local spectrum, and the consequences for the production of \( \gamma \)-rays by these particles. Starting with a power-law spectrum of electrons at acceleration, a propagation calculation is performed using a diffusion model consistent with the observed secondary to primary data, and abundances of \(^{10}\text{Be} \) and \(^{26}\text{Al} \) as found in Chapter 4, to obtain the interstellar electron spectrum. Secondary electrons and positrons are also included since these are produced in addition to \( \gamma \)-rays by cosmic ray nuclear interactions in the ISM. Note, however, that no propagation calculation is performed for cosmic ray nuclei; these particles are treated in a parametric way, using the observed positron spectrum to constrain possible forms for the average interstellar proton spectrum. Realistic models of the galactic gas distribution, magnetic field, and ambient photon populations are used in the propagation calculation. Inverse Compton, bremsstrahlung, and synchrotron production spectra are calculated using the electron and positron spectra resulting from the propagation calculations, and neutral pion decay production spectra for hadronic processes in the ISM are also obtained, and I give a consistent treatment of high energy photon production by these processes from low to very high energies. The predictions are then compared with satellite observations at keV to GeV energies (Strong & Mattix 1996; Hunter et al. 1997; Kinzer et al. 1997; Strong et al. 1998), optical Cherenkov telescope observations at TeV energies (Reynolds et al. 1993), and air shower observations at 50–1000 TeV energies (Matthews et al. 1991; Karle et al. 1995; Amenomori et al. 1997; Borione et al. 1998).

The propagation calculation for cosmic ray electrons and positrons is described in the next Section. Also addressed is the question of whether the local total cosmic ray electron spectrum is truly representative of the average interstellar electron spectrum due to the discrete nature of the primary source distribution; this has important consequences for the
injection spectrum of electrons at acceleration. It will be shown that the usual assumption of a continuous source distribution is a poor one to be made if predictions of the local total electron spectrum are made. However, from the point of view of estimates of the average interstellar spectrum, it is actually quite a reasonable assumption to make. Then, in Section 5.3, the photon production processes for IC, bremsstrahlung, synchrotron, and neutral pion decay processes are described, and photon emissivity spectra are calculated using the electron and positron spectra resulting from the propagation calculations, and the proton spectra found from the analysis of the local positron spectrum. In Section 5.4 predictions of diffuse galactic photon spectra are made. The procedure in this Section is to first construct several models involving the primary electron injection spectrum and average interstellar proton spectrum that adequately reproduce the local positron spectrum, total electron spectrum, and galactic non-thermal emission, and then to interpret these models using the observed galactic diffuse hard X-/γ-ray spectrum. The models found in this way are then contrasted with other models by various authors existing in the literature.

5.2 Propagation of Cosmic Ray Electrons and Positrons

Electrons and positron, either directly accelerated in SNRs, or produced in hadronic collisions in the ISM, are transported throughout the Galaxy after their initial acceleration/production. The propagation of these particles in the Galaxy can be adequately modelled using a diffusion model, as was done for cosmic ray nuclei in Chapter 4. However, because of the strong influence of energy losses on the electron and positron spectrum, and the spatial dependence of the gas, low energy photon, and magnetic field distributions that the electrons and positrons interact with, an analytical approach to the solution of the propagation equation is intractable. Therefore, I use the Monte Carlo method described in Chapter 2 in this Section to obtain predictions of galactic electron and positron distributions for use in diffuse photon production calculations in later Sections. However, before the propagation calculation for electrons and positrons is described, the validity of the assumption of a continuous source distribution for primary electrons is examined.

In Section 5.2.1, it is shown how the conventional approach of using relatively soft injection spectra (γ ~ 2.4), which is required to fit the locally observed total electron spectrum, can be relaxed if the inhomogeneous distribution of primary sources in the Galaxy is taken into account. Harder injection spectra are then possible for the primary source spectrum. Then,
in Section 5.2.2 the electron/positron propagation calculation is described, and predictions of the interstellar distributions of electrons and positrons in the Galaxy are made.

5.2.1 Is the Local Electron Spectrum Different from the Galactic Average?

The conventional approach of a continuous source distribution for primary electrons requires the adoption of relatively soft ($\gamma \sim 2.4 - 2.5$) injection spectra to provide adequate agreement with the locally observed spectrum up to $\sim 1$ TeV (e.g. Strong et al. 1996). However, the assumption of a continuous source distribution for electrons accelerated in primary sources breaks down when the typical diffusion distance of these particles is similar to the distance between discrete sources, and this typically occurs for energies greater than $\sim 100$ GeV. Unless an observer is located in close proximity to a source of high energy electrons, the electron spectrum seen by the observer will be steepened due to the severe radiative energy losses suffered by electrons of these energies and higher. Because the soft injection spectra required by the continuous source distribution approach differs significantly from the (harder) average electron spectral index for SNRs inferred from non-thermal radio measurements given in the SNR catalogue of Green (1995), it may be that the local electron spectrum is due to the particular location of the Sun in a region relatively devoid of sources. As such, the locally observed electron spectrum may not be representative of the true interstellar spectrum, and harder injection spectra than required by the conventional approach may be possible.

The idea that the locally observed total electron spectrum above $\sim 50 - 100$ GeV is a result of an inhomogeneous distribution of local sources is not entirely new (e.g. Shen 1970; Shen & Mao 1971; Cowsik & Lee 1979). Recent work has focused on an explanation of the local spectrum as being due to a combination of electrons accelerated in distant sources, and electrons accelerated in perhaps one or two sources in close proximity (within a few hundred pc) of the Earth (e.g. Atoyan, Aharonian & Völk 1995), or even several sources (e.g. Nishimura et al. 1995). However, the identification of these sources is putative, and is complicated by uncertainties in such things as their respective ages, and even distances. A novel approach explored by Pohl & Esposito (1998) was to consider the local spectrum to result from Poisson fluctuations in the source distribution over time. This is similar to the method used by Nishimura et al. (1995) but does not rely upon explicitly identifying particular sources as being responsible for the local spectrum.

The following qualitative argument can be used to understand how the local and interstellar average electron spectrum may differ: electrons are assumed to accelerated at strong
supernova shocks, and the nearest source region may be \( \sim 100 \) pc or more away. At energies where the energy-loss timescale is less than or comparable with the diffusion time to the nearest source, high-energy electrons will lose most of their energy before reaching the observer. The local spectrum will then be significantly steeper than the interstellar average (see Porter & Protheroe 1997 and references therein).

In this Section, I intend to quantify this argument, and show how flat injection spectra for the primary electron sources are not inconsistent with observations of the local total electron spectrum, and that the average interstellar spectrum may be flatter than the local spectrum.

Consider the time dependent diffusion equation neglecting adiabatic energy losses and gains:

\[
\frac{\partial N}{\partial t} - K \nabla^2 N + \frac{\partial}{\partial E} [b(E)N] - Q = 0
\]  

(5.1)

where all terms have been defined in Section 2.2. Now, for the moment, consider the evolution in space and time of the electron number density for electrons injected from some source \( Q \) into a region of the Galaxy in which the gas distribution, ambient photon populations, and magnetic field are homogeneous. The evolution of the electron number density will be described by the spherically symmetric form of Equation 5.1:

\[
\frac{\partial}{\partial t} N(E, R, t) - \frac{K(E)}{R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} N(E, R, t) + \frac{\partial}{\partial E} [b(E)N(E, R, t)] - Q(E, R, t) = 0
\]  

(5.2)

where \( b(E, \tau) = b(E) \) because of the assumed homogeneity for the matter distribution, magnetic field and ambient photon populations (this is an assumption that will be retained for the remainder of this Section).

The Green’s function solution for Equation 5.2 for diffusion in an infinite space can be obtained using a similar method to that employed to obtain the 1D Green’s functions in Section 2.3.2. The Green’s function has the form

\[
G(E, R, t; E', R', t') = \frac{Q_0}{|b(E)||b(E')|[4\pi (\lambda - \lambda')]^{3/2}} e^{-\frac{(R-R')^2}{4(\lambda-\lambda')}} \delta (t - t' - \tau + \tau')
\]  

(5.3)

where \( \lambda - \lambda' \equiv \lambda(E) - \lambda(E') \), with \( \lambda \) defined by Equation 2.8, and \( \tau + \tau' \equiv \tau(E) + \tau(E') \) with \( \tau \) given by

\[
\tau(E) = \int_E^\infty \frac{dE}{b(E)}.
\]  

(5.4)
Formally, the electron number density for electrons injected by a single source is then obtained by convolving the Green’s function with the source function as

\[ N(E, R, t) = \int d^3 \mathbf{R}' \int_{-\infty}^{\infty} dE' \int_{-\infty}^{t(t(E) - t)} dt' \int dR'_i G(E, R, t; E', R'_i, t') Q(E', R'_i, t'). \] (5.5)

Note that this Equation naturally takes into account the possibility of the source having some extended spatial structure. Because of the assumed spherical symmetry, \( Q \) could represent a spherical shell SNR, but this will not be considered in this Section for reasons of simplicity (see below).

Now, the electron spectrum at some point from the galactic centre, \( \mathbf{\bar{r}} \), will depend on the contribution of many sources. Let \( \mathbf{r}_i \) be the distance from the galactic centre for each of these sources. Then, assuming the spatial dimensions of the sources are small compared to their distance from the observer’s location \( \mathbf{\bar{r}} \), the total number density of electrons can be written as

\[
N(E, \mathbf{\bar{r}}, t) = \sum_i N_i(E, \mathbf{\bar{r}}, t) \\
\approx \sum_i \int_E^{\infty} dE' \int_{-t(E)}^{t_{\text{present}}(E)} dt' \int dR'_i G(E, R_i, t; E', R'_i, t') Q_i(E', t'_i) \delta(R'_i) \\
= \sum_i \int_E^{\infty} dE' \int_{-t(E)}^{t_{\text{present}}(E)} dt' G(E, R_i, t; E', t'_i) Q_i(E', t'_i) 
\] (5.6)

where \( R_i \equiv |\mathbf{\bar{r}} - \mathbf{r}_i| \), \( t_{\text{present}} \) is the time of the present epoch, and \( t(E) = E/|b(E)| \) is the energy loss time scale at energy \( E \). The replacement of the lower limit in the time integration with \( t(E) \) reflects the fact that electrons injected at times earlier than \( t(E) \) do not contribute to the number density at energy \( E \) because they lose most of their energy before they can effectively diffuse to the local region.

To specify the source function for a single source, \( Q_i \), I consider a crude model for a cosmic-ray accelerator. As already noted, I assume a source to be a mathematical point in space. A source is also assumed to be born at some time earlier than the present epoch, inject electrons for some fixed time period, and then stop. Therefore, for the \( i \)th source born at a time \( t'_i \) in the past, injecting electrons for a period \( t_{\text{life}} \), and then stopping, the source function is

\[
Q_i(E', t'_i) = A_i E'^{\gamma_i} H[t'_i; t_{\text{present}} - t'_i, t_{\text{present}} - t'_i + t_{\text{life}}] 
\] (5.7)
where \( A_i \) is a normalisation constant, \( \gamma_i \) is the injection spectral index, and \( H \) is the Heaviside function such that \( H[x; a, b] = 1 \) for \( a \leq x \leq b \) and 0 otherwise.

To calculate the local electron spectrum, the procedure is to generate configurations of sources in the Galaxy distributed throughout some source volume, \( V_{\text{src}} \), with each source having the form given by Equation 5.7, and to sum their contributions to the local spectrum using Equation 5.6. This procedure is repeated a number of times for different source configurations to examine the effect of different source distributions on the ‘local’ electron spectrum.

For the source region, I use a cylinder of maximum radius \( R_{\text{src}} = 20 \) kpc, and half-height \( z_{\text{src}} = 200 \) pc. Now, generating a source configuration for all of \( V_{\text{src}} \) is not required because electrons injected in distant parts of the Galaxy have almost no chance of diffusing to the local region. Therefore, only the region of the Galaxy, \( V_{\text{local}} < V_{\text{src}} \), for which injected electrons have a significant probability of diffusing to Earth needs to be considered. So, a configuration of \( N_{\text{src}} \) sources is generated where the sources are distributed throughout \( V_{\text{local}} \).

To determine \( V_{\text{local}} \) the maximum energy-loss timescale, \( t_{\text{max}} = \frac{E}{\rho(E)} \bigg|_{\text{max}} \) is considered. This defines a maximum diffusion time, and hence a maximum effective diffusion radius, \( R_{\text{max}} = \left[ 6K(E)t_{\text{max}} \right]^{1/2} \), beyond which electrons are unlikely to be able to diffuse to the local region. This is because the radiative cooling, and diffusion times are exactly matched at the energy for which \( t_{\text{max}} \) is found, therefore electrons injected at times earlier than \( t_{\text{max}} \), at distances greater than \( R_{\text{max}} \), usually lose their energy before they can be transported to the local region. So, I take \( V_{\text{local}} \) to be a cylindrical region centred on the Sun of radius \( R_{\text{max}} \), with half-height \( z_{\text{src}} \). Thus, \( V_{\text{local}} = 2\pi R_{\text{max}}^2 z_{\text{src}} = 12\pi K(E)t_{\text{max}}z_{\text{src}} \).

The average number of sources, \( N_{\text{src}} \), within \( V_{\text{local}} \) that could influence the local spectrum is estimated by taking into account the relative sizes of \( V_{\text{local}} \) and \( V_{\text{src}} \), and the average number of sources generated throughout the Galaxy during the time period \( t_{\text{max}} \). Given an average time between the creation of sources in the Galaxy, \( t_{\text{birth}} \), the average number of sources is

\[
N_{\text{src}} = \frac{V_{\text{local}}}{V_{\text{src}}} \frac{t_{\text{max}}}{t_{\text{birth}}} = \frac{6K(E)}{R_{\text{src}}^2} \frac{t_{\text{max}}^2}{t_{\text{birth}}}. \tag{5.8}
\]

A configuration of sources for the volume \( V_{\text{local}} \) is generated by taking \( t_{\text{birth}} = 85 \pm 30 \) years; this is the galactic average creation time of SNRs and pulsars as given by Gaensler &
Johnston (1995), an references therein. Each source is then assigned a random age distributed uniformly between the present epoch and $t_{\text{max}}$, and the lifetime of each source is taken to be $t_{\text{life}} = 10^5$ years (Atoyan, Aharonian & Völk 1995). The sources are distributed throughout $V_{\text{local}}$ assuming a constant average source number density.

For the electron propagation, the diffusion coefficient corresponding to the halo size $z_h = 4$ kpc in Section 4.2.3 is used:

$$K(E) = \begin{cases} 
4.3 \times 10^{28} \text{ cm}^2 \text{s}^{-1} & (r < 4.7 \text{ GV}) \\
4.3 \times 10^{28}(r/4.7 \text{ GV})^{0.60} \text{ cm}^2 \text{s}^{-1} & (r \geq 4.7 \text{ GV}) 
\end{cases} \quad (5.9)$$

with $r$ the magnetic rigidity. While propagating, electrons lose energy through ionisation/Coulomb losses, bremsstrahlung, inverse Compton (IC) interactions, and synchrotron radiation; explicit formulae for calculating the energy loss rate for each of these processes are given in Appendix B. The energy loss rate as a function of energy, $\dot{b}(E)$, is computed using the following parameters: neutral and ionised hydrogen number densities of 1.5 cm$^{-3}$ and 0.02 cm$^{-3}$ respectively for interactions with matter, a magnetic field strength of 6 $\mu$G for synchrotron losses, and local energy densities of 0.44 eV cm$^{-3}$, 0.33 eV cm$^{-3}$, and 0.26 eV cm$^{-3}$ for the optical to UV, mid to far infra-red, and CMBR radiation fields respectively. These parameters are a reasonable approximation of the gas densities, magnetic field and ambient radiation fields within several kpc of the local region. Note that the IC energy losses are calculated using the exact Klein-Nishina (KN) cross-section, as described in Section B.2 of Appendix B, and the wavelength dependence of the local radiation fields is taken into account when calculating these losses.

The energy-loss timescale computed using the above parameters is shown in Figure 5.1. To determine $t_{\text{max}}$, the energy at which the total energy-loss timescale in the Figure is a maximum has to be estimated. From the Figure, the total timescale is a maximum at $\sim 2$ GeV, and this corresponds to $t_{\text{max}} \approx 1.2 \times 10^7$ years.

Figure 5.2a shows a compilation of data for the local total electron spectrum (Webber et al. 1980; Golden et al. 1994; Nishimura et al. 1995 and references therein), and three representative electron spectra for different source configurations calculated using the parameters given above, and Equation 5.6 for an injection spectral index $\gamma = 2.0$ for all sources. Figure 5.2b shows three representative electron spectra calculated as for Figure 5.2a, but for each source throughout $V_{\text{local}}$ the injection spectral index is assigned by sampling from a Gaussian with central density $\gamma = 2.0$ and standard deviation $\sigma = 0.3$; this corresponds to
Figure 5.1: Energy-loss timescales for ionisation/Coulomb, bremsstrahlung, IC, and synchrotron losses for the gas densities, radiation field, and magnetic field strength given in the text. Thick and thin dotted and dashed lines show the timescales for ionisation/Coulomb and bremsstrahlung losses on neutral and ionised gas respectively. Dash-dot line shows the timescale for IC losses calculated using the exact Klein-Nishina cross-section. Triple-dot dash line shows the timescale for synchrotron losses. Solid line shows the sum of the energy losses converted to an energy-loss timescale.

the mean and spread of SNR radio spectral indices in the catalogue of Green (1996). Note that the spectra shown in the Figures are normalised to the data points around 10 GeV; this is because solar modulation effects are believed to be only minor for these energies and higher (e.g. Clem et al. 1996), and the error bars for the data points around 10 GeV are fairly tight.

Below ~ 70 GeV for Figure 5.2a and ~ 40 GeV for Figure 5.2b the calculated spectra are reasonably consistent with each other, and with the data. The underprediction below ~ 1 GeV for both of the cases considered could simply be due to the non-inclusion of secondary electrons when calculating the total spectrum (see Section 5.2.3 below), or because of the assumed homogeneity of the ISM. For whatever reason, this underprediction is of no consequence because it is independent of the source distributions used to calculate the
Figure 5.2: Local electron spectra predictions calculated as described in the text. Figure (a) corresponds to all sources having an injection spectral index $\gamma = 2.0$. Figure (b) corresponds to each source having an injection spectral index assigned from a Gaussian with central density $\gamma = 2.0$ and standard deviation $\sigma = 0.3$. Experimental data for the observed spectrum are from Webber, Simpson & Cane (1980), and the results summarised by Golden et al. (1994) and Nishimura et al. (1995).
spectra shown in the Figures. At high energies the calculated spectra deviate somewhat from pure power-laws, displaying quite ‘lumpy’ behaviour for some of the spectra shown. In fact, two of the spectra in Figure 5.2b appear to provide reasonable agreement with the observed spectrum up to the highest energies measured; this is not possible for a continuous source distribution even with a fairly soft ($\gamma = 2.4$) injection index (e.g. Porter & Protheroe 1997). Note that the spectra shown in the Figures are several of many that were calculated from different configurations of sources, but they are a representative sample of the results obtained. Also, I examined the effect of changing the diffusion coefficient to the other values obtained in Section 4.2.3, but the results were similar to those given in the Figures (the main effect was to slightly raise or lower the energy below which the calculated electron spectrum was insensitive to the source distribution, and this only varied by a few tens of GeV from the results shown in the Figures for the values of $K_0$ considered).

From the Figures it can be seen that the electron spectrum observed above $\sim 70$ GeV will be critically dependent on the source distribution. While several of the spectra appearing in the Figures seem to agree with the observed spectrum, some of the calculated spectra in the Figures are significantly higher than the observations above $\sim 50$ GeV; this could occur, for example, if the solar system were located in a region densely populated by sources. In fact, the converse is probably true for the solar system, where it is located in a region relatively sparsely populated by sources (the nearest detected putative source, Geminga, is a couple of hundred pc away, e.g. Caraveo et al. 1996). Perhaps it is not too surprising then that the local electron spectrum steepens for energies greater than $\sim 70$ GeV. Note that of all the spectra calculated, none were actually significantly steeper than the lowest spectrum shown in Figure 5.2a. In fact, most were either similar to the local spectrum, or harder than the local spectrum at high energies, as shown by the other spectra in the Figures. Therefore, over the Galaxy, it could be expected that the average interstellar spectrum is harder than the local spectrum.

So, in this Section I have shown that hard injection spectra for cosmic ray electrons are not inconsistent with the observed electron spectrum. In fact, the usual approach of a continuous source distribution using a relatively soft ($\gamma = 2.4$) injection index to obtain agreement with the local spectrum probably underestimates the source spectrum of electrons at acceleration.

In the next Section, I assume a continuous spatial distribution of sources for primary electrons. From the point of view of making predictions of diffuse photon fluxes this is a reasonable assumption because I am interested in the average interstellar electron spectrum.
Hence, if hard injection spectra are considered, requiring agreement between propagation calculations using such an assumption for the source distribution, and the local spectrum above $\sim 40$ GeV is not critical because of the discussion given in this Section.

### 5.2.2 Propagation Calculation for Cosmic Ray Electrons and Positrons

The propagation calculation for cosmic ray electrons and positrons is made within the framework of a three-dimensional (3D) diffusion model with cylindrical symmetry about $R = 0$, and symmetry about $z = 0$; a diagram of the geometry of the model is shown in Figure 5.3. For the present calculations, the Sun is assumed to be located a radial distance $R_S = 8.5$ kpc (Kerr & Lynden-Bell 1986) from the galactic centre, and the maximum radial extent of the Galaxy, $R_{\text{max}}$, is taken to be 20 kpc.

![Diagram of 3D diffusion model](image)

**Figure 5.3:** Geometry of the 3D diffusion model used for the electron and positron propagation calculations described in the text.

In the model, the galactic plane is taken to be the plane defined by $z = 0$. For the maximum dimension of the Galaxy perpendicular to the galactic plane, $z_h$, I use the three halo sizes obtained in Chapter 4: $z_h = 3, 4,$ and 5 kpc respectively. The diffusion coefficient used in the propagation calculation is assumed to have the form (c.f. Equation 4.13)
\[ K(E) = \begin{cases} K_0 & (r < 4.7 \text{ GV}) \\ K_0(r/4.7\text{GV})^{0.60} & (r \geq 4.7 \text{ GV}) \end{cases} \] (5.10)

with \( r \) the magnetic rigidity, and where \( K_0 \) is the normalisation constant determined for each of the halo sizes above, as described in Chapter 4. Note that a separate propagation calculation is carried out for each combination of \( z_h \) and \( K_0 \).

The source functions for primary electrons, and secondary electrons and positrons are as given in Chapter 3. For the injection spectral index of the primary electron source function, I consider observations of the non-thermal emission of the Galaxy. Estimates of the non-thermal power law spectral index (\( J_\nu \propto \nu^{-\beta} \)) have been made by various authors: \( \beta = 0.57 \pm 0.03 \) for 5–80 MHz in the direction of the galactic pole (Webber, Simpson & Cane 1980); \( \beta = 0.5 - 0.6 \) for 38–408 MHz over the northern galactic hemisphere (Lawson et al. 1987); \( \beta = 0.7 \) for 408–5000 MHz for low latitudes toward the inner Galaxy (Broadbent, Haslam & Osborne 1989); \( \beta = 0.85 \) in the plane, increasing to \( \beta = 1.0 \) at higher latitudes for 408–1020 MHz (Reich & Reich 1988); \( \beta = 0.6 - 1.3 \) for a high latitude band for 408–1420 MHz (Davies, Watson & Gutiérrez 1996); and \( \beta = 0.81 \pm 0.16 \) for 1.4–7.5 GHz (Platania et al. 1998). Using the well known relationship between the ambient electron spectral index, \( \Gamma \), and the non-thermal emission spectral index \( \beta = (\Gamma - 1)/2 \) (e.g. Longair 1994), \( \Gamma \) lies in the range 2.0–3.0. Now, in the energy range of the electrons (and positrons) emitting the non-thermal emission, the ambient spectrum is similar to the source spectrum (i.e. \( \Gamma \sim \gamma \)) because electron and positron energy losses are mainly dominated by bremsstrahlung. To see this, consider a simple model where electrons are injection into the Galaxy continuously with source spectrum \( Q(E) \propto E^{-\gamma} \), and lose energy at the rate \( b(E) \propto E \). Then, the number density of electrons can be written

\[ N(E) \propto \frac{1}{E} \int_{E}^{\infty} E^{-\gamma} dE \] (5.11)

and hence \( N(E) \propto E^{-\gamma} \). Thus \( \Gamma \sim \gamma \) in the energy range in which bremsstrahlung losses dominate. Now, other energy losses also contribute to the total energy loss rate, and hence the relationship between the ambient and source spectrum is not unique, particularly near the endpoints of the frequency range for the non-thermal spectrum given above. Therefore, a more relativistic range of source spectral indices at injection is probably \( \gamma \sim 1.8–2.4 \), and this is the range considered for the remainder of this Chapter.

While propagating, electrons and positrons lose energy through ionisation and Coulomb
interactions, bremsstrahlung, inverse Compton (IC), and synchrotron radiation. I model electron and positrons interactions with matter using the three component gas model for the atomic, molecular and ionised hydrogen distribution described in Section A.1 of Appendix A. A contribution to the total gas density by helium consistent with the observed abundance is also included in the gas model, as described in Section 3.3.1.4. For interactions of electrons and positrons on the ambient galactic photon fields, I consider three target photon populations: the ultraviolet (UV) to near infrared (the ‘optical’), the far infrared, and the cosmic microwave background radiation (CMBR). The models used to describe the distributions of the optical and far infrared photon populations used in this thesis are given in Section A.2 of Appendix A. For interactions by electrons and positrons with the galactic magnetic field (GMF), I use the model of the large scale galactic magnetic field given in Section A.3 of Appendix A. Energy loss formulae in both the non-relativistic and relativistic case for the various interactions electrons and positrons can undergo with the gas distribution, galactic photon, and magnetic fields are given in Appendix B; I consider ionisation loss and bremsstrahlung on both neutral and ionised gas, and IC and synchrotron losses. Note that for high energies the IC losses on the ambient galactic photon populations are in the Klein-Nishina regime, and the energy losses are calculated using Monte Carlo methods as described in Section B.2 of Appendix B.

For the propagation calculation, the Galaxy is divided up into radial bins of half-width 1 kpc centred on \( R = 1, 3, 5, 7, 9, 11, 13, 15, 17 \) and 19 kpc. These form the radial dimensions of the source regions. In the \( z \) direction, bins of half-width 200 pc starting at \( z = 0 \) kpc and extending out to \( z_h \) are used for the source region dimensions.

To obtain the probability matrix for the \( i \)th radial bin and \( j \)th \( z \) bin, I take 111 energy bins at intervals of \( \Delta \log E = 0.1 \) with mid-bin energy starting at \( 10^{-3.0} \) GeV. Particles are injected uniformly within the source region, and the Monte Carlo procedure outlined in Section 2.3.2 is followed. Because of the geometry of the propagation model, the Monte Carlo procedure is modified in the following way to simulate diffusion in 3D: a uniformly sampled normal deviate, \( \zeta \), is multiplied by the standard deviation

\[
\sigma = \min \left( \sqrt{6K \Delta t}, \sigma_{\text{max}} \right)
\]

where all terms have been explained in Section 2.3.2. Since the diffusion is in 3D, steps in the \( x \), \( y \), and \( z \) directions are generated:
\[ \Delta x = \sigma \zeta \cos \phi \sin \theta \]
\[ \Delta y = \sigma \zeta \sin \phi \sin \theta \]
\[ \Delta z = \sigma \zeta \cos \theta \]

where \( \phi \) and \( \theta \) are spherical polar angles. The angles \( \phi \) and \( \theta \) are found by sampling from a uniform distribution on the interval \([0, 2\pi]\), and from a uniform \( \cos \theta \) distribution on the interval \([-1, 1]\) respectively. Then, the new position is found by adding \( \Delta x \) to the \( x \) position of the particle, \( \Delta y \) to the \( y \) position, and so forth. The rest of the Monte Carlo procedure is as described in Section 2.3.2.

Based on the discussion in Section 2.3.3, a value of \( \sigma_{\text{max}} = 50 \) pc is used, and \( N = 10^5 \) particles are injected at each of the source energy bins. The observing regions are taken to be annuli of radial half-width \( \Delta R_{\text{obs}} \) and half-thickness in the \( z \) direction \( \Delta z_{\text{obs}} \) centred on \( R = 1, 3, 5, 7, 9, 11, 13, 15, 17, \) and 19 kpc. The half-thickness in the \( z \) direction is taken to \( \Delta z_{\text{obs}} = 100 \) pc from the same discussion that motivated the choice for \( \sigma_{\text{max}} \). Now, for \( \Delta R_{\text{obs}} \) a value as small as used for \( \Delta z_{\text{obs}} \) is not required because the variation of the particle density gradient in the radial dimension is not as rapid as in the \( z \) direction. Therefore, a value for \( \Delta R_{\text{obs}} \) needs to be chosen that is smaller than the source region radial half-width, but large enough so that the counting statistics of particles in the observing region are maximised (recall the discussion in Section 2.3.3). So, I take \( \Delta R_{\text{obs}} = 250 \) pc, which should be sufficiently small so that the particle gradient is approximately constant over the observing region, but is large enough that a reasonably large number of particles are counted during a run of the simulation code.

### 5.2.3 Calculated Electron and Positron Spectra

The predicted interstellar electron and positron spectra are obtained from the probability matrix and the appropriate source distribution using Equation 2.5. For secondary electrons and positrons, the convolution of the probability matrix and the source distribution is over all source elements because the extent of the gas and cosmic ray nuclei distributions means the entire halo is effectively the source region for these particles. For primary electrons, the convolution is only over the source element closest to the galactic plane because the sources of these particles are assumed to be confined to a thin disk about \( z = 0 \).
For each of the annuli centred on \( R = 1, 3, 5, 7, 9, 11, 13, 15, 17, \) and 19 kpc the electron and positron distributions in space and energy are obtained as described above. These distributions are taken to represent the average distribution of electrons and positrons for the each of these annuli, and are used in the emissivity calculations in Section 5.3. In the following, calculated electron and positron spectra for the radial bin centred on \( R = 9 \) kpc are presented since this corresponds to the local region. For the secondary positron spectrum, I compare my predictions with available data to ensure the propagation model adequately reproduces the observations, and to examine the spectrum of cosmic ray nuclei producing these particles; this has direct consequences for the production of diffuse \( \gamma \)-rays and will be discussed shortly. The predicted total electron spectrum is discussed, and compared with observations.

5.2.3.1 Secondary Electron and Positron Spectra

The interstellar spectra of secondary positrons for the local region calculated for the propagation model parameters \( z_h = 4 \) kpc and \( K_0 = 4.3 \times 10^{28} \) cm\(^2\) s\(^{-1}\) are shown in Figure 5.4a. The corresponding positron fractions, \( e^+/(e^+ + e^-)_{\text{obs}} \), are shown in Figure 5.4b; these have been calculated using the parameterisation of the local total electron spectrum given by Protheroe (1982), which is shown in Figure 5.5 for reference. In addition to the propagation model predictions, available data for the positron spectrum and fraction are also shown (Golden et al. 1994; Barwick et al. 1998 and references therein).

In both Figures, the positron spectra have been calculated using source spectra obtained in Section 3.3.1.4 using the Protheroe (1982) and Mori (1997) median proton spectra. Also, a positron spectrum (and fraction) is shown which has been calculated using a ‘hard’ proton spectrum that can be parameterised locally as

\[
J_p(E_p) = 1.2E_p^{-2.55},
\]  

(5.16)

in the secondary positron source spectrum calculation. The relevance of the hard proton spectrum will be discussed shortly.

The calculated positron spectra for both the Protheroe (1982) and Mori (1997) median incident proton spectra agree fairly well with the most recent measurements between \( \sim 8 \) GeV up to \( \sim 50 \) GeV (Barwick et al. 1998); good agreement between the predicted positron spectra and observations is not expected below this because solar modulation effects cause
Figure 5.4: Local positron (a) spectrum and (b) fraction. Solid line in each Figure shows the prediction for $K_0 = 4.3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$, $z_h = 4 \text{ kpc}$, and the Protheroe (1982) incident proton spectrum for the positron production spectrum calculation. Dashed line shows the prediction for the same $K_0$ and $z_h$ but with the Mori (1997) median incident proton spectrum. Dash-dotted line shows the prediction for the hardest incident proton spectrum (for the same $K_0$ and $z_h$) agreeing with the highest energy data point of Barwick et al. (1998). Note the discrepancy between the calculated spectra and the observations below $\sim 5-8 \text{ GeV}$ in (a) is due to solar modulation effects.
the observed spectrum to be lower than the true interstellar spectrum (see e.g. Moraal, Jokipii & Mewaldt 1991, or Clem et al. 1996). There is a factor $\sim 1.4$ difference between these predictions, however, above $\sim 1$ GeV. This is a direct result of the $\sim 1.4$ factor difference at high energies between the Protheroe (1982) and Mori (1997) proton spectra used for the positron production spectrum calculations, as discussed earlier in Section 3.3.1.2. From the point of view of propagation model studies this is an important point because the differences in the predictions of various models (e.g. Protheroe 1982; Wong & Ng 1986) are generally smaller than that caused by the adoption of the two proton spectra in the present calculations. Although a detailed study of propagation model predictions is not the purpose of the present work, this is an interesting point that should be borne in mind in future studies of the secondary positron spectrum.
For the predictions of the positron fraction for the Protheroe (1982) and Mori (1997) median proton spectra, the agreement between the predictions and observations is fairly good, although for energies below $\sim 5$ GeV charge-sign dependent modulation effects may alter the measured fraction from the true interstellar value (Clem et al. 1996). At high energies, the better agreement is obtained for the Mori (1997) median proton spectrum. However, the positron fraction depends on the assumed total electron spectrum, so a slight adjustment in the parameterisation given in Figure 5.5 could allow the prediction for the Protheroe (1982) proton spectrum to agree better with the data. Both the predicted spectra and fractions for these ‘standard’ proton spectra used in the source spectrum calculations are in reasonable agreement with the data, but the Mori (1997) median proton spectrum is adopted for diffuse $\gamma$-ray calculations later because it appears to provide the better fit of the two proton spectra to both the most recent high energy positron spectra, and fraction observations.

The relevance of the hard proton spectrum given by Equation 5.16 is now discussed. The motivation for considering such an ambient proton spectrum, which is clearly harder than the locally observed spectrum, is the recent interpretation of diffuse $\gamma$-rays above $\sim 500$ MeV observed by EGRET. Several authors (Aharonian & Atogyan 1996; Protheroe & Stannev 1997) suggest that the EGRET results can be explained in terms of hadronic interactions between recently accelerated cosmic rays and gas clouds located in close proximity to the cosmic ray accelerators. Generally, a relatively flat (e.g. $J_p(E_p) \propto E_p^{-(2.3-2.4)}$) incident proton spectrum is required in the inner Galaxy by these authors to fit the $\gamma$-ray data.

Now, the locally observed proton spectrum is observed to have a spectrum $J_p(E_p) \propto E_p^{-2.76}$ (e.g. Webber 1983), which is significantly steeper than required in the inner Galaxy by these authors to explain the $\gamma$-ray results. This fact can be reconciled with the ‘hard proton spectrum’ hypothesis if the proton spectrum in the outer Galaxy, or away from the source regions, is dominated by diffusive transport which steepens the proton spectrum enough to agree with the local spectrum.

Instead of assuming an ad-hoc flat cosmic ray spectrum in the inner Galaxy, I estimate an ‘average’ hard proton spectrum for the Galaxy by considering the highest ambient proton spectrum allowed by the error bars on the highest energy data point for the positron spectrum given by Barwick et al. (1998). To understand why this can work, consider the typical diffusion distance, $r_{\text{diff}} \sim [K(E)\Delta t]^{1/2}$, for positrons with energies $\sim 10-100$ GeV. This is of the order 1 to several kpc if $\Delta t$ is estimated using Figure 5.1. So, positrons of these
Figure 5.6: Total secondary electron and positron distributions at (a) 10 GeV, and (b) 100 GeV as a function of position in the Galaxy. The secondary electron and positron source function has been calculated assuming the ‘hard’ proton spectrum discussed in the text, and the propagation calculations have been carried out using the parameters $z_h = 4.0$ kpc and $K_0 = 4.3 \times 10^{28}$ cm$^2$ s$^{-1}$. Note these distributions are quite broad, when compared with the spatial distribution of the secondary electron and positron source function (Figure 3.12).
energies essentially sample the interstellar proton spectrum on galactic scales. This can be seen by considering the total distributions of secondary electrons and positrons at 10 and 100 GeV respectively shown in Figures 5.6a and 5.6b; when compared with the spatial source distribution of these particles (see Figure 3.12) these distributions are quite broad in $R$ and $z$ and indicate effective diffusion from regions of relatively high gas density (e.g. the inner Galaxy) throughout the Galaxy. Therefore, contributions to the locally observed positron spectrum come from regions toward the inner Galaxy, as well as locally, and from the outer Galaxy. This would sample an ‘average’ interstellar proton spectrum that could be slightly flatter than locally observed, but not as flat as invoked to explain the inner Galaxy $\gamma$-ray data alone. By considering the flattest positron source spectrum, and hence flattest proton spectrum, that can be accomodated by the local positron spectrum observations after propagation, a useful estimate can be made of this ‘average’ proton spectrum, and this is the origin of the parameterisation given by Equation 5.16. Note that Moskalenko & Strong (1998a) find using a 3D propagation code for nuclei, and electrons and positrons, that an ambient proton spectral index of $\sim 2.6$ agrees with the high energy positron data also, which is entirely consistent with the simplified calculations presented here.

The effect on the predicted positron spectrum due to different $z_h / K_0$ combinations for the propagation model can be seen in Figure 5.7. At high energies, there is virtually no difference in the predicted spectra for different values of $z_h$ and $K_0$, mainly because energy losses tend to be the dominant influence on the spectrum at these energies. At lower energies, down to $\sim 0.01$ GeV, diffusion effects affect the model predictions, and the calculated spectra for the three $z_h / K_0$ combinations differ by $\sim 10\%$ with the predictions for the combination $z_h = 3$ kpc and $K_0 = 3.2 \times 10^{28}$ cm$^2$ s$^{-1}$ being the lowest; at lower energies energy losses again become the dominant influence on the spectrum.

Variations in the predicted electron and positron spectra in the $\sim 0.5 - 10$ GeV range are important because the non-thermal spectrum from $\sim 50$ MHz up to GHz frequencies, and bremsstrahlung and IC $\gamma$-rays below $\sim 0.1 - 3$ GeV and $\sim 0.05 - 1$ GeV, respectively, are generated by electrons and positrons in this energy range. Because the relevant observations of the galactic non-thermal spectrum used in Section 5.4.1 to constrain the likely source models lie in the above frequency range, it is important to consider propagation affects on the predicted non-thermal spectrum; this will be examined further in Section 5.4.1.
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Figure 5.7: Dashed line shows the positron spectrum for the parameters \( z_h = 3.0 \) kpc and \( K_0 = 3.2 \times 10^{28} \) cm\(^2\) s\(^{-1}\); Solid line shows the positron spectrum for the parameters \( z_h = 4.0 \) kpc and \( K_0 = 4.3 \times 10^{28} \) cm\(^2\) s\(^{-1}\); Dash-dotted line shows the positron spectrum for the parameters \( z_h = 5.0 \) kpc and \( K_0 = 5.3 \times 10^{28} \) cm\(^2\) s\(^{-1}\).

5.2.3.2 Total Electron Spectra

The interstellar total electron spectra obtained from the propagation calculations for the combination \( z_h = 4 \) kpc and \( K_0 = 4.3 \times 10^{28} \) cm\(^2\) s\(^{-1}\) are shown in Figures 5.8a and 5.8b, together with a compilation of measurements of the local spectrum (Webber, Simpson & Cane 1980; Golden et al. 1994; Nishimura et al. 1995 and references therein). The total spectra shown in the Figures have been calculated for primary injection spectral indices (i) \( \gamma = 1.8 \), and (ii) \( \gamma = 2.4 \), which are the upper and lower limits of the range for \( \gamma \) considered in this thesis and serve to illustrate the variation in the predicted spectra for the range of injection spectra. The secondary electron and positron contribution in each Figure has been calculated assuming the (a) Mori (1997) median proton spectrum, and (b) the hard proton spectrum given by Equation 5.16. In each Figure, the normalisation for the primary spectrum has been made by normalising the total spectrum (primary + secondary) to the
data points around 10 GeV; the reason for normalising the predicted spectra to the data around these energies was discussed in Section 5.2.1.

Generally, the inclusion of the secondary component improves the agreement between the predicted spectrum and the spectrum derived from non-thermal radio observations below \(\sim 1\) GeV for hard injection spectra, and causes soft injection spectra to not agree as well. For energies above the normalisation point, soft injection spectra provide probably the best fit to the data up to TeV energies, but still overpredict the observed spectrum at the highest energies. However, as was seen in Section 5.2.1 agreement with the locally observed spectrum can be achieved if a departure from the cylindrical symmetry assumed in the propagation model is made, and the discrete nature of the source distribution is taken into account. The spectra in the Figures are reasonably consistent with the observed spectrum up to \(\sim 30\)–40 GeV which is close to the energy threshold for which the distribution of sources was not found to have an effect on the local spectrum (see Figures 5.2a and 5.2b).

Several features of relevance for later Sections are shown in the total electron distributions for (a) 1 GeV, and (b) 1000 GeV given in Figures 5.9. At low energies, the distribution is quite broad due to diffusion effects, while at high energies the distribution is narrow, tracing the primary electron spatial source distribution. The broad extent of the low energy electron distribution is particularly important for predictions of high latitude IC \(\gamma\)-rays because the ambient photon populations that serve as targets for this process extend right out to the halo boundary (see e.g. Figure A.3c). The narrow distribution of high energy electrons shows that the effects of anisotropic corrections to the IC scattering formula (Moskalenko & Strong 1998b) will not affect the model predictions significantly at high energies, although these become less important at high energies where the main IC target is the (isotropic) CMBR field.

Note that variations in the predicted total spectrum below \(\sim 5\) GeV due to different \(z_h/K_0\) combinations (not shown) are similar to those for the interstellar positron spectrum shown in Figure 5.7. As noted earlier, these will have some influence on the predicted non-thermal spectrum, and bremsstrahlung and IC \(\gamma\)-ray fluxes.
Figure 5.8: Local interstellar total electron spectra for injection spectral indices (i) $\gamma = 1.8$, and (ii) $\gamma = 2.4$ respectively. Solid lines show the total spectrum (primary + secondary), while the dashed lines show only the primary spectrum. Total secondary electron and positron spectra (dotted lines) are calculated assuming the (a) Mori (1997) median proton spectrum, and (b) the hard proton spectrum obtained in Section 5.2.3.1. The total spectra have been normalised to the data points around 10 GeV, as described in the text. Data references: as in the legend of Figure 5.2.
Figure 5.9: Total electron distributions at (a) 1 GeV, and (b) 1000 GeV as a function of position in the Galaxy. The secondary electron and positron contribution has been calculated assuming the Mori (1997) ambient proton spectrum, and the propagation calculations have been made using the model parameters $z_h = 4.0$ kpc and $K_0 = 4.3 \times 10^{28}$ cm$^2$ s$^{-1}$. 
5.3 Diffuse Radiation Production

The energy lost by cosmic ray electrons and positrons while propagating is channelled into heating the ISM, and the production of diffuse radiation. Also, the cosmic ray nuclei responsible for the production of the secondary electron and positron component of the total electron spectrum produce neutral pions through interactions with the gas distribution, which subsequently decay to $\gamma$-rays.

In this Section, I give the formulae used to calculate the emissivities of photons arising from bremsstrahlung, IC, and synchrotron interactions for electrons and positrons, and the emissivity of photons produced by hadronic interactions in the ISM. The emissivities of these processes are then calculated using the spectra of electrons and positrons from the propagation calculations described in Section 5.2.2, and the cosmic ray proton spectra from Section 3.3.1.1. Several examples of these emissivity spectra for the local region are then shown.

The general references for this Section are Gould (1969), and Blumethal & Gould (1970) for bremsstrahlung and IC interactions, Pacholczyk (1970) for synchrotron radiation, and those given in Appendix D for the production of neutral pions.

5.3.1 Neutral Pion Decay

Following the discussion of the production of secondary electrons and positrons from hadronic interactions in the ISM (see Section 3.3.1), the emissivity of photons of energy $E_{\gamma}$ per interstellar H-atom at position $\vec{R}$ produced by inelastic collisions between cosmic ray nuclei and gas atoms at rest in the ISM can be written

$$Q_{\text{hadrons}}(E_{\gamma}, \vec{R}) = \int_{E_p}^{\infty} J_p(E_p, \vec{R}) \epsilon^M(E_p) \frac{d\sigma_{pp-2\gamma}}{dE_{\gamma}} dE_p \quad (5.17)$$

where $J_p(E_p, \vec{R})$ (GeV$^{-1}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$) is the incident cosmic ray proton spectrum, $\epsilon^M(E_p)$ is the nuclear enhancement factor taking into account cosmic ray nuclei heavier than protons, and the ISM composition assumed in this thesis (see Section 3.3.1.4), and $d\sigma_{pp-2\gamma}/dE_{\gamma}$ (GeV$^{-1}$ cm$^2$) is the production cross-section of $\gamma$-rays from proton-proton collisions in the ISM, and is given in Appendix D. The threshold proton energy for this process, $E_p^{th_1}$, is found by equating $s^{1/2} = [2m_p(E_p^{th_1} + m_p)]^{1/2} = [4m_p^2 + E_{\pi^0}^2 - m_{\pi^0}^2]^{1/2} + E_{\pi^\pm}$ with $E_{\pi^0} = \gamma_c(E_{\pi^0}^{min} - \beta_cE_{\pi^0}^{min})$ where $s$ is the square of the CMS energy, $m_p$ and $m_{\pi^0}$ are the masses of the proton and neutral pion respectively, $E_{\pi^0}^*$ is the pion energy in the CMS, $\gamma_c = s^{1/2}/2m_p$ and $\beta_c$ are...
the Lorentz factor and velocity of the CMS with respect to the IS, and \( E_\gamma^{\text{min}} = E_\gamma + m_{e^0}^2/4E_\gamma \), and \( p_{e^0}^{\text{min}} \) are the minimum pion energy and momentum in the IS respectively.

### 5.3.2 Bremsstrahlung

For electron and positron interactions with matter, the emissivity of photons of energy \( E_\gamma \) at positron \( \vec{R} \) produced in bremsstrahlung interactions with gas atoms of species \( s \) can be written

\[
Q_{\text{brem},s}(E_\gamma, \vec{R}) = \int_{E_e^{\text{th}}}^{\infty} J_e(E_\gamma, \vec{R}) \frac{d\sigma_{\text{brem},s}(E_\gamma, E_e)}{dE_\gamma} dE_e
\]

where \( J_e(E_\gamma, \vec{R}) \) (GeV\(^{-1}\) cm\(^{-2}\) s\(^{-1}\) sr\(^{-1}\)) is the incident total cosmic ray electron spectrum, and \( E_e^{\text{th}} = \max(E_\gamma, E_e^{\text{min}}) \) where \( E_e^{\text{min}} \) is the minimum energy bound of the incident electron spectrum which is taken to be \( 10^{-3} \) GeV, the lowest electron/positron energy in Section 5.2.2. The production cross-section of bremsstrahlung photons on gas atoms of species \( s \), \( d\sigma_{\text{brem},s}/dE_\gamma \) (GeV\(^{-1}\) cm\(^{-2}\)), is given by

\[
\frac{d\sigma_{\text{brem},s}(E_\gamma, E_e)}{dE_\gamma} = \frac{3\alpha \sigma_T}{8\pi E_\gamma} \frac{1}{E_\gamma} F_s(E_\gamma, E_e)
\]

\[
F_s(E_\gamma, E_e) = \left[ 1 + \left( \frac{\gamma}{\gamma_e} \right)^2 \right] \phi_{1,s}(E_\gamma, E_e) - \frac{2}{3} \frac{\gamma}{\gamma_e} \phi_{2,s}(E_\gamma, E_e)
\]

where \( \alpha \) and \( \sigma_T \) are the fine structure constant and the Thompson cross-section respectively, \( \gamma \equiv (E_e - E_\gamma)/m_e c^2 \) and \( \gamma_e \equiv E_e/m_e c^2 \) are the final and initial Lorentz factors of the incident electron/positron, and \( \phi_{1,s} \) and \( \phi_{2,s} \) are functions that describe the electron/positron scattering on gas atoms of species \( s \), and are calculated as described in Appendix C.

Equation 5.20 is sufficient for the treatment of bremsstrahlung production by electrons with kinetic energies \( \sim 2 \) MeV and higher. At lower energies more complicated expressions provide a better description (Strong, Moskalenko & Reimer 1998), but since I only calculate the electron/positron spectrum down to 1 MeV the present treatment of the production of bremsstrahlung photons is probably adequate. At any rate, the contribution by bremsstrahlung to the total spectrum has been shown to be at most a few percent around 1 MeV and lower (e.g. Porter & Protheroe 1997; Strong, Moskalenko & Reimer 1998) so any error induced in the total diffuse \( \gamma \)-ray predictions will be negligible.

I consider bremsstrahlung production on neutral hydrogen and helium, and ionised hydrogen; it is important to include ionised hydrogen because of its possible contribution at
high galactic latitudes due to its large scale height perpendicular to the galactic plane.

5.3.3 Inverse Compton Scattering

For electrons or positrons interacting with low energy photons through IC scattering events, the emissivity of photons of energy $E_\gamma$ at position $\vec{R}$ can be written

$$Q_{IC}(E_\gamma, \vec{R}) = \frac{4\pi}{c} \int_{E_e^h}^{\infty} J_e(E_e, \vec{R}) \frac{dP_{IC}(E_\gamma, E_e, \vec{R})}{dE_\gamma} dE_e$$

(5.21)

where $J_e(E_e, \vec{R})$ (GeV$^{-1}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$) is the incident total electron spectrum, and $E_e^h$ is the threshold electron energy for production of scattered photons off target photons with energy $\epsilon$,

$$E_e^h = \frac{E_\gamma + \left(E_\gamma^2 + (m_e c^2)^2 E_\gamma / \epsilon\right)^{1/2}}{2}.$$  

(5.22)

The production rate of IC photons, $dP_{IC}/dE_\gamma$ (GeV$^{-1}$ s$^{-1}$), is given by

$$\frac{dP_{IC}(E_\gamma, E_e, \vec{R})}{dE_\gamma} = c \int_{0}^{\infty} n(\epsilon, \vec{R}) \frac{d\sigma_{IC}(E_\gamma, \epsilon, E_e)}{dE_\gamma} d\epsilon$$

(5.23)

with $n(\epsilon, \vec{R})$ (GeV$^{-1}$ cm$^{-3}$) the number density of target photons at $\vec{R}$, $d\sigma_{IC}/dE_\gamma$ (GeV$^{-1}$ cm$^2$) is the production cross-section of IC photons, and it has been assumed that the target photon distribution is isotropic. The production cross-section of IC photons is given by

$$\frac{d\sigma_{IC}(E_\gamma, \epsilon, E_e)}{dE_\gamma} = \frac{3\sigma_T (m_e c^2)^2}{4E_e^2 \epsilon} F(p, q)$$

(5.24)

where

$$F(p, q) = 2q \ln q + (1 + 2q)(1 - q) + \frac{1}{2} \frac{(pq)^2}{1 + pq} (1 - q)$$

(5.25)

$$p = \frac{4E_e}{(m_e c^2)^2}$$

(5.26)

$$q = \frac{E_\gamma}{p(E_e - E_\gamma)}.$$  

(5.27)

In this thesis, the target photon populations are modelled as dilute blackbody distributions (see Section A.2, Appendix A). Therefore, the target photon number density distribution, $n(\epsilon, \vec{R})$, is just
\[ n(e, \vec{R}) = \frac{\omega(\vec{R})}{(kT)^4 (hc)^3} \frac{e^2}{e^2/kT - 1} \]  

(5.28)

where \( \omega(\vec{R}) \) (eV cm\(^{-3}\)) is the energy density of the target photon field at position \( \vec{R} \), \( k \) is Boltzmann’s constant and \( T \) is the temperature of the blackbody.

Note the assumption made in this thesis of an isotropic target photon distribution may result in some uncertainty for diffuse IC \( \gamma \)-ray predictions below \( \sim 10 \) GeV (e.g. Moskalenko & Strong 1998b); this will be discussed further in Section 5.4.2.

### 5.3.4 Synchrotron Radiation

For electron or positron interactions with a magnetic field, the emissivity of photons of energy \( E_\gamma \) at position \( \vec{R} \) can be written

\[ Q_{\text{sync}}(E_\gamma, \vec{R}) = \frac{4\pi}{c} \int_0^\infty J_e(E_e, \vec{R}) \frac{dP_{\text{sync}}(E_\gamma, E_e)}{dE_\gamma} dE_e \]

(5.29)

where \( J_e(E, \vec{R}) \) (GeV\(^{-1} \) cm\(^{-2} \) s\(^{-1} \) sr\(^{-1} \)) is the incident total electron spectrum, and \( dP_{\text{sync}}/dE_\gamma \) (GeV\(^{-1} \) s\(^{-1} \)) is the production rate of synchrotron photons. This is given by

\[ \frac{dP_{\text{sync}}(E_\gamma, E_e)}{dE_\gamma} = \frac{\sqrt{3}e^3 cB(\vec{R})}{(m_e c^2) h E_\gamma} F(E_\gamma/h\nu_c) \]

(5.30)

where \( F(x) = x \int_x^\infty K_{5/3}(x) dx \) with \( K_{5/3} \) the modified Bessel function of order \( 5/3 \), and the critical frequency, \( \nu_c \), is given by

\[ \nu_c = \frac{3eB(\vec{R}) c E_e^2}{4\pi (m_e c^2)^3} \]

(5.31)

and \( B (\mu \text{G}) \) is the magnetic field strength at position \( \vec{R} \), and \( e \) is the magnitude of the electronic charge (\( e \equiv |e| \)).

The calculation of the synchrotron emissivity using Equation 5.29 is correct provided \( \nu_s \ll c/\tau_e \) where \( \nu_s \sim \gamma^2 \nu_c \) with \( \gamma_c = E_e/m_e c^2 \) (e.g. Blumenthal & Gould 1970), which for the typical electron/positron energies and magnetic field strengths considered in this thesis is always satisfied.

### 5.3.5 Photon Emissivities for Sample Electron and Proton Spectra

Using the formulae given in Sections 5.3.2 to 5.3.4, I calculate the photon emissivity spectra for bremsstrahlung, IC and synchrotron radiation using the electron and positron distributions obtained in Section 5.2.3.
For each of the annuli centred on \( R = 1, 3, 5, 7, 9, 11, 13, \) and 15 kpc for which the electron and positron spectra are obtained from the propagation calculations, the photon emissivity is obtained by integrating the incident electron and positron spectra together with the appropriate production function as described earlier. The photon emissivity from neutral pion decays is also calculated for each of these annuli using the incident proton spectra given in Section 5.2.3.1, weighted by the spatial distribution of secondary electron and positron sources (Equation 3.39), and then integrated with the appropriate production function.

For the purposes of illustration, I consider the emissivity spectra for the annulus centred on \( R = 9 \) kpc; this corresponds to the emissivity distribution for the processes described above in the local region.

For cosmic ray interactions with matter, Figures 5.10a and 5.10b show the emissivity spectra of bremsstrahlung and neutral pion decay produced photons per interstellar H-atom in the galactic plane. The primary electron injection spectral indices used to calculate these emissivity spectra are: (i) \( \gamma = 1.8 \); (ii) \( \gamma = 2.0 \); (iii) \( \gamma = 2.2 \); and (iv) \( \gamma = 2.4 \). The incident proton spectra used for the secondary electron and positron contribution to the total electron spectrum for either Figure, and the neutral pion decay photon emissivity, are: (a) the Mori (1997) median proton spectrum; and (b) the maximal incident proton spectrum (the 'hard' proton spectrum) consistent with the observed positron spectrum, as found in Section 5.2.3.1 (Equation 5.16). Also shown in the Figures are emissivity spectra per interstellar H-atom for the local region estimated from COS-B data (Strong et al. 1988), COMPTEL data (Strong et al. 1996), and EGRET data (Strong & Mattox 1996). Note the electron and positron spectra used in Figures 5.10 and 5.10b correspond to the halo size and diffusion coefficient normalisation \( z_h = 4.0 \) kpc and \( K_0 = 4.3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1} \) respectively; the effect of varying these parameters in the propagation calculations, and the ensuing effect on the predicted emissivity spectra, will be discussed shortly.

In general, soft injection spectra (\( \gamma \sim 2.4 \)) agree better with the COMPTEL data, while harder injection spectra (\( \gamma \sim 2.0 - 2.2 \), even \( \gamma \sim 1.8 \)) agree better with the EGRET data below \( \sim 500 \) MeV. There is some uncertainty in the COMPTEL spectrum below 30 MeV (Strong 1997), which could amount to a factor of two overestimate of the true emissivity spectrum. If this is taken into account then the \( \gamma = 2.2 \) spectrum agrees with the COMPTEL data as well down to \( \sim 3 \) MeV, although below this the predicted emissivity for this injection spectrum still underpredicts the data. At higher energies, the predictions are consistent with the COS-B data, but underpredict the EGRET data to some degree. Recent explanations
of this apparent high energy excess favour an IC origin (e.g. Pohl & Esposito 1998), and so the excess in the emissivity spectra for matter interactions could be due to the assumptions about the IC contribution (which must be subtracted) made during the analysis procedure used to interpret the $\gamma$-ray data, from which the emissivity spectra are derived. The problem of the high energy excess will be examined further when predictions of the diffuse intensity spectra are made in Section 5.4.2.

The variation of the predicted secondary electron and positron spectra for the halo size and diffusion coefficient normalisations was shown in Section 5.2.3.1; a similar variation also obviously occurs for the total predicted electron spectra. For the case $z_h = 3.0$ kpc and $K_0 = 3.2 \times 10^{28}$ cm$^2$ s$^{-1}$ the secondary positron spectrum below $\sim 5$ GeV was lowest, while for the case $z_h = 5.0$ kpc and $K_0 = 5.3 \times 10^{28}$ cm$^2$ s$^{-1}$ the predicted spectrum below $\sim 5$ GeV
Figure 5.11: Local $\gamma$-ray emissivity for cosmic ray interactions with matter calculated for different halo size/diffusion coefficient normalisation values. Dashed curves are electron bremsstrahlung spectra for an injection spectral index $\gamma = 2.4$ and the parameters $z_h = 3.0$ kpc and $K_0 = 3.2 \times 10^{28}$ cm$^2$ s$^{-1}$ (lower), $z_h = 4.0$ kpc and $K_0 = 4.3 \times 10^{28}$ cm$^2$ s$^{-1}$ (middle), and $z_h = 5.0$ kpc and $K_0 = 5.3 \times 10^{28}$ cm$^2$ s$^{-1}$ (upper). Dotted curve is the photon emissivity for decaying neutral pions resulting from cosmic ray interactions with gas in the ISM calculated assuming the Mori (1997) median proton spectrum. Data as in the legend of Figure 5.10.

was highest; at higher energies no significant variation with different $z_h/K_0$ combinations was found. Electrons and positrons of $\sim 5$ GeV produce bremsstrahlung $\gamma$-rays of energies $\sim 1-2$ GeV, and so a variation in the predicted emissivity spectrum for cosmic ray interactions with matter would be expected for energies where the bremsstrahlung contribution dominates. Figure 5.11 shows the local emissivity spectrum for interactions with matter calculated for the injection spectral index $\gamma = 2.4$ and the Mori (1997) median proton spectrum, and the three $z_h/K_0$ combinations found in Chapter 4. Propagation effects cause the predicted emissivity to vary by $\sim 30\%$ below $\sim 50$ MeV; above this the neutral pion decay contribution dominates. Although the variation for different $z_h/K_0$ combinations is not insignificant, it will be shown in Section 5.4.1 that the combination $z_h = 4$ kpc and $K_0 = 4.3 \times 10^{28}$ cm$^2$ s$^{-1}$ provide a good fit to the galactic non-thermal emission for the greatest number of possible
source models. Therefore, only the predicted emissivities for this $z_h/K_0$ combination will be considered further in this Section.

![Graphs](image)

**Figure 5.12:** Synchrotron and IC emissivity spectra for the annulus centred on $R = 9.0$ kpc for injection spectral indices (a) $\gamma = 1.8$, (b) $\gamma = 2.0$, (c) $\gamma = 2.2$, and (d) $\gamma = 2.4$ for an exponential cut-off in the injection spectrum at 100 TeV. Total electron spectra have been calculated assuming the Mori (1997) median proton spectrum, and the model parameters $z_h = 4.0$ kpc and $K_0 = 4.3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$.

For synchrotron and IC interactions, I show the variation of the predicted emissivity
spectra with distance perpendicular to the galactic plane in Figures 5.12a to 5.12d for the annulus centred on \( R = 9 \) kpc. In these Figures, I have calculated the emissivities for primary injection spectral indices (i) \( \gamma = 1.8 \), (ii) \( \gamma = 2.0 \), (iii) \( \gamma = 2.2 \), and (iv) \( \gamma = 2.4 \) for an exponential cut-off in the injection spectrum at 100 TeV; note that a cut-off in the injection spectrum is expected for energies \( \sim 100 - 1000 \) TeV if electrons are accelerated by SNR (e.g. Lagage & Cesarsky 1983; Hillas 1984). Self-consistency for both the synchrotron and IC spectra is ensured by using the same ambient photon field and magnetic field model for the propagation calculation and the emissivity spectra calculation. As can be seen in the Figures, synchrotron radiation can contribute significantly in the hard X-ray regime, being at least comparable to the IC contribution for energies \( < 10 - 100 \) keV in the galactic plane for all but the softest injection spectra considered. Note that for the energies where synchrotron radiation could be an important contributor to the spectrum, the synchrotron emissivity falls off quicker with distance perpendicular to the plane than the IC emissivity. This is because the scale heights of the electron distributions producing the photons are vastly different: the IC spectrum is produced by \( < 0.1 - 1 \) GeV energy electrons which have an extensive distribution (see Figure 5.9a), while the synchrotron spectrum is produced by TeV energy electrons which are closely confined to the galactic plane (see Figure 5.9b).

Variation of the primary injection spectral index from \( \gamma = 2.4 \) to \( \gamma = 1.8 \) increases the synchrotron emission, and tends to flatten the IC spectrum. Furthermore, the shape and cut-off in the synchrotron and IC spectra are directly related to the form of the cut-off, and the cut-off energy, in the primary injection spectrum. The variation with the form of the high energy cut-off (e.g. abrupt or exponential) can be seen in Figures 5.13a and 5.13b, which have been calculated assuming a cut-off energy of 1000 TeV and an injection spectral index \( \gamma = 2.0 \). The variation of the cut-off in the synchrotron and IC spectra with cut-off energy can be seen by directly comparing Figures 5.12b and 5.13a which are calculated for an exponential cut-off with cut-off energies at 100 TeV and 1000 TeV respectively. For an abrupt cut-off, the synchrotron and IC emission is slightly higher than for the exponential cut-off for the energy decade below which the emissivity spectrum for either production process steepens due to the cut-off. This is directly related to the form of the electron spectrum at high energies, and the lower prediction for the exponential cut-off is because the primary electron spectrum begins to steepen at a lower energy than the abrupt cut-off case.

Since both synchrotron and IC emissivity spectra display features that can be directly related to the form of the primary injection spectrum near its inevitable high energy cut-off,
Figure 5.13: Synchrotron and IC emissivity spectra for a $\gamma = 2.0$ injection spectrum, and an (a) exponential cut-off, and an (b) abrupt cut-off, in the injection spectrum at 1000 TeV.

This can be useful as a direct test of the average primary injection spectrum at acceleration. This allows more stringent constraints to be placed on models of galactic electron sources because they must simultaneously agree with experimental data in both of the energy regimes in which synchrotron and IC X-/$\gamma$-rays dominate the emission.

5.4 Diffuse Photon Intensity Spectra

Using the emissivity spectra for bremsstrahlung, synchrotron, IC and $\pi^0$-decay photons calculated for each of the annuli in Section 5.3.5, I calculate the diffuse intensity spectra for these production processes, and compare my predictions with observation in this Section. The diffuse non-thermal galactic emission is first used to select possible source models, since this is a constraint that must be satisfied by any model. Then, predictions of the diffuse X-/$\gamma$-ray spectrum are made for these source models, which are then compared with satellite, Cherenkov telescope, and air shower array results.

Observations of diffuse intensity spectra are usually obtained for a given region of sky specified by some galactic longitude and latitude intervals. To calculate the intensity spectra for a particular region of sky, the intensity in a single direction must first be obtained. For
the intensity of diffuse photons of energy $E_{\gamma}$ in the direction of galactic longitude $l$ and $b$ resulting from the interactions described in Section 5.3, the following integral needs to be evaluated:

\[
J_{\gamma}(E_{\gamma}, l, b) = \int_0^{r_{\text{max}}(l, b)} \left[ Q_{\text{hadm} \rightarrow 2\gamma}(E_{\gamma}, l, b, r) + \sum_{s=\text{H}, \text{He}, \text{HII}} \frac{n_s}{n_H} Q_{\text{brems},s}(E_{\gamma}, l, b, r) \right] \\
\times \left[ n_{\text{H}}(l, b, r) + n_{\text{H}_2}(l, b, r) + n_{\text{H}^{\text{II}}}(l, b, r) \right] e^{-r/r_{\text{CMBR}}(E_{\gamma})} dr \\
+ \frac{1}{4\pi} \int_0^{r_{\text{max}}(l, b)} \left[ \sum_{i=1,8} Q_{\text{IC},i}(E_{\gamma}, l, b, r) + Q_{\text{syn},i}(E_{\gamma}, l, b, r) \right] e^{-r/r_{\text{CMBR}}(E_{\gamma})} dr
\]

(5.32)

where $Q_{\text{hadm} \rightarrow 2\gamma}$ is given by Equation 5.17, $Q_{\text{brems},s}$ is given by Equation 5.18, $n_{\text{H}}$ and $n_{\text{H}_2}$ are atomic and molecular hydrogen number densities and are obtained from the gas model described in Appendix A, $Q_{1\text{C},i}$ is given by Equation 5.21 and the sum is over the eight blackbody components of the ISRF model described in Appendix A, $Q_{\text{syn},i}$ is given by Equation 5.29, and the integration in each term is along the line-of-sight in the direction defined by $l, b$, out to a distance $r_{\text{max}}$. The exponential factor in both of the integrands takes into account attenuation of the diffuse flux at high energies ($E_{\gamma} \geq 50$ TeV or so) by pair production on the CMBR ($\gamma + \gamma_{\text{CMBR}} \rightarrow e^+ + e^-$). Formally, this is included for all processes, but in practice only needs to be calculated for the $\pi^0$-decay and IC contributions because bremsstrahlung and synchrotron radiation are negligible at the energies where attenuation on the CMBR is important. The attenuation path length, $r_{\text{CMBR}}(E_{\gamma})$, is calculated as a function of incident photon energy as described by Protheroe (1986).

Having obtained the diffuse intensity in a single direction specified by $l$ and $b$, it is straightforward to obtain the intensity over a region specified by the longitude and latitude intervals $l_i \leq l \leq l_u$ and $b_l \leq b \leq b_u$ respectively. This is obtained by averaging the quantity

\[
J_{\gamma}(E_{\gamma}, l) = \frac{1}{(b_u - b_l)} \int_{b_l}^{b_u} J_{\gamma}(E_{\gamma}, l, b) db
\]

(5.33)

over the entire longitude interval. In practice, the intensity needs to be calculated for a number of discrete directions within the specified longitude and latitude intervals. Assuming a spherical geometry centred on the Sun, the sky within the specified longitude and latitude interval is divided into pixels of half-width $\Delta l/2$ and $\Delta b/2$ centred on equally spaced $l$ and $b$ values such that the region of sky is entirely covered by the pixels. Then, the following
summation is performed to obtain the intensity over the region of sky for the longitude and latitude intervals above:

\[ J_\gamma(E_\gamma) = \frac{\sum_{i=1}^{l_i} \sum_{j=1}^{b_j} J_\gamma(E_\gamma, l_i, b_j) \cos b_j \Delta l}{\sum_{i=1}^{l_i} \sum_{j=1}^{b_j} \cos b_j \Delta l} \]  

(5.34)

where \( l_i \) and \( b_j \) are the mid-pixel longitude and latitudes respectively. For the calculations presented in Sections 5.4.1 and 5.4.2 I use pixels of half-width \( \Delta l/2 = 0.5^\circ \) and \( \Delta b/2 = 0.5^\circ \).

By way of explanation, the relationship between galactic coordinates \( l \) and \( b \), and the cylindrical coordinates used in the propagation model, and models of the gas distribution, radiation and magnetic fields described in Appendix A is given by

\[ R^2 = R'^2 + R_S^2 - 2R'R_S \cos l \]  

(5.35)

\[ z = r \sin b \]  

(5.36)

\[ R' = r \cos b \]  

(5.37)

where \( R \) is measured from the galactic centre to an arbitrary point in the galactic plane, \( R' \) is the distance from the solar system to the same point in the plane, \( R_S \) is the distance of the solar system to the galactic centre, and \( z \) is the distance perpendicular to the galactic plane. For the line-of-sight integrations in Equation 5.32 the maximum integration distance, \( r_{\text{max}} \), as a function of \( l \) and \( b \) is found by first considering the maximum possible distance \textit{in the plane} as a function of galactic longitude. This is found by rearranging Equation 5.35, and taking the positive root of the resulting quadratic equation for \( R' \) with \( R = R_{\text{max}} \) where \( R_{\text{max}} \) is the maximum radial extent of the Galaxy

\[ R'_{\text{max}}(l) = R_S \cos l + \sqrt{R_S^2 \sin^2 l + R_{\text{max}}^2}. \]  

(5.38)

Now if \( \tan b = z_{\text{max}}/R'_{\text{max}}(l) \) where \( z_{\text{max}} \) is the maximum extent of the Galaxy perpendicular to the galactic plane, obviously \( r_{\text{max}} = \sqrt{R'_{\text{max}}(l)^2 + z_{\text{max}}^2} \). Otherwise, if \( \tan b < z_{\text{max}}/R'_{\text{max}}(l) \) then \( r_{\text{max}} = R'_{\text{max}} \sqrt{1 + \tan^2 b} \), and if \( \tan b > z_{\text{max}}/R'_{\text{max}}(l) \) then \( r_{\text{max}} = z_{\text{max}} \sqrt{1 + (\tan^2 b)^{-1}} \).

5.4.1 Non-thermal Radio

The non-thermal emission of the Galaxy in the radio band (~ 10 MHz to ~ 1 GHz) is generated by synchrotron radiating electrons and positrons with energies ~ 0.5 to ~ 5 GeV
for a magnetic field strength $\sim 6 \mu G$. In the direction of the galactic poles, this emission is essentially generated by the local electron and positron distribution, which for the energy range above contains contributions from sources within a few kpc of the local region. To see this note that the typical diffusion distance for electrons and positrons of these energies, $r_{\text{diff}} \sim [K(E)\Delta t]^{1/2} \equiv [K(E)\Delta E/b(E)]^{1/2}$, is of the order several kpc (for $K(E)$ and $b(E)$ as in Section 5.2.1). Thus, the local electron and positron spectrum in this energy range samples the respective source distributions of these particles on kpc scales. Now the main dominant contribution to the non-thermal emission is produced by primary electrons, since these make up $\sim 90\%$ of the total cosmic ray electron spectrum. Therefore, predictions of the polar non-thermal emission can be used as a constraint on possible models for the primary injection spectrum within several kpc of the local region.

Figures 5.14a and 5.14b show the predicted non-thermal intensity spectrum in the direction of the galactic pole calculated using Equation 5.34 for different primary injection spectra, and ambient proton spectra. Also shown are data points that have been obtained for the brightness temperature at the pole for 408 MHz given by Broadbent et al. (1990), $T_{\text{408 MHz}} = 12.3$ K, and the range of synchrotron indices at 38 MHz and 1420 MHz given by Lawson et al. (1987) and Reich & Reich (1988) respectively. In these Figures, the model predictions are made for the $z_h/K_0$ combination $z_h = 4.0$ kpc and $K_0 = 4.3 \times 10^{28}$ cm$^2$ s$^{-1}$; non-thermal spectra for other combinations will be examined shortly.

It can be seen from the Figures that soft primary injection spectra ($\gamma \sim 2.4$) provide a poor fit to the non-thermal data at all frequencies. For harder injection spectra ($\gamma \sim 1.8 - 2.0$), however, the situation is different. If the primary spectra are considered on their own then the agreement with the data for these injection spectral indices is only reasonable for $\gamma = 2.0$, but when the secondary electron and positron contribution is included the agreement is good for even $\gamma = 1.8$. Note it is the inclusion of the secondary electron and positron component that makes it possible to obtain agreement at low frequencies for hard injection spectra; including the secondary component only worsens the agreement for soft injection spectra. It should be emphasised that the inclusion of the secondary component of the non-thermal emission cannot be avoided because the predictions for the secondary electron and positron spectra are absolute. So, the non-thermal emission due to secondary electrons and positrons provides a natural ‘base line’ for the galactic emission, upon which the contribution by primary electrons is then included.

Although slight differences at low frequencies occur, the agreement for primary injection
Figure 5.14: Non-thermal intensity in the direction of the north galactic pole. Thin solid, dashed, and dash-dotted lines show the non-thermal intensity spectra for primary electrons corresponding to injection spectral indices $\gamma = 1.8, 2.0$ and 2.4 respectively. Triple dot-dashed line shows the non-thermal intensity spectrum for secondary electrons and positrons for the (a) Mori (1997) median proton spectrum, and (b) hardest proton spectrum allowed by the local positron spectrum data (Section 5.2.3.1). Thick solid, dashed, and dash-dotted lines show the non-thermal intensity spectrum for the sum of the intensities corresponding to the injection spectra above, and the contribution by secondary electrons and positrons.

indices $\gamma \sim 1.8 - 2.0$ for both ambient proton spectra is good (Figure 5.14a for the Mori (1997) median spectrum, and Figure 5.14b for the hard proton spectrum). This is because of the normalisation procedure for the total spectrum, and both proton spectra result in
similar fluxes of secondary electrons and positrons below $\sim 1$ GeV anyway.

In the earlier work of Porter & Protheroe (1997) only primary electrons were considered, and it was not possible to fit the polar non-thermal spectrum across the full frequency range covered by the observations for any of the injection spectra considered; it can be seen now that including secondary electrons and positron allows the full frequency range of the observations to be fitted only for hard injection spectra. So, within several kpc of the local region, it appears hard ($\gamma \sim 1.8 - 2.0$) spectral indices are favoured by observations of the polar non-thermal emission.

\[
\begin{align*}
\nu \ (\text{Hz}) & \quad \nu \ J(\nu) \ (\text{erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}) \\
10^{-11} & \quad 10^{-10} & \quad 10^{-9} & \quad 10^{-8} & \quad 10^{-7} & \quad 10^{-6} \\
& & & & & \\
\end{align*}
\]

Figure 5.15: Non-thermal intensity in the direction of the north galactic pole for primary injection index $\gamma = 2.0$. Solid line shows the intensity for the halo size/diffusion coefficient normalisation $z_h = 3.0$ kpc and $K_0 = 3.2 \times 10^{28}$ cm$^2$ s$^{-1}$, dashed line shows the intensity for $z_h = 4.0$ kpc and $K_0 = 4.3 \times 10^{28}$ cm$^2$ s$^{-1}$, and dash-dotted line shows the intensity for $z_h = 5.0$ kpc and $K_0 = 5.3 \times 10^{28}$ cm$^2$ s$^{-1}$. The secondary electron and positron contribution is calculated using the Mori (1997) median proton spectrum.

Variations in the non-thermal predictions for different $z_h/K_0$ combinations can be expected because the electron and positron spectra below $\sim 5 - 10$ GeV are affected by propagation effects. These cause the predicted electron and positron spectra to be slightly higher or lower than those used to calculate Figures 5.14a and 5.14b (e.g. Figure 5.7). Additionally, the scale height of the electron and positron distributions producing the non-thermal emission, and the GMF, is similar to the size of the halo. Therefore, the predicted non-thermal emission can be expected to scale to some degree with the size of the halo used in the propagation calculations.
To examine the variations in the non-thermal predictions due to different combinations of \( z_h \) and \( K_0 \), I calculate the non-thermal emission for the combinations of \( z_h \) and \( K_0 \) found in Chapter 4 for the injection spectral index \( \gamma = 2.0 \), since this agrees best over the frequency range covered by the observations. In Figure 5.16 I show the model predictions for the three \( z_h/K_0 \) combinations. The predicted spectrum is higher for the large \( z_h/K_0 \) combination, and lower for the small \( z_h/K_0 \) combination. Now, while obviously for a \( \gamma = 2.0 \) injection spectrum the middle \( z_h/K_0 \) combination gives the best agreement, the sort of variations in the predictions evident in the Figure mean it might be possible that varying the diffusion model parameters will allow harder or softer injection spectra to give better agreement with the data. Realistically, with the sort of variations that can be seen in the Figure only \( \gamma = 1.8 \) and \( \gamma = 2.2 \) injection spectra need to be investigated further. Note that for the middle \( z_h/K_0 \) combination \( \gamma = 1.8 \) already provides a reasonable fit to the data.

Figures 5.16a and 5.16b show the predicted non-thermal intensities for (a) \( \gamma = 1.8 \) and (b) \( \gamma = 2.2 \) injection spectra. It can be seen that the agreement for a large \( z_h/K_0 \) combination for the \( \gamma = 1.8 \) spectrum is as reasonable as for the middle combination, only slightly outside the error bars for the highest frequency data point. For a \( \gamma = 2.2 \) injection spectrum, reasonable agreement with the data is only obtained for the smallest \( z_h/K_0 \) combination. Note that the secondary electron and positron contribution for these Figures is calculated using the Mori (1997) median proton spectrum; the hard proton spectrum gives similar results.

Having investigated the variations in the predictions of the non-thermal spectrum due to the primary injection spectral indices, ambient proton spectra used for the secondary electron and positron source function, and to different combinations of the propagation model halo size and diffusion coefficient normalisation, the likely source models can be found. Generally, hard injection spectra are favoured. For the middle \( z_h/K_0 \) combination \( \gamma = 2.0 \) is in good agreement with the data, while a \( \gamma = 1.8 \) index is in reasonable agreement. Also, for a \( \gamma = 1.8 \) index a large \( z_h/K_0 \) combination provides as good agreement as the middle combination. For a softer injection spectrum (\( \gamma = 2.2 \)) a small \( z_h/K_0 \) combination is required to give reasonable agreement with the data.

For the predictions of diffuse X-/\( \gamma \)-rays in the next Section, I consider the injection spectral indices \( \gamma = 1.8 \) and \( \gamma = 2.0 \) for the middle \( z_h/K_0 \) combination as the ‘best fit’ model to the non-thermal emission since the predictions for \( \gamma = 2.0 \) actually go through all of the non-thermal data points; the other combinations of injection index and \( z_h/K_0 \) are investigated as special cases.
5.4. DIFFUSE PHOTON INTENSITY SPECTRA

![Graph (a)](image-a.png)

![Graph (b)](image-b.png)

Figure 5.16: Non-thermal intensity in the direction of the north galactic pole for primary injection indices (a) $\gamma = 1.8$, and (b) $\gamma = 2.2$. The line-styles for different $z_h/K_0$ combinations are the same as in the legend of Figure 5.15. In each Figure the secondary electron and positron contribution is calculated using the Mori (1997) median proton spectrum.

5.4.2 X-rays and $\gamma$-rays

‘Best Fit’ Source Model Spectra

The inner Galaxy intensity spectra predicted for the ‘best fit’ source models for the polar non-thermal emission are shown in Figures 5.17a–d. Also shown in the Figures are observations by OSSE for $l = 0^\circ$ and $l = 25^\circ$ (Kinzer et al. 1997), and COMPTEL (Strong
et al. 1998), and EGRET (Strong & Mattox 1996; Strong, Moskalenko & Reimer 1998) for the same region of the Galaxy. Note that the OSSE data contain positron annihilation components (continuum + line), and some contribution by point sources. The unique spectral shapes of the positron annihilation components allow them to be reliably separated from the other components, and a useful estimate of the observed spectrum with these removed is obtained by a linear connection between the spectra < 300 keV and above ~ 1 MeV in the Figures. Purcell et al. (1996) showed that the < 300 keV spectrum, measured toward the galactic centre and corrected for discrete sources resolved by the SIGMA imaging γ-ray telescope (e.g. Paul et al. 1991), is approximately the same shape and intensity as at 25° longitude. Therefore, even though there may still be some unresolved point source contribution to the sub-MeV spectrum, the spectrum observed for l = 25° (with positron annihilation components ‘removed’) is probably the best estimate of the diffuse emission for these energies.

In the Figures, the theoretical predictions have been calculated for (a) a γ = 1.8 injection spectral index, and the Mori (1997) median proton spectrum, (b) a γ = 1.8 injection spectral index, and the hard proton spectrum (Equation 5.16), (c) a γ = 2.0 injection spectral index, and the Mori (1997) median proton spectrum, and (d) a γ = 2.0 injection spectral index, and the hard proton spectrum. For the primary injection spectra, exponential and abrupt cut-offs in the injection spectrum at 100 TeV and 1000 TeV respectively have been used to show the range of spectra that could occur for different high energy cut-offs in the injection spectrum.

The models are consistent with the data from ~ 0.1 – 1 GeV, and slightly underpredict the data for energies above this. Below ~ 0.1 GeV down to ~ 10 MeV the underpredictions by the models with a γ = 1.8 injection spectrum is a factor ~ 2, while the models with γ = 2.0 underpredict the data a factor ~ 2.5. Some of this can be ascribed to the uncertainties in the UV/optical radiation field model, for in this energy regime IC scattering of UV/optical, and infrared, photons are the dominant contributions to the IC flux, which tends to be the major γ-ray production process operating in this energy range. The uncertainty in the UV/optical component of the interstellar radiation field is probably the greatest due to the approximations used in the absorption calculation, and with estimates of the total galactic luminosity of this component varying by a factor of two or more (e.g. Chi & Wolfendale 1991; Strong & Youssefi 1991). Indeed, a recent estimate of the UV/optical component (Strong, Moskalenko & Reimer 1998) gives an energy density ~ 3 times that used in this thesis for the
Figure 5.17: Diffuse intensity spectra for the inner Galaxy ($-30^\circ \leq l \leq 30^\circ$, $|b| \leq 5^\circ$) for primary injection spectrum (a) $\propto E^{-1.8}$ and Mori median proton spectrum, (b) $\propto E^{-1.8}$ and the hard proton spectrum, (c) $\propto E^{-2.0}$ and Mori median proton spectrum, and (d) $\propto E^{-2.0}$ and the hard proton spectrum. Theoretical predictions: dashed line (IC); dotted line ($\pi^0$-decay); dash-dotted line (bremsstrahlung); triple-dot dashed lines (synchrotron); solid lines (total). Lower triple-dot dashed line above 10 keV calculated for an abrupt cut-off at 100 TeV; next triple-dot dashed line for an exponential cut-off at 100 TeV; other triple-dot dashed lines for same cut-offs at 1000 TeV. Data: stars and crosses, OSSE data for $l = 0^\circ$ and $l = 25^\circ$ respectively (Kinzer et al. 1997); open boxes, COMPTEL (Strong et al. 1998); solid boxes, EGRET (Strong & Mattox 1996; Strong, Moskalenko & Reimer 1998).

inner 3 kpc of the Galaxy. Additionally, anisotropic corrections to the IC scattering formula (Moskalenko & Strong 1998b) that affect the predictions of IC $\gamma$-rays produced by scattering
off starlight photons (see Moskalenko & Strong 1998b for a detailed description) mean that the model predictions in the energy regime in which IC scattering of UV/optical photons dominates are only lower bounds on the true flux. Given these uncertainties, it is probably reasonable to allow that the model predictions below $\sim 10$ GeV for the IC flux can vary by a factor of at least two, and possibly more. Note that at higher energies IC scattering of UV/optical, and infra red photons is in the Klein-Nishina regime, and scattering of CMBR photons dominates (e.g. Figure B.1). Therefore, the uncertainties in the IC flux predictions discussed above do not apply at high energies.

![Diffuse intensity spectra](image)

Figure 5.18: Diffuse intensity spectra for the inner Galaxy ($-30^\circ \leq l \leq 30^\circ$, $|b| \leq 5^\circ$) for primary injection spectrum $\propto E^{-2.6}$ and Mori median proton spectrum, with the IC fluxes from UV to optical, and infra red, photons arbitrarily increased by a factor of three. Theoretical predictions: dashed line (IC); dotted line ($\pi^0$-decay); dash-dotted line (bremsstrahlung); triple-dot dashed lines (synchrotron); solid lines (total). Lower triple-dot dashed line above 10 keV calculated for an abrupt cut-off at 100 TeV; next triple-dot dashed line for an exponential cut-off at 100 TeV; other triple-dot dashed lines for same cut-offs at 1000 TeV. Data: as for Figure 5.17a–d.

If the uncertainty in the IC flux below $\sim 10$ GeV is taken into account, then the fit of all the models shown in Figures 5.17a–d would be reasonable from $\sim 1$ MeV up to $\sim 10$ GeV. An example of this is shown in Figure 5.18 where the IC predictions for the UV to optical,
and infra red, photon populations have arbitrarily been increased by a factor of 2.5 for model (c) ($\gamma = 2.0$, Mori 1997 median proton spectrum). The total spectrum below $\sim 10$ MeV is now quite close to the observed spectrum. For model (a) and (b) it should be noted that a similar increase would probably make the total spectra below $\sim 10$ MeV agree quite well with the observations.

Examining the spectrum below $\sim 1$ MeV, it can be seen in Figures 5.17a–d that the contribution by synchrotron radiation varies with injection spectral index and high energy cut-off, as expected from the discussion in Section 5.3.5. For models (a) and (b), the predictions for either form of high energy cut-off considered, for a cut-off energy at 1000 TeV, are significantly above the OSSE data for $l = 0^\circ$ and energies below $\sim 200$ keV, and probably $\sim 1$ MeV if the positron annihilation contribution were removed from the observed spectrum. If the $l = 25^\circ$ OSSE spectrum below $\sim 300$ keV is considered which, as noted earlier, is probably the best estimate of the diffuse emission at these energies, then reasonable agreement with the spectrum below $\sim 300$ keV requires a cut-off energy $\sim 200$ TeV or so for models (a) and (b). For models (c) and (d), the predicted spectra actually agree quite well with the OSSE data for $l = 25^\circ$ for either form of cut-off at 1000 TeV. So, while for models (a) and (b) the maximum cut-off energy is constrained to be below $\sim 200$ TeV (assuming a synchrotron origin for the diffuse emission), energies as high as $\sim 1000$ TeV are allowed for models (c) and (d).

Considering now a slightly larger region of the inner Galaxy, I examine the model predictions for diffuse $\gamma$-rays with energies up to $\sim 1000$ TeV. Figures 5.19a–d show the model predictions for the inner Galaxy ($-60^\circ \leq l \leq 60^\circ$, $|b| \leq 10^\circ$). It can be seen for models (a) and (b) that the Tibet array upper limit excludes cut-off energies much higher than $\sim 20$ TeV or so. For cut-offs this low, a synchrotron origin explanation of the OSSE data in Figures 5.19a–b is obviously not plausible, and some other hypothesis needs to be considered to explain the unusual spectrum below $\sim 1$ MeV. However, models (c) and (d) only slightly conflict with the Tibet upper limit for a cut-off energy of 1000 TeV. So, a cut-off energy $\sim 1000$ TeV toward the inner Galaxy is not excluded for a $\gamma = 2.0$ injection spectrum, and an explanation of the sub-MeV diffuse spectrum in terms of synchrotron radiation is still a viable hypothesis, but requires acceleration up to $\sim 1000$ TeV in the cosmic ray sources.

I now examine the model predictions for the outer Galaxy. Figures 5.20a–d show the calculated spectra for the region $50^\circ \leq l \leq 220^\circ$ and $|b| \leq 10^\circ$. Also shown are the Tibet array upper limit (Amenomori et al. 1997), and the Utah-Michigan array (Matthews et al.
Figure 5.19: Diffuse high energy intensity spectra for the inner Galaxy ($-60^\circ \leq l \leq 60^\circ$, $|b| \leq 10^\circ$) for primary injection spectrum (a) $\propto E^{-1.8}$ and Mori median proton spectrum, (b) $\propto E^{-1.8}$ and the hard proton spectrum, (c) $\propto E^{-2.0}$ and Mori median proton spectrum, and (d) $\propto E^{-2.0}$ and the hard proton spectrum. Theoretical predictions: dashed line (IC); dotted line ($\pi^0$-decay); dash-dotted line (bremsstrahlung); solid lines (total). Lower dashed line above $\sim 30$ TeV calculated for an abrupt cut-off at 100 TeV; next dashed line for an exponential cut-off at 100 TeV; other dashed lines for same cut-offs at 1000 TeV. Data: solid circles, EGRET data for $-60^\circ \leq l \leq 60^\circ$ and $|b| \leq 10^\circ$ (Hunter et al. 1997); solid boxes, COS-B data for $91^\circ \leq l \leq 110^\circ$ and $|b| \leq 10^\circ$ (Paul et al. 1978); open stars, COS-B data for $116^\circ \leq l \leq 136^\circ$ and $|b| \leq 10^\circ$ (Paul et al. 1978); upper limit T, Tibet air shower array for $20^\circ \leq l \leq 55^\circ$ and $|b| \leq 5^\circ$ (Amenomori et al. 1997).

1991) and CASA-MIA (Borione et al. 1998) upper limits. Note that the Utah-Michigan and CASA-MIA upper limits are actually given as upper limits on the ratio of $\gamma$-rays to cosmic
Figure 5.20: Diffuse high energy intensity spectra for the outer Galaxy (50° ≤ l ≤ 220°, |b| ≤ 10°) for primary injection spectrum (a) ∝ E^{-1.8} and Mori median proton spectrum, (b) ∝ E^{-1.8} and the hard proton spectrum, (c) ∝ E^{-2.0} and Mori median proton spectrum, and (d) ∝ E^{-2.0} and the hard proton spectrum. Theoretical predictions: as in the legend of Figure 5.19. Data: solid boxes, COS-B data for 315° ≤ l ≤ 36° and |b| ≤ 10° (Paul et al. 1978); open stars, COS-B data for 16° ≤ l ≤ 36° and |b| ≤ 10° (Paul et al. 1978); upper limit T, Tibet air shower array for 140° ≤ l ≤ 225° and |b| ≤ 5° (Amenomori et al. 1997); upper limit UM, Utah-Michigan array for 30° ≤ l ≤ 220° and |b| ≤ 10° (Matthews et al. 1991); upper limit CM, CASA-MIA array for 50° ≤ l ≤ 220° and |b| ≤ 10° (Borione et al. 1998).

rays, J_γ/J_{CR}, in their respective papers, and I have converted these to diffuse γ-ray intensity upper limits using the all-particle cosmic ray spectrum given by Stanev (1992). For this
region of the Galaxy, models (a) and (b) are clearly in conflict with the Tibet upper limit, and probably require an even lower cut-off energy than in the inner Galaxy to satisfy this constraint on the diffuse spectrum. For a cut-off energy at \( \sim 100 \text{ TeV} \) the CASA-MIA upper limit would provide a way of constraining the form of the cut-off (abrupt or exponential) in the injection spectrum, but the cut-off energy imposed by the Tibet upper limit precludes this. For models (c) and (d), cut-off energies higher than \( \sim 100 \text{ TeV} \) violate the CASA-MIA upper limit. Note that models (b) and (d) can be excluded outright, at least for the outer Galaxy, because the neutral pion decay \( \gamma \)-ray flux violates the CASA-MIA upper limit. So, for the outer Galaxy electrons can be accelerated up to \( \sim 10 \text{ TeV} \) energies at most if they have very hard injection spectra (\( \gamma \sim 1.8 \)), and up to \( \sim 100 \text{ TeV} \) energies if their injection spectrum is softer (\( \gamma \sim 2.0 \)). Furthermore, an average proton spectrum slightly harder in the outer Galaxy than observed locally is ruled out unless some ad-hoc break in the spectrum is introduced.

Upper limits on the diffuse galactic \( \gamma \)-ray spectrum at TeV energies are also given by the Whipple telescope (Reynolds et al. 1993), and the HEGRA array (Karle et al. 1995). The model predictions for the regions of sky covered by these experiments, which are shown in Figures 5.21a–b, are at least 1.5 orders of magnitude below these upper limits. So, no further constraints on the models considered in this Section are possible using these experimental upper limits.

I now examine the model predictions at high galactic latitudes, and compare with observations. Figures 5.22 show the model predictions for high galactic latitudes (\( |b| > 70^\circ \)), and the total (extragalactic + galactic) diffuse spectrum measured by HEAO-1 (Kinzer et al. 1997), COMPTEL (Bloemen et al. 1998), and EGRET (Strong, Moskalenko & Reimer 1998). Below a few MeV, the model predictions account for, at most, only a few percent of the total spectrum, which is presumably due to Seyfert I and II galaxies and a contribution by supernovae at these energies (e.g. Zdziarski 1996). For energies greater than \( \sim 100 \text{ MeV} \), the predicted galactic spectrum for all source models comprises at least \( \sim 50\% \) of the total spectrum. Above \( \sim 4 \text{ GeV} \) models (a) and (b) actually account for close to 100\% of the observed spectrum. However, the only constraint that can be made on the source models is that harder spectra than \( \gamma = 1.8 \) will be inconsistent with the highest energy EGRET data, which is a useful cross-check with the non-thermal predictions made in Section 5.4.1 in essentially the same direction. These estimates of the contribution to the total high latitude spectrum by the galactic component are significantly larger than previously estimated.
Figure 5.21: Diffuse high energy intensity spectra for the region of sky covered by the Whipple Cherenkov telescope (Reynolds et al. 1993), and the HEGRA array (Karle et al. 1995), for primary injection spectrum (a) \( \propto E^{-1.8} \) and Mori median proton spectrum, (b) \( \propto E^{-1.8} \) and the hard proton spectrum, (c) \( \propto E^{-2.0} \) and Mori median proton spectrum, and (d) \( \propto E^{-2.0} \) and the hard proton spectrum. Theoretical predictions: as in the legend of Figure 5.19.

(Sreekumar et al. 1998), and have implications for studies of the extragalactic background beyond the scope of the present work.

Other Source Models

I now consider the ‘special case’ source models found in Section 5.4.1. Since the main
Figure 5.22: Diffuse high latitude intensity spectra \((-180^\circ \leq l \leq 180^\circ, |b| > 70^\circ)\) for primary injection spectrum (a) \(\propto E^{-1.8}\) and Mori median proton spectrum, (b) \(\propto E^{-1.8}\) and the hard proton spectrum, (c) \(\propto E^{-2.0}\) and Mori median proton spectrum, and (d) \(\propto E^{-2.0}\) and the hard proton spectrum. Theoretical predictions: as in the legend of Figure 5.19. Data, total intensity spectra for high latitudes: closed circles, HEAO-1 (Kinzer et al. 1997); open boxes, COMPTEL (Bloemen et al. 1998); solid boxes, EGRET (Strong, Moskalenko & Reimer 1998); thick dashed line: IC spectrum derived by Chen, Dwyer & Kaaret (1997).

Variation in the other possible source models is the size of the halo, I have first calculated the high latitude spectrum for the other possible source models because any variations in the halo size should be most visible in the direction of the galactic poles. Figures 5.23a and 5.23b show the high latitude spectra for an (a) \(\gamma = 1.8\) injection spectrum with \(z_h = 5\) kpc, and
a (b) $\gamma = 2.2$ injection spectrum with $z_h = 3$ kpc. In both these Figures, the hard proton spectrum is used for the neutral pion decay contribution since this would give the greatest contribution (if any) to the diffuse flux.

The increased halo size for the $\gamma = 1.8$ injection spectrum causes little difference with the predictions for model (b) in the previous Section, and it can be expected that there will be little difference between model (b) and the predictions for the same injection spectrum but with an increased halo size. For a $\gamma = 2.2$ injection spectrum, the contribution to the high latitude spectrum is much smaller than for the other models considered previously. In fact, the total contribution for this injection spectrum is of the order $\sim 30\%$ around 1 GeV, and at most a few percent for energies lower than this. Around 1 GeV this is practically half that estimated for the other models considered.

![Image](image.png)

Figure 5.23: Diffuse high latitude intensity spectra ($-180^\circ \leq l \leq 180^\circ$, $|b| > 70^\circ$) for primary injection spectrum (a) $\propto E^{-1.8}$ and $z_h = 5$ kpc, (b) $\propto E^{2.2}$ and $z_h = 3$ kpc. Theoretical predictions: as in the legend of Figure 5.19; the neutral pion decay contribution has been calculated using the hard proton spectrum given by Equation 5.16. Data: as in the legend of Figure 5.22.

It is unlikely that there will be a significant contribution to the sub-MeV emission for a $\gamma = 2.2$ injection spectrum, which can be seen by simply examining how rapidly the synchrotron emissivity decreases as the injection spectrum decreases from $\gamma = 1.8 \rightarrow 2.4$ (Figures 5.12a–d). Therefore, any useful constraints on this source model will be found by examining the high energy spectra in the inner and outer Galaxy, as was done for models.
(a) to (d) in Figures 5.19 and 5.20. Since a hard proton spectrum in the outer Galaxy is not tenable (see Figures 5.20b and 5.20d) I only consider a standard proton spectrum for the inner and outer Galaxy spectrum. Figures 5.24a and 5.24b show the predictions for this model for the inner and outer Galaxy. Toward the inner Galaxy, no constraints are possible on the cut-off energy in the injection spectrum, and the predicted spectrum does not agree well with the EGRET data points above $\sim 1$ GeV. For the outer Galaxy, the predicted spectra for a cut-off energy at 1000 TeV violate the CASA-MIA upper limit. Electrons with energies higher than $\sim 500$ TeV cannot be accelerated in the outer Galaxy. This is the least constrained of all the source models considered in this Chapter.

![Graph](image)

Figure 5.24: Diffuse high energy intensity spectra for the (a) inner Galaxy ($-60^\circ \leq l \leq 60^\circ$, $|b| \leq 10^\circ$), and (b) outer Galaxy ($50^\circ \leq l \leq 220^\circ$, $|b| \leq 10^\circ$) for a primary injection spectrum $\propto E^{-2.2}$ and Mori median proton spectrum. Theoretical predictions: as in the legend of Figure 5.19. Data: as in the legends of Figures 5.19 and 5.20.

### 5.5 Discussion

To summarise briefly before discussing the implications of the calculations presented in Section 5.4.2, if primary electrons are accelerated with very hard injection spectra ($\gamma \sim 1.8$) in the inner Galaxy, then the maximum energies attainable are at most of the order $\sim 20$ TeV. For the outer Galaxy, electrons accelerated with this spectral index cannot be accelerated to energies higher than $\sim 10$ TeV. At least for this source model, an explanation of
5.5. DISCUSSION

the observed sub-MeV spectrum in terms of synchrotron radiation is not viable, and some other hypothesis must be advanced to account for the peculiar spectrum observed by OSSE. For slightly softer injection spectra ($\gamma \sim 2.0$), electrons can be accelerated up to energies $\sim 1000$ TeV, while in the outer Galaxy acceleration up energies higher than $\sim 100$ TeV is not possible. For this injection spectrum, it is possible to postulate a significant contribution to the sub-MeV spectrum by synchrotron radiation, but only if electrons are accelerated to energies $\sim 1000$ TeV. If even softer injection spectra are considered ($\gamma \sim 2.2$), the primary injection spectrum in the inner Galaxy is unconstrained, and synchrotron emission at sub-MeV energies is negligible. However, in the outer Galaxy electrons cannot be accelerated to energies higher than $\sim 500$ TeV.

5.5.1 Interpretations of the Galactic Hard X-ray Continuum Radiation

The most difficult aspect of the galactic diffuse $\gamma$-ray spectrum to explain is the sub-MeV emission observed by OSSE. Instead of a continuation of the $\propto E_\gamma^{-2.0}$ power-law of the observed spectrum at higher energies, the OSSE spectrum is $\propto E_\gamma^{-2.5}$. This has proved a challenge for conventional models of the diffuse $\gamma$-ray emission (e.g. Strong et al. 1996) because the predicted $\gamma$-ray spectra for these models give flatter spectra below $\sim 1$ MeV, instead of the steep spectra required to explain the OSSE data. Several authors have interpreted the sub-MeV spectrum as being electron bremsstrahlung, and suggested this is generated by a new population of low energy electrons (Skibo et al. 1996; also Purcell et al. 1996), or electrons accelerated in-situ by the interstellar plasma turbulence (Schlickeiser et al. 1997). However, there are problems with the energetics of the Skibo et al. (1996) model, where the power required to maintain the new low-energy population of electrons against Coulomb losses in the ISM exceeds that supplied by galactic SNRs by about an order of magnitude. Other explanations have been advanced: synchrotron radiation by ultra high energy electrons and positrons (Schlickeiser & Mörsberger 1997), unresolved point sources, probably SNRs (Strong, Moskalenko & Reimer 1998), and shock accelerated electrons with energies $\sim 100$ TeV and higher (Porter & Protheroe 1997); Pohl (1999) has ruled out an inverse bremsstrahlung origin. Of these, the last two appear to be the most promising, and the generation of the sub-MeV spectrum by shock accelerated electrons has been explored further in this thesis.

Only if primary electrons are accelerated with a $\gamma \sim 2.0$ injection spectrum, and up to energies approaching $\sim 1000$ TeV in the inner Galaxy, is a synchrotron origin viable for
the sub-MeV spectrum. Therefore, for the other injection spectra considered, some other hypothesis must be advanced. In these cases, an unresolved point source component is probably the best explanation because the domination by diffuse emission must change over to domination by a source component below some energy. Strong, Moskalenko & Reimer (1998) suggest that this occurs around \( \sim 1 \) MeV, and this is consistent with the calculations presented for the injection spectra other than \( \gamma \sim 2.0 \). Even if the average injection spectrum of electrons is \( \gamma \sim 2.0 \), the rather high maximum acceleration energy required (\( \sim 1000 \) TeV) to explain the sub-MeV spectrum as purely due to synchrotron emission could be problematic. This is because EGRET detected SNRs lying in the galactic plane, such as IC 443, do not appear to generate any discernable \( \gamma \)-ray flux in the TeV energy range (Buckley et al. 1998). Baring et al. (1998) suggest that cosmic ray acceleration to TeV energies and beyond in shell-type SNRs, and generation of \( \gamma \)-ray emission in the GeV range, are anti-correlated because circumstances favouring intense \( \gamma \)-ray production in the GeV and sub-TeV energy ranges (e.g. high gas density) limit particle acceleration in these sources to energies well below \( \sim 100 \) TeV (the only SNR detected with TeV \( \gamma \)-ray emission, SN 1006, is located \( \sim 1 \) kpc outside the galactic plane). Therefore, a high electron injection energy in the inner Galaxy might be an unlikely hypothesis because of the high ISM density in this region. So, it might be that even for a \( \gamma \sim 2.0 \) injection spectrum a significant contribution by an unresolved point source component could exist in the sub-MeV spectrum, although not to the same degree as required for the \( \gamma = 1.8 \) spectrum. In any case, an explanation of the sub-MeV spectrum purely in terms of synchrotron radiation is probably not viable.

### 5.5.2 The Diffuse Photon Spectrum Above \( \sim 1 \) GeV

In the energy range \( \sim 1 \) MeV to \( \sim 1 \) GeV the interpretation of the diffuse \( \gamma \)-ray spectrum is relatively uncontroversial, although the relative importance of the IC and bremsstrahlung components have varied over time; currently it is believed IC \( \gamma \)-rays comprise the bulk of the spectrum where electron/positron interactions dominate the diffuse spectrum. For energies \( > 1 \) GeV the observed spectrum displays an anomalous excess over the conventional picture where the neutral pion decay component dominates the emission. In the work of Porter & Protheroe (1997), it was shown how variations in the primary injection spectrum could produce a contribution by IC \( \gamma \)-rays at high energies that dominated the neutral pion decay component. However, at the time the limitations of the propagation model used for the electron propagation only allowed relatively soft injection spectra \( \gamma \sim 2.2 - 2.4 \) to be considered.
In this thesis, I have shown how harder spectral indices for the primary injection spectrum are possible if the discrete nature of the primary source distribution is taken into account, and are actually favoured by non-thermal radio observations. The model predictions for these hard injection spectra provide fairly good agreement with the EGRET spectrum up to $\sim 10$ GeV (e.g. Figures 5.19a–d), even when a hard average interstellar proton spectrum is considered. That the EGRET ‘excess’ spectrum can be naturally explained in terms of a hard primary injection spectrum hypothesis, and not as being due to e.g. pulsars (Pohl et al. 1997), is a conclusion reached by several other authors (Pohl & Esposito 1998; Strong, Moskalenko & Reimer 1998). Note that the observed EGRET spectrum is not fitted well by one of the source models considered ($\gamma = 2.2$), and some other hypothesis must be found to explain the excess spectrum in this case.

If it is accepted that the high energy $\gamma$-ray spectrum is due to IC $\gamma$-rays from electrons accelerated with hard injection spectra, then a natural corollary is the resulting $\gamma$-ray spectrum must exhibit a cut-off, related to the cut-off in the injection spectrum, to ensure agreement with the various upper limits provided by air shower arrays. For the source model with $\gamma = 1.8$, I have shown that the high energy cut-off in the injection spectrum must be around $\sim 10$ TeV for the entire galactic plane. For a $\gamma = 2.0$ source model, the cut-offs in the inner and outer Galaxy differ, being $\sim 1000$ TeV and $\sim 100$ TeV respectively. Recently, Baring et al. (1998) presented models for shock acceleration in SNRs which produce very flat electron spectra quite similar to the $\gamma \sim 1.8 - 2.0$ spectra found in the present study. If the $\gamma \sim 1.8$ source spectrum is considered, the limits on the gamma-ray spectrum in the galactic plane support the view of Baring et al. (1998) that SNRs located in the galactic plane cannot be the sources of galactic cosmic rays up to $\sim 10^6$ GeV; in their model, cosmic rays with energies $\sim 100$ TeV and higher are accelerated in high latitude sources, such as SN 1006. For a $\gamma = 2.0$ injection spectrum, cosmic ray acceleration up to $\sim 100$ TeV is allowed throughout the Galaxy, and probably higher in the inner Galaxy. Both of these models point to different origins for the highest energy galactic cosmic rays: for a $\gamma = 1.8$ source spectrum, cosmic rays with energies up to the knee must be accelerated in galactic halo sources, while for a $\gamma = 2.0$ source spectrum the inner Galaxy is probably the source of high energy cosmic rays.
5.6 Summary

The propagation of cosmic ray electrons and positrons, and the production of diffuse $\gamma$-rays in the Galaxy has been considered in this Chapter. It has been shown how the conventional assumption of relatively soft injection spectra for primary electrons at acceleration to obtain agreement with the locally observed electron spectrum is unnecessary from the point of view for predictions of diffuse $\gamma$-rays. The propagation calculation for electrons and positrons was then described, and predictions of the interstellar secondary positron spectrum, and fraction, and the total electron spectrum obtained using the propagation model were compared with observations. Diffuse photon production spectra were then calculated using the predicted electron and positron spectra, and the interstellar proton spectra obtained from the fitting procedure to the local positron spectrum. The synchrotron emissivities were then used to obtain predictions of the non-thermal radio spectrum in the direction of the galactic poles, which were used to constrain the possible models for the electron injection spectrum. Then, predictions of the diffuse galactic $\gamma$-ray spectrum were made, and compared with relevant observations from keV to TeV energies and beyond. In Table 5.1, the models considered in this Chapter are listed, and their agreement with various experimental results is summarised.

Constraints on the maximum acceleration energy for all of the models considered are provided in one way or another by the gamma-ray spectrum above $\sim 1$ GeV. In particular, it
was found that electrons cannot be accelerated to energies higher than $\sim 10$ TeV in the disk of the Galaxy if their average injection spectrum is $\propto E^{-1.8}$, while for a $\propto E^{-2.0}$ average injection spectrum electrons cannot be accelerated to energies higher than $\sim 1000$ TeV in the inner Galaxy, or higher than $\sim 100$ TeV in the outer Galaxy. Therefore, for a $\gamma = 1.8$ injection spectrum the sub-MeV $\gamma$-ray spectrum cannot be explained as synchrotron emission, and a significant contribution by a point source component below the detection threshold of OSSE is probably the case. For a $\gamma = 2.0$ injection spectrum, the sub-MeV spectrum can be explained almost completely as synchrotron emission, but requires acceleration up to $\sim 1000$ TeV.

Of the source models considered in this Chapter, an average injection spectrum $\propto E^{-1.8}$ and a high energy cut-off at 10 TeV throughout the galactic plane, and an average injection spectrum $\propto E^{-2.0}$ and a high energy cut-off at 1000 TeV in the inner Galaxy, and 100 TeV in the outer Galaxy, agree best with the observations of the galactic non-thermal radio emission, and diffuse galactic $\gamma$-ray spectrum. The first of these models can be interpreted as providing support for recent models of cosmic ray acceleration and $\gamma$-ray production in SNRs: namely, most of the galactic cosmic rays up to $\sim 10^6$ GeV are not accelerated in sources in the galactic disk, but in high latitude sources such as SN 1006.
Chapter 6

Summary and Conclusions

In this thesis, I have examined several aspects of the production and propagation of cosmic ray electrons and positrons in the Galaxy, and the diffuse photon fluxes produced by these particles through interactions in the Galaxy during propagation.

The description of cosmic ray transport in the Galaxy used in this thesis relied upon the diffusion-convection model introduced in Chapter 2. For physically interesting parameters of this model, it is intractable to solve the propagation equations analytically, and a numerical method based on Monte Carlo simulation techniques was described to do this. It was shown that the Monte Carlo method is particularly reliable and efficient, reproducing accurately the particle distributions for two cases of the propagation model for which analytical solutions exist.

The sources of cosmic ray electrons and positrons in the Galaxy were then discussed in Chapter 3. Electron acceleration in sources such as SNRs was modelled using first order Fermi acceleration at a planar shock, and a simple leaky box model for a cosmic ray accelerator. It was shown how simple geometrical considerations on the size of the acceleration region can alter the form of the injection spectrum at high energies, introducing a natural cut-off in the injection spectrum, that differ from the situation when such considerations are not taken into account. Electron and positron production through inelastic collisions between cosmic ray nuclei and gas in the ISM was described, and a model of the production process was adopted for the calculation of the production spectrum of these particles. The distributions of the sources of electrons and positrons were constructed using available information on the SNR distribution, galactic gas distribution, and cosmic ray distribution derived from observations of γ-rays.
The transport of galactic cosmic ray nuclei was then considered in Chapter 4 using a simplified form of the propagation model in 1D suitable for the treatment of these particles. It was shown how convection could be neglected as playing a significant role in the transport of cosmic rays in the Galaxy. An analytical treatment of the 1D propagation model was then used to obtain constraints on the halo size and diffusion coefficient that could be used in the propagation calculations for cosmic ray electrons and positrons. It was shown how the analytical approach used in this Chapter actually provided similar results to more complicated treatments of the propagation of nuclei in the Galaxy, and was therefore quite useful, at least as a first approximation.

The propagation of cosmic ray electrons and positrons, and the production of diffuse $\gamma$-rays was then considered in Chapter 5. A 3D diffusion model was used for the propagation calculation, and distributions of cosmic ray electrons and positrons in the Galaxy were obtained using the source functions from Chapter 3. Diffuse photon production spectra were obtained using the calculated electron and positron spectra, and interstellar proton spectra obtained from the fitting procedure to the local positron spectrum. Possible models for the sources of primary electrons in the Galaxy were constrained using the observed galactic non-thermal radio emission; it was found that the injection spectrum of primary electrons at acceleration was probably very hard, being between $\gamma = 1.8$ and $\gamma = 2.0$. Then, predictions of the galactic diffuse $\gamma$-ray spectrum were made for these models and compared with relevant observations. It was found that if the average injection spectrum at acceleration of primary electrons is $\propto E^{-1.8}$ then the maximum acceleration energy of these particles in the galactic plane is $\sim 10$ TeV. If, however, the average injection spectrum is $\propto E^{-2.0}$ then primary electrons can be accelerated at most up to 1000 TeV in the inner Galaxy, and 100 TeV in the outer Galaxy. The first model considered, with a $\propto E^{-1.8}$ average injection spectrum and high energy cut-off at 10 TeV, was interpreted as providing support for recent models of shock acceleration in SNRs that propose the majority of galactic cosmic rays up to $\sim 10^6$ GeV are not accelerated in SNRs located in the galactic disk, but in high latitude sources such as SN 1006.

While the calculations presented in this thesis have been made with a detailed propagation model, and a full treatment of photon production processes, etc., the very nature of the conclusions reached points the way for further work in this area. For example, the conclusion that most of the galactic cosmic rays up to $\sim 10^6$ GeV are produced in high latitude sources requires the extension of the primary source function given in Chapter 3
outside of the galactic disk. Additionally, the propagation model, gas model, etc. presently adopt a cylindrically symmetric geometry, which obviously ignores spatial structure details of the Galaxy, such as spiral arms. With increased computing resources, this restriction on the geometry could be relaxed, paving the way for a truly three-dimensional treatment of cosmic ray sources, propagation and γ-ray production in the Galaxy.

The treatment of the propagation of cosmic ray nuclei in the Galaxy was performed using analytical methods in the present work. By extending the Monte Carlo method to treat non-continuous energy loss/gain processes, the propagation of cosmic ray nuclei, and electrons and positrons can be treated using the same model. In this way, the propagation parameters of the model (e.g. halo size, diffusion coefficient) can be self-consistently determined directly from cosmic ray abundance data, rather than the analytical method used in Chapter 4 which relied upon model dependent interpretations of surviving fractions of cosmic ray radioisotopes. With a self-consistent approach to the propagation of cosmic ray nuclei, other models with different energy dependencies for the diffusion coefficient can be explored. For example, re-acceleration models, and the model proposed by Biermann and coauthors (Biermann 1997, and references therein) assume a spatial diffusion coefficient $\propto E^{-1/3}$, which is a result of a Kolmogorov spectrum of turbulence in the ISM. The details of these models are beyond the scope of the present work, but they are motivated by more physical arguments than those used to arrive at the diffusion coefficient energy dependence used in this thesis.

A first attempt has been made to deduce information directly from the observed diffuse γ-ray spectrum of the Galaxy on the distribution, and probable injection spectrum of electrons at acceleration in SNRs. With improved observations by experiments at high energies, it may be possible to obtain even further constraints on the sources of galactic cosmic rays.
Appendix A

Distributions of Physical Parameters

The propagation calculations presented in Chapter 5, and the calculations of the source functions for secondary electrons and positrons, and diffuse photon fluxes from synchrotron, bremsstrahlung, neutral pion decays, and inverse Compton interactions in the Galaxy rely upon detailed distributions for the galactic gas distribution, ambient photon populations, and galactic magnetic field to make them as realistic as possible. In this Appendix, the details of the models used to describe these physical parameters in this thesis are described. Note that each of the models described below is described assuming a cylindrical coordinate system with symmetry about $R = 0$.

A.1 Galactic Matter Distribution

The distribution of gas in the Galaxy consists of three components: the distribution of atomic hydrogen (HI), that of molecular hydrogen (H$_2$), and that of ionised hydrogen (HII).

For the atomic hydrogen distribution perpendicular to the galactic plane, I follow the work of Dickey & Lockman (1990) who have described it in considerable detail. They summarise the following for the average distribution: in the inner few kpc of the Galaxy, but not in the galactic nucleus, the distribution is quite thin and can be approximated by a single Gaussian with dispersion $\sigma_{\text{HI}} = 70$ pc. From $\sim 4$ kpc out to $\sim 9$ kpc the distribution has fairly constant properties and is adequately approximated by a linear combination of two Gaussians with central densities $n_{\text{m1}} = 0.395 \text{ cm}^{-3}$ and $n_{\text{m2}} = 0.107 \text{ cm}^{-3}$ and dispersions
\( \sigma_{\text{HI}_1} = 212 \text{ pc} \) and \( \sigma_{\text{HI}_2} = 530 \text{ pc} \), respectively, and an exponential with maximum density in the galactic disk \( n_{\text{HI}_2} = 0.064 \text{ cm}^{-3} \) and scale height \( z_{\text{HI}} = 403 \text{ pc} \). Beyond the solar circle the atomic hydrogen layer undergoes a `flaring'. Following Bregman, Kelson & Ashe (1993) this is described by modifying the scale heights of the three components used for the distribution between \( \sim 4 \) and \( \sim 9 \text{ kpc} \) by a factor \( R/R_S \) where \( R_S = 8.5 \text{ kpc} \) is the distance of the Sun from the galactic centre, and \( R \) is the galactocentric distance beyond the solar circle. Note that there is considerable uncertainty associated with the vertical distribution of atomic hydrogen in the galactic nucleus. For simplicity I have assumed that the vertical distribution used for \( R < 4 \text{ kpc} \) extends to \( R = 0 \).

For the radial distribution of atomic hydrogen, several surveys exist in the literature (see e.g. Burton 1988 for a summary). I use the radial profile presented by Gordon & Burton (1976) (their Figure 4), scaled for the adopted value of \( R_S \). The radial profile is normalised to a density of \( 0.57 \text{ cm}^{-3} \) at \( R_S \), corresponding to the sum of the densities at \( R_S \) and \( z = 0 \) for the three vertical components used for the \( 4 \leq R \leq 9 \text{ kpc} \) region of the Galaxy described above. The distribution of atomic hydrogen in the Galaxy can be described by the following function:

\[
\begin{align*}
n_{\text{HI}}(R, z) = n_{\text{HI}}(R) \times & 
\begin{cases} 
\exp \left( - \left[ \frac{z}{0.06 \text{ kpc}} \right]^2 \right) & \text{if } R < 4 \text{ kpc} \\
\sum_{i=1,2} \exp \left( - \left[ \frac{z}{\sigma_{\text{HI}_i}} \right]^2 \right) + \exp \left( - \frac{|z|}{0.40 \text{ kpc}} \right) & \text{if } 4 \leq R \leq 9 \text{ kpc} \\
\sum_{i=1,2} \exp \left( - \left[ \frac{z}{\sigma_{\text{HI}_i} (R/R_S)} \right]^2 \right) + \exp \left( - \frac{|z|}{0.40 \text{ kpc}} (R/R_S) \right) & \text{if } R > 9 \text{ kpc}
\end{cases}
\end{align*}
\]

where the radial distribution function, \( n_{\text{HI}}(R) \), is shown as the solid line in Figure A.1.

Several surveys of the large-scale distribution of CO, which acts as a tracer of molecular hydrogen, in the Galaxy exist in the literature (e.g. Sanders, Solomon & Scoville 1984; Cohen, Dame & Thaddeus 1986; Bronfman et al. 1988). The survey of Bronfman et al. (1988) is the most complete because it covers both the northern and southern galactic hemispheres to provide essentially homogeneous coverage of the inner Galaxy.

For the distribution of molecular hydrogen, I use the average of the northern and southern distributions given by Bronfman et al. (1988). The number density of molecular hydrogen can be represented by the following function:
\[ n_{n_2}(R, z) = X Q_{\text{CO}}(R) \exp\left( -\ln 2 \left[ \frac{z - z_0(R)}{z_1/2(R)} \right]^2 \right) \]  \hspace{1cm} (A.2)

where \( X \) is the conversion factor relating molecular hydrogen column density, \( N(H_2) \), to the velocity integrated CO intensity, \( W(\text{CO}) \),

\[
\frac{N(H_2)}{W(\text{CO})} \equiv n_{n_2}/Q_{\text{CO}} \equiv X \\
= (2.8 \pm 0.4) \times 10^{20} \text{ (cm}^{-2} \text{ (K km s}^{-1})^{-1}\text{),} \tag{A.3}
\]

and \( Q_{\text{CO}} \), the CO volume emissivity, and \( z_0 \) and \( z_{1/2} \) are given in Table A.1.

<table>
<thead>
<tr>
<th>( R ) (kpc)</th>
<th>( Q_{\text{CO}} ) (K km s(^{-1}) kpc(^{-1}))</th>
<th>( z_0 ) (pc)</th>
<th>( z_{1/2} ) (pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.25</td>
<td>1.5 ± 0.4</td>
<td>39 ± 26</td>
<td>77 ± 30</td>
</tr>
<tr>
<td>2.75</td>
<td>3.3 ± 0.5</td>
<td>36 ± 9</td>
<td>80 ± 10</td>
</tr>
<tr>
<td>3.25</td>
<td>5.8 ± 0.5</td>
<td>0 ± 4</td>
<td>61 ± 4</td>
</tr>
<tr>
<td>3.75</td>
<td>5.5 ± 0.5</td>
<td>-8 ± 4</td>
<td>65 ± 5</td>
</tr>
<tr>
<td>4.25</td>
<td>8.4 ± 0.5</td>
<td>1 ± 3</td>
<td>71 ± 3</td>
</tr>
<tr>
<td>4.75</td>
<td>9.0 ± 0.8</td>
<td>-10 ± 5</td>
<td>72 ± 5</td>
</tr>
<tr>
<td>5.25</td>
<td>9.6 ± 0.6</td>
<td>-1 ± 4</td>
<td>82 ± 4</td>
</tr>
<tr>
<td>5.75</td>
<td>8.6 ± 0.4</td>
<td>-4 ± 3</td>
<td>83 ± 4</td>
</tr>
<tr>
<td>6.25</td>
<td>9.1 ± 0.5</td>
<td>-19 ± 3</td>
<td>73 ± 3</td>
</tr>
<tr>
<td>6.75</td>
<td>7.9 ± 0.4</td>
<td>-22 ± 3</td>
<td>63 ± 4</td>
</tr>
<tr>
<td>7.25</td>
<td>9.2 ± 0.5</td>
<td>-14 ± 3</td>
<td>58 ± 4</td>
</tr>
<tr>
<td>7.75</td>
<td>7.7 ± 0.5</td>
<td>-9 ± 5</td>
<td>72 ± 7</td>
</tr>
<tr>
<td>8.25</td>
<td>5.0 ± 0.3</td>
<td>-4 ± 5</td>
<td>80 ± 9</td>
</tr>
<tr>
<td>8.75</td>
<td>3.6 ± 0.6</td>
<td>13 ± 6</td>
<td>66 ± 10</td>
</tr>
<tr>
<td>9.25</td>
<td>4.8 ± 0.6</td>
<td>-4 ± 3</td>
<td>23 ± 5</td>
</tr>
<tr>
<td>9.75</td>
<td>1.7 ± 0.5</td>
<td>-20 ± 77</td>
<td>147 ± 139</td>
</tr>
</tbody>
</table>

Table A.1: Parameters for the molecular hydrogen number density distribution (Equation A.2) taken from the work of Bronfman et al. (1988). The parameters, \( Q_0 \), \( z_0 \) and \( z_{1/2} \) are given for radial bins of half-width 0.25 kpc centred on the \( R \) values given in the Table. The uncertainties on the parameters are taken directly from Bronfman et al. (1988), but are not used to calculate the number density distribution.

Ionised hydrogen generally only comprises a small amount of the total hydrogen mass in the galactic disk. However, the HII distribution has a rather large scale height perpendicular to the galactic plane (\( \sim 1 \) kpc), and this results in a HII column density perpendicular to the galactic disk of the same order of magnitude as that of HI and H\(_2\). It is important to include the HII component in the gas model because electron and positron energy losses at low energies are dominated by interactions with matter (neutral and ionised). The larger
extent of the HII layer (in comparison with the neutral gas layer) leads to substantial losses at low energies even at relatively large values of $z$.

The articles of Reynolds (1990a, 1990b, 1993) give a comprehensive picture of the extended HII distribution perpendicular to the galactic plane. Generally, the scale height is $\sim 1 - 1.5$ kpc, with number density in the plane $\sim 0.02$ cm$^{-3}$ locally. The radial distribution of this component decreases with galactocentric radius with a scale length $\sim 20$ kpc (Cordes et al. 1991). Additionally, there is a component concentrated about $R \sim 4$ kpc which represents HII regions, possibly associated with regions of intense star formation. This component has a scale height $\sim 10$ times smaller than the extended layer, and a density $\sim 10$ times higher.

Adopting the model of Cordes et al. (1991), the ionised hydrogen distribution in the Galaxy may be described by the following expression:

\[
n_{\text{HII}}(R,z) = 0.025 \exp \left( -\frac{|z|}{1 \text{ kpc}} \right) \exp \left( -\frac{R}{20 \text{ kpc}} \right)^2 + 0.2 \exp \left( -\frac{|z|}{0.15 \text{ kpc}} \right) \exp \left( -\frac{R-4}{2 \text{ kpc}} \right)^2
\]

(A.4)

where the first term represents the extended layer, and the second term represents the component concentrated about $R \sim 4$ kpc.

The distributions of atomic, molecular and ionised hydrogen calculated using Equations A.1 - A.4 are shown in Figures A.1a and A.1b.

### A.2 Interstellar Radiation Field

The ambient radiation fields of the Galaxy are of main interest for the purposes of $\gamma$-ray calculations, and electron and positron energy losses. The interstellar radiation field (ISRF) is generally divided into three components: ultraviolet (UV) and optical (covering the wavelength range 0.1 to 8.0 $\mu$m), infrared (8.0 to 1000.0 $\mu$m) and the cosmic microwave background radiation (CMBR); other potential low energy target photon fields, such as the galactic non-thermal background, can be reasonably excluded from consideration (see Section A.2.1 below).

The UV to optical radiation field is attributed to stellar emission by different populations of stars in the Galaxy, while the infrared field is generated by reprocessing of the UV/optical component by dust and other particles associated with gas in the ISM; the CMBR is the remnant signature of the big bang fireball first discovered by Penzias & Wilson (1965).
Figure A.1: Number density of hydrogen atoms in the Galaxy. Figure (a) shows the radial profile of atomic (solid line), molecular (triple-dot dashed line) and ionised (dot dashed line) components. Figure (b) shows the vertical profile for galactocentric radii $4\, \text{kpc} \leq R \leq 9\, \text{kpc}$: dotted lines show the individual components of the atomic hydrogen distribution, triple-dot dashed line shows the molecular hydrogen profile and the dot dashed line the ionised component.

Several calculations of the UV/optical and infrared galactic radiation fields exist in the literature (Mathis, Mezger & Panagia 1983; Bloemen 1985; Cox, Krügel & Mezger 1986;
Chi & Wolfendale 1991; Strong, Moskalenko & Reimer 1998). In this Section, I present the
distribution of ambient photon fields used in this thesis. Aspects of this calculation are based
on the work of Chi & Wolfendale (1991) for the UV to optical, and Cox, Krügel & Mezger
(1986) for the mid to far infrared, but several modifications have been made in the present
work to ensure self-consistency with the treatment of both the stellar and infrared radiation
fields that yield different galactic photon distributions than in earlier calculations.

For accurate calculations of IC interactions between cosmic ray electrons and these low
energy photon fields, the spectral and spatial distributions must be known. For the CMBR
this is trivial because it possesses an essentially uniform spatial distribution over the galactic
volume, and spectral distribution of an almost perfect blackbody radiating at a temperature
$T_{\text{CMBR}} = 2.728 \pm 0.004$ (Fixsen et al. 1996). From the relation between the energy density
and temperature for a blackbody radiator, $u = a T^4$ with $a = 4\pi c$ and $\sigma = 8\pi^2 k^4/60h^3 c^2 \equiv
3.544 \times 10^7 \text{ eV cm}^{-2} \text{ K}^{-4}$, the energy density of the CMBR throughout the Galaxy is
estimated as $0.262 \text{ eV cm}^{-3}$.

The distributions of the UV/optical and the infrared are, however, not as simple to cal-
culate. This is because the UV/optical emission depends on the distribution of the different
stellar components emitting photons in this wavelength range, and the distribution of dust
and other particles in the Galaxy that absorbs these photons. The infra red photon popula-
tion is due to reprocessed starlight by dust in the ISM, and so depends on the distribution
of dust and other particles associated with gas in the Galaxy.

The basic equation used to calculate the spectral and spatial distribution of the UV/optical
and infrared radiation fields at position $\vec{r}$ and wavelength $\lambda$ is

$$u(\vec{r}, \lambda) = \frac{1}{4\pi c} \int \frac{Q(\vec{r}, \lambda)}{|\vec{r} - \vec{r'}|^2} \exp \left( -\int_{\vec{r'}}^{\vec{r}} \kappa(\vec{r''}, \lambda) \, dl \right) d^3 r' \quad (A.5)$$

where $Q(\vec{r}, \lambda)$ is the volume emissivity, $\kappa(\vec{r''}, \lambda)$ is the absorption function and $l \equiv |\vec{r} - \vec{r'}|$. The
integration is over the entire galactic volume. The calculation of this integral is described
in general terms below, and the energy density distributions for the different radiation fields
are then calculated.

To evaluate Equation A.5 for some volume element $\Delta V_{\text{local}}$ centred on $\vec{r}$, I assume cylin-
drical symmetry, as in the propagation calculations, and employ Monte Carlo techniques to
calculate the volume integration: $N$ different volume elements, $\Delta V'$, centred on positions $\vec{r}'$
are chosen uniformly distributed throughout the Galaxy. If any of the $\Delta V'$ intersect with
ΔV_{local}, they are rejected, and a new \( \bar{r}’ \) is chosen so that all of the volume elements generated are outside \( \Delta V_{local} \) (this ensures that singularities in the integration, such as when \( |\bar{r} - \bar{r}'| \to 0 \), do not occur). The emissivity of the volume elements centred on each of the \( \bar{r}' \), \( Q \Delta V' \), is then calculated. The emissivity of each volume element are then summed together, weighted by \( 1/|\bar{r} - \bar{r}'|^2 \) and the absorption (if any) along the line-of-sight between \( \bar{r} \) and \( \bar{r}' \). The average energy density throughout \( \Delta V_{local} \) is then obtained by dividing the resulting sum by \( N \) and \( 4\pi c \). In practice, it was found that choosing spherical regions of radius \( r_{local} \sim 100 \text{ pc} \), and \( N = 10^4 \) was sufficient. For convenience, each of the \( \Delta V' \) was also calculated using a radius \( r' = r_{local} \).

For the emissivity distribution of the stars producing the UV to optical ambient photon population, I adopt the standard Bahcall & Soneira (1980) spatial distribution for the exponential disk originally considered by Mathis, Mezger & Panagia (1983):

\[
Q_i(R, z, \lambda) = Q_i(R_S, 0, \lambda) \exp \left( -\frac{(R - R_S)}{d_{R_i}} - \frac{|z|}{d_{z_i}} \right) \tag{A.6}
\]

where the subscript runs \( i = 1, \ldots, 4 \) for the four stellar components considered to produce this photon field (to be described below), \( R_S \) is the galactocentric distance of the Sun, \( Q_i(R_S, 0, \lambda) \) is the local spectral emissivity of the \( i \)th stellar component, and \( d_{R_i} \) and \( d_{z_i} \) are the scale heights of the \( i \)th component in the radial direction, and that perpendicular to the galactic plane. Mathis, Mezger & Panagia (1983) considered four stellar components to be responsible for the emission in the 0.1 \( \mu \text{m} \) to 8.0 \( \mu \text{m} \) wavelength range: UV emission by early type stars, emission by two disk star components and by red giants. The radial scale lengths and scale heights for these components are listed in Table A.2.

<table>
<thead>
<tr>
<th>( W_i )</th>
<th>( T_i ) (K)</th>
<th>( d_{R_i} ) (kpc)</th>
<th>( d_{z_i} ) (kpc)</th>
<th>( R_c ) (kpc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0 \times 10^{-17}</td>
<td>30000</td>
<td>2.5</td>
<td>0.06</td>
<td>4.0</td>
</tr>
<tr>
<td>1.0 \times 10^{-14}</td>
<td>7500</td>
<td>4.0</td>
<td>0.19</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0 \times 10^{-13}</td>
<td>4000</td>
<td>4.0</td>
<td>0.27</td>
<td>0.0</td>
</tr>
<tr>
<td>4.0 \times 10^{-13}</td>
<td>3000</td>
<td>1.3</td>
<td>0.05</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Table A.2: Parameters of the four component stellar model used for the UV to optical radiation field calculation.

The local emissivity spectrum of each stellar component, \( Q(R_S, 0, \lambda) \), is parameterised by a diluted blackbody spectrum with dilution factor \( W_i \) and temperature \( T_i \):
\[ Q_i(R, \lambda) = W_i e^{\frac{8\pi \hbar c}{\lambda^3} \left( \frac{1}{\exp(hc/\lambda kT_i) - 1} \right)} \quad (A.7) \]

where \( W_i \) and \( T_i \) are also given in Table A.2 for each stellar component. Note that stellar component 1, while identified by Mathis, Mezger & Panagia (1983), was not actually parameterised at all; representing it as a diluted blackbody spectrum here is an approximation, which appears to provide a reasonable fit with the local spectrum given by Mathis, Mezger & Panagia (1983) (compare Figure A.2c at small wavelengths with Figure 1 of Mathis, Mezger & Panagia 1983).

For the absorption of starlight by dust in the ISM, I use the average extinction cross-section per interstellar H-atom in the diffuse ISM given by Désert, Boulanger & Puget (1990) (their Figure 2), with an albedo for the dust particles of 0.63 (Mezger, Mathis & Panagia 1982). When combined with the gas model given in Section A.1, this gives the extinction cross-section as a function of position in the Galaxy.

The emissivity distribution of the mid to far infrared photon population was calculated by fitting three blackbody curves to the local spectrum with temperatures and dilution coefficients \( T_i = 300 \) K, 100 K and 25 K, and \( W_i = 1.5 \times 10^{-9}, 5.0 \times 10^{-8}, 2.0 \times 10^{-4} \), respectively (see Figure A.2c). The first two components represent emission by polycyclic aromatic hydrocarbons (PAHs) and other particles in the ISM (e.g. Désert, Boulanger & Puget 1990), and the last represents emission by cold dust associated with atomic and molecular hydrogen in the ISM. The decomposition of the mid to far infrared spectrum into a three component blackbody model is slightly ad-hoc, with the main approximation being that of the mid infrared (\( \sim 10 \) to \( \sim 60 \) \( \mu \)m) by the first two blackbody components: only in an approximate way do these represent emission by PAHs and other particles radiating at temperatures of a few hundred Kelvins. However, this wavelength range of the ISRF is less intense than that at shorter and longer wavelengths and so contributes little to the overall energy density of the infrared radiation field. The structure found in this wavelength range in more detailed models (e.g. Dwek et al., 1997) is therefore not important for calculations of electron and positron energy losses, which justifies the approximate modelling of this portion of the spectrum.

The local emissivity spectrum for the mid to far infra-red component is scaled according to the hydrogen distribution given for the gas model described in Section A.1 to obtain the emissivity as a function of position in the Galaxy. Note that absorption only occurs for the stellar component of the ISRF, and so the exponential damping term in Equation A.5
Figure A.2: Wavelength dependence in the galactic plane of the interstellar radiation field at (a) $R = 0$ kpc, (b) $R = 4$ kpc, and (c) $R = 8.5$ kpc. Dashed line gives the contribution by the stellar component, while the dotted and tripe-dot dashed lines give the contribution by the dust related component and the CMBR respectively. For the spectrum corresponding to the local region, the data points below 100 $\mu$m are estimates given by Cox & Mezger (1989) based on IRAS data and 5 $\mu$m data from Matsumoto et al. (1988). The data points at 151 and 241 $\mu$m are based on COBE/DIRBE results from Boulanger & Désert (1992), normalised to the 100 $\mu$m IRAS point (see Strong & Youssefi 1996).
taking this into account does not have to be evaluated when calculating the radiation field
distribution in the mid to far infra red.

Figures A.2a-c show the spectral distribution of the galactic radiation field in the galactic
plane from 0.1 $\mu$m up to the waveband dominated by emission by the CMBR for different
galactocentric radii. The spectrum is dominated by emission from the stellar component in
the inner Galaxy, but has roughly equal contributions by the stellar and mid to far infra red
components for larger radii. Figures A.3a-c show contour maps of the (a) UV and optical,
(b) mid to far infra red, and (c) total ISRF energy density in the $(R, z)$ plane; the contour
increments are given in the Figure legend in units of eV cm$^{-3}$.

### A.2.1 Other Potential Target Photon Populations

The Galaxy also possesses a population of radio photons due to synchrotron radiating elec-
trons, thermal bremsstrahlung and other processes. In principle, this is a potential target
population for inverse Compton scattering. However, as shown below, the number and en-
ergy densities of radio photons are generally too low to be of any significance. To illustrate
this, consider the non-thermal radio emission in the direction of the galactic pole. Broadbent
et al. (1990) obtain a brightness temperature of 12.3 K at 408 MHz for the non-thermal
component, and this corresponds to an intensity of $J_{408\text{MHz}} \approx 6.3 \times 10^{-19}$ erg cm$^{-2}$ s$^{-1}$
sr$^{-1}$ Hz$^{-1}$. The non-thermal emission at lower and high frequencies can be characterised
by a power-law dependence with frequency for the variation of the brightness temperature:
$T_B \propto \nu^{-\beta}$. At 38 MHz, Lawson et al. (1987) give a range $\beta = 0.5 - 0.6$, and at 1420 MHz
Reich & Reich (1988) give $\beta = 0.85 - 1.1$. The number and energy densities of non-thermal
radio photons can then be estimated as

$$
\frac{4\pi}{hc} \int_{20\text{MHz}}^{\infty} \frac{J(\nu)}{\nu} d\nu \approx 0.4 \text{ (cm}^{-3}\text{)}
$$

(A.8)

$$
\frac{4\pi}{c} \int_{20\text{MHz}}^{\infty} J(\nu) d\nu \approx 4 \times 10^{-13} \text{ (eV cm}^{-3}\text{)}
$$

(A.9)

where a low frequency cut off at 20 MHz has been used because the radio spectrum is expected
to be affected by absorption effects below $\sim 10 - 20$ MHz (e.g. Webber 1983). From this
it can be seen that the population of non-thermal radio photons will contribute negligibly
to any inverse Compton losses suffered by cosmic ray electrons during their propagation
Figure A.3: Energy density distribution of the (a) UV to optical, (b) mid to far infra red, and (c) total ISRF (not including the contribution by the CMBR). Units are eV cm$^{-3}$, and the contour intervals are in 0.1 eV cm$^{-3}$ steps starting at 0.1 eV cm$^{-3}$ for the first contour. For each diagram the lowest contour is the rightmost shown.
through the Galaxy. Obviously other low energy populations, such as produced by thermal bremsstrahlung, will contribute similarly negligible amounts because they are comparable in magnitude, or less, to the non-thermal population (see Figure A.4).

![Graph showing the frequency dependence and approximate relative strength of galactic synchrotron, free-free, and dust emission as well as the CMBR - taken from Platania et al. (1998).](image)

Figure A.4: Frequency dependence and approximate relative strength of galactic synchrotron, free-free, and dust emission as well as the CMBR – taken from Platania et al. (1998).

### A.3 Galactic Magnetic Field

The galactic magnetic field (GMF) can be separated into two components: the large scale field (the ‘regular’ component), and the random magnetic field. The regular component is probably associated with the spiral arm structure of the Galaxy (e.g. Beck et al. 1996 and references therein), while the random component is due to the turbulent nature of the ISM. Typically, the random galactic magnetic field varies in direction and magnitude over distances ~ 10 – 100 pc (Rand & Kulkarni 1989; Ohno & Shibata 1993).

At any point in the Galaxy, the magnetic field possesses both length and direction, and hence is a vector quantity. For the GMF, the total magnetic field is given by the sum of the regular and random field component vectors,

\[ \vec{B}_{\text{tot}} = \vec{B}_{\text{reg}} + \vec{B}_{\text{rand}}. \]  

(A.10)
For the calculation of synchrotron losses for cosmic ray electrons and positrons, the magnitude of \( \vec{B}_{\text{tot}} \) is required. At any point in the Galaxy, \((x, y, z)\), this is given by

\[
|B_{\text{tot}}(x, y, z)| = \left[ (B_{x_{\text{reg}}} - B_{x_{\text{rand}}})^2 + (B_{y_{\text{reg}}} - B_{y_{\text{rand}}})^2 + (B_{z_{\text{reg}}} - B_{z_{\text{rand}}})^2 \right]^{1/2}.
\]  (A.11)

The \( x, y, \) and \( z \) components of the random field are found by first sampling the spherical polar angles, \( \phi \) and \( \theta \) from a uniform distribution on the interval \([0, 2\pi]\), and a uniform \( \cos \theta \)
distribution on the interval \([-1, 1]\), and then obtaining the individual components as

\[
B_{x_{\text{rand}}} = |\vec{B}_{\text{rand}}| \zeta \cos \phi \sin \theta
\]  (A.12)

\[
B_{y_{\text{rand}}} = |\vec{B}_{\text{rand}}| \zeta \sin \phi \sin \theta
\]  (A.13)

\[
B_{z_{\text{rand}}} = |\vec{B}_{\text{rand}}| \zeta \cos \theta
\]  (A.14)

where \( \zeta \) is a uniformly sampled normal deviate, and \(|\vec{B}_{\text{rand}}|\) is taken to be 5 \( \mu \)G (Rand & Kulkarni 1989). For the propagation calculations described in Chapter 5, the typical diffusion distance of electrons or positrons is always greater than \( \sim 50 \) pc. This is similar to the typical distance over which the random component varies. So, it is reasonable to simply generate new values for \( \vec{B}_{\text{rand}} \) for the propagation calculations, rather than use the approach of other authors (e.g. Honda 1987) which use a cubic lattice embedded in the Galaxy, and specifically assign a direction and magnitude of the random component for each cell of the lattice.

For the regular component, I use an axisymmetric model with the following parameterisation that fits the data summarised by Beck et al. (1996):

\[
B_{\text{reg}}(R, z)/(\mu \text{G}) = \begin{cases} 
6 \exp \left( -\frac{|l|}{\bar{z}_B} \right) & , R \leq 4 \text{ kpc} \\
(0.73(R - R_S)/\text{kpc} + 3.0) \times \sin(2\pi \left[ \frac{R}{5.8 \text{kpc}} - 1.361 \right]) \exp \left( -\frac{|l|}{\bar{z}_B} \right) & , 4 \text{ kpc} \leq R < 8.5 \text{ kpc} \\
3.0 \sin(2\pi \left[ \frac{R}{5.8 \text{kpc}} - 1.361 \right]) \exp \left( -\frac{|l|}{\bar{z}_B} \right) & , 8.5 \text{ kpc} \leq R < 13.5 \text{ kpc} \\
0 & , R \geq 13.5 \text{ kpc} 
\end{cases}
\]  (A.15)

where \( R_S \) is the galactocentric distance of the Sun from the galactic centre, and \( \bar{z}_B = 1.0 \) kpc (e.g. Giller et al. 1994). The positive sign in Equation A.15 corresponds to the clockwise direction when looking at the galactic plane from the north. Note that there is no explicit
\( z \) component of the regular component in Equation A.15, thus the only \( z \) component of the GMF for the model used in this thesis is given by the random component. Figure A.5 shows a plot of Equation A.15 in the galactic plane.

![Graph](image.png)

Figure A.5: Regular component of the GMF model used in this thesis. Data points: taken from the review by Beck et al. (1996).
Appendix B

Energy Losses of Cosmic Ray Electrons and Positrons

Cosmic-ray electrons and positrons lose energy during propagation in the Galaxy by interactions with gas in the ISM (ionisation/Coulomb and bremsstrahlung), the galactic magnetic field (synchrotron), and ambient radiation fields (inverse Compton). In this Appendix, I give the formulae used to calculate the electron and positron energy losses in the propagation calculations described in Chapter 5.

B.1 Energy Losses due to Interactions with Matter in the ISM

Electrons and positrons interact with the galactic gas distribution, and lose energy through ionisation and Coulomb interactions, and bremsstrahlung losses. To treat ionisation losses in neutral gas I use the Bethe-Bloch formula (e.g. Ginzburg 1979; Longair 1992)

\[
- \left( \frac{dE_e}{dt} \right)_{\text{in}} = \frac{3}{4} \sigma_T c (m_e c^2) \times \sum_{s=\text{H,He}} \frac{n_s Z_s}{\beta} \left\{ \ln \left( \frac{m_e c^2 \gamma_e^2 \beta^2 E_e}{2 I_s^2} \right) - \left( \frac{2}{\gamma_e} - \frac{1}{\gamma_e^2} \right) \ln 2 + \frac{1}{\gamma_e^2} + \frac{1}{8} \left( 1 - \frac{1}{\gamma_e} \right)^2 \right\}
\]

(B.1)

where \( \sigma_T \) is the Thomson cross-section, \( Z_s \) is the nucleus charge, \( n_s \, (\text{cm}^{-3}) \) is the number density of gas of species \( s \), \( I_s \) is the ionisation potential, \( E_e = \gamma m_e c^2 \) is the incident electron
energy with $\gamma_e$ and $m_e c^2$ the electron Lorentz factor and rest mass respectively, and $\beta = v/c$ with $v$ the electron/positron velocity. For the ionisation potentials I take $I_H = 13.6 \text{ eV}$ and $I_{He} = 24.6 \text{ eV}$.

For Coulomb interactions in a fully ionised medium in the cold plasma limit I use (Ginzburg 1979; also Strong & Moskalenko 1998)

$$- \left( \frac{dE_e}{dt} \right)_{\text{i}} = \frac{3}{4} \sigma_e c (m_e c^2) \frac{Z n}{\beta} \left\{ \ln \left( \frac{E_e (m_e c^2)}{4 \pi r_e^2 c^2 Z n} \right) - \frac{3}{4} \right\}$$  \quad (\text{B.2})

where $r_e$ is the electron radius, and $Z n \equiv n_e \text{ (cm}^{-3} \text{)}$ is the electron number density.

Bremsstrahlung losses in a neutral gas can be obtained by integrating over the bremsstrahlung luminosity (e.g. Blumenthal & Gould 1970)

$$- \left( \frac{dE_e}{dt} \right)_{\text{b,n}} = c \beta \sum_{s=\text{H,He}} n_s \int_0^{E_e} E_\gamma \frac{d\sigma_{\text{brem},s}(E_\gamma, E_e)}{dE_\gamma} dE_\gamma$$  \quad (\text{B.3})

where $E_e \text{ (GeV)}$ is the energy of the incident electron, and $d\sigma_{\text{brem},s}/dE_\gamma \text{ (GeV}^{-1} \text{ cm}^2 \text{)}$ is the bremsstrahlung production cross-section (see Equation 5.20) for interactions with gas species $s$.

For neutral gas species, a useful approximation of Equation B.3 is given by Ginzburg (1969)

$$- \left( \frac{dE_e}{dt} \right)_{\text{b,n}} = 4 \alpha r_e^2 c \beta \sum_{s=\text{H,He}} n_s Z_s (Z_s + 1) E_e \begin{cases} \left[ \ln \left( \frac{2E_e}{m_e c^2} \right) - \frac{1}{3} \right] & \frac{E_e}{m_e c^2} \leq 100 \\ \ln \left( 191 Z_s^{-1/3} \right) + \frac{1}{18} & \frac{E_e}{m_e c^2} \geq 1000 \end{cases}$$  \quad (\text{B.4})

where $\alpha$ is the fine structure constant. A linear connection is used to approximate the energy losses between the low and high energy regimes. For bremsstrahlung losses in an ionised medium, the following formula is used (Ginzburg 1979):

$$- \left( \frac{dE_e}{dt} \right)_{\text{b,i}} = 4 \alpha r_e^2 c Z (Z + 1) n E_e \left[ \ln \left( \frac{2E_e}{m_e c^2} \right) - \frac{1}{3} \right].$$  \quad (\text{B.5})

### B.2 Energy Losses from Inverse Compton Scattering

Electrons and positrons interact with the low energy ambient photon populations in the Galaxy, and lose energy through inverse Compton (IC) scattering events. If the scattering process is in the classical (Thomson) regime ($E_e \epsilon \ll m_e c^2$ with $E_e$ and $\epsilon$ the energies of the
incident electron and target photon, respectively), then the energy loss rate is given by the well-known expression

\[- \left( \frac{dE_e}{dt} \right)_{\text{IC}, T} = \frac{4}{3} \sigma \epsilon u \left( \frac{E}{m_e c^2} \right)^2 \tag{B.6}\]

where \( u \) is the energy density of the target radiation field. However, for the ambient radiation fields, and electron energies, considered in this thesis the interaction between an incident electron and target photon is not always in the classical regime. Therefore, the exact Klein-Nishina cross-section must be used to calculate the electron energy loss rate. For a target photon population with number density \( n(\epsilon) \) \((\text{GeV}^{-1} \text{ cm}^{-3})\) the energy loss rate is (Blumenthal & Gould 1970)

\[- \left( \frac{dE_e}{dt} \right)_{\text{IC}} = \int_0^\infty \left\{ \int_\epsilon^{E_{\gamma, \text{max}}} (E_{\gamma} - \epsilon) n(\epsilon) \left( \frac{d\sigma_{\text{IC}}(E_{\gamma}, \epsilon, E_e)}{dE_{\gamma}} \right) dE_{\gamma} \right\} d\epsilon \tag{B.7}\]

where \( \epsilon \) \((\text{GeV})\) is the energy of the target photon, the maximum electron energy,

\[E_{\gamma, \text{max}} = \frac{4\epsilon \gamma_{e}^2 (m_e c^2)}{m_e c^2 + 4\epsilon \gamma_{e}^2}, \tag{B.8}\]

with \( \gamma_e = E_e/(m_e c^2) \) is given by the kinematics of the process, \( E_{\gamma} \) \((\text{GeV})\) is the scattered photon energy, and \( d\sigma_{\text{IC}}/dE_{\gamma} \) \((\text{GeV}^{-1} \text{ cm}^2)\) is given by Equation 5.24.

The energy loss time-scale, \( E/|\frac{dE}{dt}| \), calculated using Equations B.6 and B.7, for IC scattering events on the spectral distribution of the local ISRF (see Figure A.2c) is shown in Figure B.1. Even at incident electron/positron energies as low as \( \sim 10 \) GeV the time-scale calculated using Equation B.7 starts to deviate from the classical formula. This illustrates the importance of using the KN formula for calculating the IC energy loss even for relatively modest electron/positron energies.

### B.3 Energy Losses from Synchrotron Radiation

Electrons and positrons gyrating in a magnetic field lose energy through synchrotron radiation. This process can be understood as Compton scattering by the electrons or positrons of the virtual photons of the static magnetic field in the classical (Thomson) regime. A good description can be found in the paper of Blumenthal & Gould (1970). Equation B.6 can then be used to calculate the energy loss due to synchrotron radiation with the ambient photon energy density, \( u \), replaced by the magnetic field energy density:
Figure B.1: Inverse Compton energy loss time-scale calculated for the local ISRF. Thin solid line shows the total time-scale calculated assuming the scattering events remain in the classical (Thompson) regime (Equation B.6), while the thick solid line shows the total time-scale calculated assuming the full KN cross-section (Equation B.7). Also shown are the time-scales corresponding to IC losses calculated using the KN cross-section for the UV to optical (dashed), mid to far infra red (triple-dot dashed), and CMBR (dotted) components of the local ISRF.

\[ -\left(\frac{dE}{dt}\right)_{\text{sync}} = \frac{4}{3} \sigma_t e B_{\perp}^2 \frac{E}{8\pi} \left(\frac{E}{m_e c^2}\right)^2 \]  

(B.9)

where \(B_{\perp}^2/8\pi\) is the magnetic field energy density, and the angle average over random directions of the electron/positron motion with respect to the direction of the magnetic field has been taken.
Appendix C

Evaluating the Scattering Functions for Bremsstrahlung

The calculation of the bremsstrahlung production spectrum (Equation 5.20) requires the evaluation of the atomic scattering functions $\phi_{1,s}$ and $\phi_{2,s}$ to describe the interaction between electrons/positrons and atoms of gas species $s$. In this Appendix, I describe the calculation of $\phi_{1,s}$ and $\phi_{2,s}$ for atomic hydrogen, helium, and ionised hydrogen. These gas species are the predominant targets in the ISM for the production of bremsstrahlung photons. The general references for this Appendix are the papers by Gould (1969), and Blumenthal & Gould (1970), and the references therein.

C.1 Ionised Hydrogen

For ionised hydrogen the scattering system is essentially a collection of unshielded charges. Hence, for an unshielded charge, $Ze$, $\phi_1 = \phi_2 = Z^2 \phi_U$ applies where

$$
\phi_U(E_\gamma, E_e) = 4 \left\{ \ln \left( \frac{2E_e}{m_e c^2} \right) - \frac{1}{2} \right\},
$$

(C.1)

$E_\gamma$ is the photon energy, $E_e$ is the incident electron energy and $m_e c^2$ is the rest mass of the electron.

C.2 Atomic Hydrogen and Helium

For atomic hydrogen and helium the evaluation of $\phi_1$ and $\phi_2$ is more involved. Following the general procedure outlined in Blumenthal & Gould (1970), each of $\phi_1$ and $\phi_2$ is calculated
as

\[ \phi_i(E_\gamma, E_e) = 4 \int_{q_{\text{min}}}^{\infty} G_i(q; E_\gamma, E_e) \xi(q) dq \]  \hspace{1cm} (C.2)

where \( i = 1, 2 \) and

\[ q_{\text{min}} = \frac{E_\gamma}{2E_e(E_e - E_\gamma)} \]  \hspace{1cm} (C.3)

is the minimum momentum transfer, \( \xi \) the ‘atomic scattering factor’ and \( G_i \) are functions defined by Blumenthal & Gould (1970). \( \xi \) can be expanded in terms of powers of the atomic form factor \( W(q) \), i.e.,

\[ \xi(q) = c_0 + \sum_p c_p(1 - W(q)^p), \]  \hspace{1cm} (C.4)

with \( c_0 \) and \( c_p \) constants. The scattering functions are then given by

\[ \phi_i(E_\gamma, E_e) = c_0 \phi_i(E_\gamma, E_e) + 4 \sum_p c_p \left( \delta_i + \int_{q_{\text{min}}}^{1} G_i(q; E_\gamma, E_e)(1 - W(q)^p) dq \right) \]  \hspace{1cm} (C.5)

where \( \delta_1 = 1 \) and \( \delta_2 = 5/6 \), and the functions \( G_i \) are

\[ G_1(q; E_\gamma, E_e) = q^{-3}[q - q_{\text{min}}]^2 \]  \hspace{1cm} (C.6)

\[ G_2(q; E_\gamma, E_e) = q^{-4}\left[q^3 - 6q_{\text{min}}^2 q \ln(q/q_{\text{min}}) + 3q_{\text{min}}^2 q - 4q_{\text{min}}^3\right] \]  \hspace{1cm} (C.7)

with \( q_{\text{min}} \) given by Equation C.3.

For hydrogen-like atoms or ions (nucleus of charge \( Z \), one electron), \( \xi \) is

\[ \xi(q) = (Z - 1)^2 + 2Z(1 - W_Z(q)), \]  \hspace{1cm} (C.8)

where

\[ W_Z(q) = \frac{1}{(1 + a_Z^2 q^2)^2} \]  \hspace{1cm} (C.9)

and

\[ a_Z = \frac{1}{2\alpha Z} \]  \hspace{1cm} (C.10)
with $\alpha$ the fine structure constant. Thus, the sum in Equation C.4 is up to $p = 1$, and obviously $c_0 = (Z - 1)^2$ and $c_1 = 2Z$.

For helium-like atoms, $\xi$ is

$$\xi(q) = (Z - 2)^2 + 4Z(1 - W_1(q)) - 2(1 - W_1(q)^2) \quad \text{(C.11)}$$

where $W_1(q)$ is found by setting $Z = 1$ in Equation C.9, and with $a_Z$ replaced by

$$a_Z = \frac{1}{2\alpha(Z - 5/16)}. \quad \text{(C.12)}$$

Therefore, the sum in Equation C.4 is up to $p = 2$, and $c_0 = (Z - 2)^2$, $c_1 = 4Z$ and $c_2 = 2$. Note that these forms for $\xi$ has been found by assuming the two-electron wave function of helium-like atoms can be approximated by the product of one-electron Hylleras or Hartree functions.

For large values of $\Delta = a q_{\text{min}}$, however, the scattering functions for both hydrogen and helium approach the unshielded expression

$$\phi_1 = \phi \to (Z^2 + n)\phi_U \quad \text{(C.13)}$$

where $n$ is the number of electrons.
Appendix D

\textbf{γ-Ray, Electron and Positron Production Cross-Sections}

The production of \( \gamma \)-rays, and electrons and positrons from interactions between cosmic ray nuclei and gas atoms at rest is predominantly through the processes \( pp \rightarrow \pi^0 X \) whereupon the neutral pion decays almost instantly into two \( \gamma \)-rays (mean lifetime \( \sim 10^{-16} \) s and branching ratio 98.789\%, Particle Data Group 1990), and \( pp \rightarrow \pi^\pm X \) and the charged pions decay to muons and then electrons or positrons (mean lifetime \( \sim 2.6 \times 10^8 \) s and branching ratio 99.988\%, Particle Data Group 1990) respectively. In this Appendix, the calculation of the production cross-section for these processes is described.

The production of \( \gamma \)-rays is a simple two-stage process

\[
p + p \rightarrow \pi^0 + X
\]
\[
\downarrow
\]
\[
\gamma + \gamma
\]

while the production of secondary electrons or positrons are slightly more complicated processes

\[
p + p \rightarrow \pi^\pm + X
\]
\[
\downarrow
\]
\[
\mu^\pm + \nu_\mu/\bar{\nu_\mu}
\]

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\[ e^\pm + \nu_e/\bar{\nu}_e + \nu_\mu/\bar{\nu}_\mu. \]

The calculation of the first stage of either production process requires the adoption of a model to describe the hadronic interaction, and subsequent production of pions. The interaction model used in the present calculations is described in Section D.1. In Section D.2 the treatment of the second (and third in the case of charged pions) stage for the decay of pions into secondary particles is described. The full production cross-section for all processes is then calculated in Section D.3.

### D.1 Models for Meson Production

The hadronic interaction model used to obtain the production cross-section of neutral and charged pions is described in this Section. A detailed comparison of the pion distributions of various interaction models with accelerator data has been performed by Dermer (1986b). Within the limited statistics of the available data, Dermer has shown that an isobar model (Stecker 1970) provides the best representation of the pion production process for energies from threshold up to an incident proton kinetic energy (KE) \( \sim 3 \) GeV. For incident proton KEs higher than \( \sim 7 \) GeV, scaling models (e.g. Stephens & Badhwar 1981) have been shown to provide an accurate picture of the pion production process (Dermer 1986a). For proton KEs intermediate to these two regimes, it has been conventional to linearly interpolate the pion production spectrum between the predictions of the isobar dominated regime below \( \sim 3 \) GeV, and the scaling model dominated regime above \( \sim 7 \) GeV (Dermer 1986b). The reason for this is no complete secondary pion production data exist in the intermediate energy range with which to compare against model predictions. For the present calculations, this convention has been followed and the pion production spectrum for incident proton KEs 3–7 GeV is linearly interpolated between the model predictions at the boundaries of this energy range. The isobar model used to obtain the pion distribution below 3 GeV (proton KE) is described next, and then the description of the scaling model for the pion distribution above 7 GeV is given.

The isobar model, first described in detail by Stecker (1970), is based on the theory that pion production is mediated through the excitation of the \( \Delta_{3/2} \) isobar. For example, if the secondary particle is a neutral pion, the interaction describing this process is
\[ p + p \rightarrow \Delta^+ + X \]
\[ \downarrow \]
\[ p + \pi^0 \]

whereupon the neutral pion subsequently decays to two photons. In this model the production spectrum of secondary pions with energy \( E^*_\pi \) in the centre-of-momentum system (CMS) by incident protons with laboratory system (LS) energy \( E_p \) is given by the expression (Stecker 1970; Dermer 1986a)

\[
\frac{dN(E^*_\pi, E_p)}{dE^*_\pi} = \Gamma \left[ \arctan \left( \frac{s^{1/2} - m_p - m_\Delta}{\Gamma} \right) - \arctan \left( \frac{m_p + m_\pi - m_\Delta}{\Gamma} \right) \right]^{-1} \tag{D.1}
\]

\[
\times \int_{m_\pi + m_\pi}^{s^{1/2} - m_p} \frac{F^*_\pi(E^*_\pi; E_p, m_\Delta)}{(m_\Delta - m_\Delta^0)^2 + \Gamma^2} dm_\Delta \text{ (GeV}^{-1})
\]

where \( s = 2m_p(E_p + m_p) \) is the total CMS energy squared, \( m_p \) and \( m_\pi \) are the proton and pion masses, and \( m_\Delta^0 \) and \( \Gamma \) are the most probable mass and width of the delta isobar resonance. The integration in Equation D.2 is over the isobar mass spectrum which is a Breit-Wigner form with peak \( m_\Delta^0 \) and width \( \Gamma \). The only resonance considered is the \( \Delta_{3/2}(1232) \) for which \( m_\Delta^0 = 1.232 \text{ GeV} \) and \( \Gamma = 0.115/2 \text{ GeV} \) (Particle Data Group 1992). The function \( F^*_\pi(E^*_\pi; E_p, m_\Delta) \) is the energy spectrum of pions in the CMS, and has the form

\[
F^*_\pi(E^*_\pi; E_p, m_\Delta) = \frac{1}{2\beta_\Delta^* \gamma_\Delta^* p'_\pi} \times H[E^*_\pi; \gamma_\Delta^*(E^*_\pi - \beta_\Delta^* p'_\pi), \gamma_\Delta^*(E^*_\pi + \beta_\Delta^* p'_\pi)] \text{ (GeV}^{-1}) \tag{D.2}
\]

where \( H[x; a, b] \) is the Heaviside function and is equal to 1 if \( a \leq x \leq b \) and 0 otherwise, \( \gamma_\Delta^* = (s + m_\Delta^2 - m_p^2)/2m_\Delta s^{1/2} \) is the Lorentz factor of the isobar in the CMS, and \( E'_\pi = (m_\Delta^2 + m_\pi^2 - m_p^2)/2m_\Delta \), and \( p'_\pi \) are the pion energy and momentum respectively in the rest frame of the isobar.

For the present calculations, the production spectrum of pions in the lab system (LS) is required. The corresponding expressions to Equation D.2 and D.3 describing the production
spectrum of pions with energy \( E_\pi \) in the LS by incident protons with energy \( E_p \) are (Dermer 1986b)

\[
\frac{dN(E_\pi, E_p)}{dE_\pi} = \Gamma \left[ \arctan \left( \frac{s^{1/2} - m_p - m_0}{\Gamma} \right) - \arctan \left( \frac{m_p + m_\pi - m_0}{\Gamma} \right) \right]^{-1} (D.3)
\]

\[
\times \int_{m_p + m_\pi}^{s^{1/2} - m_p} \frac{F_\pi(E_\pi; E_p, m_\Delta)}{(m_\Delta - m_0^2 + \Gamma^2) d\Delta} \text{ (GeV}^{-1})
\]

and

\[
F_\pi(E_\pi; E_p, m_\Delta) = \frac{1}{2p_\pi} \left\{ \frac{1}{2\beta_\Delta \gamma_\Delta} H[E_\pi; \gamma_\Delta(E'_\pi - \beta_\Delta p'_\pi), \gamma_\Delta(E'_\pi + \beta_\Delta p'_\pi)] \right. \\
+ \frac{1}{2\beta_\Delta \gamma_\Delta} H[E_\pi; \gamma_\Delta(E'_\pi - \beta_\Delta p'_\pi), \gamma_\Delta(E'_\pi + \beta_\Delta p'_\pi)] \right\} \text{ (GeV}^{-1}) \tag{D.4}
\]

where the Lorentz factors of the forward (+) and backward (−) moving isobars are \( \gamma_\Delta = \gamma_c \gamma_\Delta(1 \pm \beta_\Delta \gamma_\Delta) \), and \( \gamma_c = s^{1/2}/2m_p \) is the Lorentz factor of the CMS with respect to the LS. In this model the outgoing isobars produced in the collision are assumed to travel collinear to the initial direction of the colliding protons in the CMS, and decay isotropically.

The model described by Stephens & Badhwar (1981) is based on scaling arguments that imply asymptotic forms in the high energy limit for the differential cross-sections for secondary production in inelastic collisions. The Lorentz invariant cross-sections \( E_\pi^* (d^3 \sigma^* / dp_\pi^3) \) for charged and neutral pion production in \( pp \) collisions inferred from experimental data at \( E_p \geq 13.5 \text{ GeV} \) are given by Badhwar, Stephens & Golden (1977) and Stephens & Badhwar (1981). In this model the normalised production spectrum of pions with energy \( E_\pi \) in the LS by incident protons with energy \( E_p \) is obtained by integrating over the LS polar emission angle

\[
\frac{dN(E_\pi, E_p)}{dE_\pi} = \frac{2\pi p_\pi}{\langle \zeta \sigma_\pi(E_p) \rangle} \int_0^\Delta d(\cos \theta^*_{\pi}) E_\pi^* \left( \frac{d^3 \sigma^*}{dp_\pi^3} \right) \text{ (GeV}^{-1}) \tag{D.5}
\]

where \( \langle \zeta \sigma_\pi(E_p) \rangle \) is the multiplicity weighted cross-section for the process \( pp \rightarrow \pi X \) in the scaling model, and \( \cos \theta^*_{\pi} = [\beta_c E_\pi - E^*_{\max}(s)]/\beta_c \gamma_c p_\pi \), and \( E^*_{\max}(s) = (s - m_X^2 + m_\pi^2)/2s^{1/2} \) is the maximum energy of the meson in the CMS, and \( m_X \) depends on the interaction channel: \( m_X = 2m_p \) for \( pp \rightarrow \pi^0 X \), \( m_X = 2m_p + m_\pi \) for \( pp \rightarrow \pi^- X \), \( m_X = m_p + m_n \) for \( pp \rightarrow \pi^+ X \) and \( m_X = m_d \) for \( pp \rightarrow \pi^+ d \). The Lorentz invariant cross-sections have the form
\[ E^*_\pi \left( \frac{d^2 \sigma^*}{dp^*_\pi} \right) = A F_\pi(E_p)(1 - x_\pi)^Q \exp \left[ -\frac{Bp_{\pi\perp}}{1 + 4m_p^2/s} \right] \]  

\quad (D.6)

where

\[ F_{\pi^0}(E_p) = (1 + 23E_p^{-2.6})(1 - 4m_p^2/s)^R \quad (D.7) \]

\[ F_{\pi^\pm}(E_p) = (1 + 4m_p^2/s)^{-R} \]

\[ Q = \frac{C_1 + C_2p_{\pi\perp} + C_3p_{\pi\perp}^2}{[1 + 4m_p^2/s]^{1/2}} \]

\[ x_\pi = \left[ x_\pi^2 + (4/s)(p_{\pi\perp}^2 + m_\pi^2) \right]^{1/2} \]

\[ x_{\pi\perp} = \frac{\gamma_{\pi\beta}p_{\pi\perp}}{[(s - m_\pi^2 - m_\pi^2)^2 - 4m_\pi^2m_\pi^2]^{1/2}/2s^{1/2}} \]

and the constants \( A, B, C_1, C_2, C_3 \) and \( R \) are equal to: 140, 5.43, 6.1, -3.3, 0.6 and 2 respectively for neutral pions; 153, 5.55, 5.3667, -3.5, 0.8334 and 1 respectively for positively charged pions; and 127, 5.3, 7.0334, -4.5, 1.667 and 3 respectively for negatively charged pions. Note that the pion energy distribution for both the isobar and scaling models is normalised so that \( \int_0^\infty dE_\pi dN(E_\pi, E_p)/dE_\pi = 1. \)

### D.2 Treatment of Pion Decay to Secondaries

The interaction model was described in the previous Section. To treat the decay of the pions produced in the hadronic process, the secondary particle distributions must be used. For neutral pions, the decay \( \pi^0 \rightarrow 2\gamma \) is a two body decay with an isotropic distribution in the CMS. The LS distribution of \( \gamma \)-rays resulting from the decay of neutral pions is therefore (e.g. Stecker 1970)

\[ F(E_\gamma, E_{\pi0}) = \frac{2}{p_{\pi0}}, \quad \frac{1}{2}(E_{\pi0} - p_{\pi0}) \leq E_\gamma \leq \frac{1}{2}(E_{\pi0} + p_{\pi0}) \quad (D.8) \]

where the factor 2 accounts for two photons per decay. For charged pions, the muons created in the decay are fully polarised, and this results in an asymmetry in the decay to electrons and positrons. Firstly, the LS distribution of muons resulting from the decay of charged pions is

\[ F(E_\mu, E_{\pi\pm}) = \frac{m_{\pi\pm}}{2p_{\pi\perp}^2p_\mu}, \quad m_\mu \gamma_\mu^- \leq E_\mu \leq m_\mu \gamma_\mu^+ \quad (D.9) \]
where $E_\mu$ is the energy of the muon in the LS, $\gamma_\mu^\pm = \gamma_e \gamma_\mu^e (1 \pm \beta_e \beta_\mu^e)$, $\gamma_\mu^e = (m_e^2 + m_\mu^2)/2m_e m_\mu \approx 1.039$ and $\beta_\mu^e \approx 0.2712$ are the muon Lorentz factor and velocity in the rest frame of the pion. Now the distribution of electrons/positrons in the muon rest frame, assuming a massless electron, is (e.g. Dermer 1986a)

\[ F_\xi^e (E_e', \cos \theta') = \frac{2\epsilon^2 (3 - 2\epsilon)}{m_\mu} \left[ 1 - \xi \left( \frac{1 - 2\epsilon}{3 - 2\epsilon} \cos \theta \right) \right] \]  

(D.10)

where $\epsilon = 2E_e'/m_\mu$, $E_e'$ is the electron/positron energy in the muon rest frame, and $\theta'$ is the angle between the direction of the electron momentum and the polarization direction of the parent muon. In Equation D.10, $\xi = \pm 1$ for $e^\pm$, and $\xi = 0$ for an isotropic distribution in the muon rest frame. The LS distribution of electrons/positrons is obtained by integrating over the polar emission angle as (e.g. Moskalenko & Strong 1998)

\[ F_\xi (E_e, E_\mu) = \int_{\cos \theta_{\text{min}}(E_e, E_\mu)}^1 F_\xi^e (E_e', \cos \theta') \frac{\partial (E_e', \cos \theta ')}{\partial (E_e, \cos \theta)} d(\cos \theta) \]  

(D.11)

where $\cos \theta_{\text{min}}(E_e, E_\mu) = \max \left\{ -1, \left( E_\mu - m_\mu^2 / 2E_e \right) / p_\mu \right\}$, $E_\mu = m_\mu \gamma_\mu$ and $p_\mu = E_\mu \beta_\mu$ are the muon energy and momentum in the LS with $\gamma_\mu$ and $\beta_\mu$ the muon Lorentz factor and velocity respectively, and

\[ \frac{\partial (E_e', \cos \theta')}{\partial (E_e, \cos \theta)} = \begin{vmatrix} \gamma_\mu (1 - \beta_\mu \cos \theta) & -\beta_\mu \gamma_\mu E_e \\ 0 & \frac{1 - \beta_\mu^2}{(1 - \beta_\mu \cos \theta)} \end{vmatrix} = \frac{\gamma_\mu (1 - \beta_\mu^2)}{1 - \beta_\mu \cos \theta} = \frac{1}{\gamma_\mu (1 - \beta_\mu \cos \theta)} \]  

(D.12)

where $E_e' = E_e \gamma_\mu (1 - \beta_\mu \cos \theta)$, $\cos \theta' = (\cos \theta - \beta_\mu)/(1 - \beta_\mu \cos \theta)$. The compound distribution function $F(E_e, E_{e^\pm}) = F(E_e, E_\mu) F(E_\mu, E_{\pi^\pm})$ can be obtained by combining the individual decay distributions given by Equations D.9 through D.11. Therefore, the distribution of electrons/positrons from the decay of charged pions can be evaluated as (e.g. Dermer 1986a; Moskalenko & Strong 1998)

\[ F_\xi (E_e, E_{e^\pm}) = \frac{m_{\pi^\pm}}{p_{\pi^\pm} \gamma_\mu} \begin{cases} P_\xi (\gamma_\mu^+) - P_\xi (\gamma_\mu^-) & 0 \leq E_e < E_1 \\ P_\xi (\gamma_1) - P_\xi (\gamma_\mu^-) + P_\xi (\gamma_\mu^+) - Q_\xi (\gamma_1) & E_1 \leq E_e < E_2 \\ Q_\xi (\gamma_\mu^+) - Q_\xi (\gamma_\mu^-) & E_2 \leq E_e < E_3 \\ Q_\xi (\gamma_\mu^+) - Q_\xi (\gamma_1) & E_3 \leq E_e \leq E_4 \end{cases} \]  

(D.13)
where \( \gamma^\pm \) was given earlier, \( \gamma_1 = E_e/m_\mu + m_\mu/4E_e \), and \( E_1 = m_\mu/2\gamma^+_\mu(1 + \beta^+_\mu) \), \( E_2 = m_\mu/2\gamma^-_\mu(1 + \beta^-_\mu) \), \( E_3 = m_\mu/2\gamma^-_\mu(1 - \beta^-_\mu) \), \( E_4 = m_\mu/2\gamma^+_\mu(1 - \beta^+_\mu) \), and the functions \( P \) and \( Q \) are

\[
P_\xi(\gamma) = \frac{m_\mu}{2} \int_1^\gamma d\gamma \int_{-1}^1 \frac{F'_\xi(E'_e, \cos \theta)}{\gamma_\mu(1 - \beta_\mu \cos \theta)} d(\cos \theta)
\]

(D.14)

\[
Q_\xi(\gamma) = \frac{m_\mu}{2} \int_1^\gamma d\gamma \int_{-1}^1 \frac{F'_\xi(E'_e, \cos \theta)}{(E_\mu - m_\mu/2E_e)/\gamma_\mu(1 - \beta_\mu \cos \theta)} d(\cos \theta).
\]

(D.15)

Performing the integrations for \( P \) and \( Q \) gives (Moskalenko & Strong 1998)

\[
P_{\pm}(\gamma) = \frac{4}{9} \left( \frac{E_e}{m_\mu} \right)^2 \left\{ \frac{E_e}{m_\mu} [-32\gamma^3(1 \pm \beta) + \gamma (24 \pm 32\beta)] + \gamma^2 (27 \pm 9\beta) \mp 9 \ln[\gamma (1 \pm \beta)] \right\}
\]

(D.16)

\[
P_0(\gamma) = \left( \frac{E_e}{m_\mu} \right)^2 \left\{ \left( \frac{E_e}{m_\mu} \right) \left( -\frac{128}{9} \gamma^3 + \frac{32}{3} \gamma \right) + 12\gamma^2 \right\}
\]

(D.17)

\[
Q_+(\gamma) = \frac{1}{12} \left\{ \left( \frac{E_e}{m_\mu} \right)^3 \left[ 16 \ln \left( \frac{\gamma + 1}{\gamma - 1} \right) - 64\gamma (1 - \beta) \right]
\]

+ \left( \frac{E_e}{m_\mu} \right)^2 \left[ 48\gamma^2 (1 - \beta) + 24 \ln \left( \frac{\beta}{1 + \beta} \right) - 2 \ln(\gamma \beta) + 10 \ln[\gamma (1 + \beta)] \right] \right\}
\]

(D.18)

\[
Q_-(\gamma) = \frac{1}{36} \left\{ \left( \frac{E_e}{m_\mu} \right)^3 \left[ -512\gamma^3 (1 - \beta) + \gamma (576 - 320\beta) - 48 \ln \left( \frac{\gamma + 1}{\gamma - 1} \right) \right]
\]

+ \left( \frac{E_e}{m_\mu} \right)^2 \left[ 288\gamma^2 (1 - \beta) - 72 \ln \left( \frac{\beta}{1 + \beta} \right) \right] + 6 \ln(\gamma \beta) + 30 \ln[\gamma (1 + \beta)] \right\}
\]

(D.19)

\[
Q_0(\gamma) = \frac{1}{18} \left\{ \left( \frac{E_e}{m_\mu} \right)^3 \left[ -128\gamma^3 (1 - \beta) + \gamma (96 - 32\beta) \right]
\]

+ \left( \frac{E_e}{m_\mu} \right)^2 \left[ 108\gamma^2 (1 - \beta) + 15 \ln[\gamma (1 + \beta)] \right] \right\}
\]

(D.20)

### D.3 Calculation of Production Cross-Sections

Using the hadronic interaction model described in Section D.1, and the treatment of the pion decay given in Section D.2, the production cross-sections for \( \gamma \)-rays, electrons and
positrons are obtained. Given the pion energy distribution for an incident proton energy \( E_p \), \( dN(E_\pi, E_p)/dE_\pi \), the cross-section for the reaction \( pp \rightarrow \pi X, < \zeta\sigma_\pi(E_p) > \), and the LS distributions of the appropriate secondary particles, the cross-section for the processes \( pp \rightarrow e^\pm X \) or \( pp \rightarrow \gamma X \) may be written (e.g. Moskalenko & Strong 1998)

\[
\frac{d\sigma(E_\gamma, E_p)}{dE_\gamma} = \langle \zeta\sigma_\pi(E_p) \rangle \int_{E_\pi^{\min}(E_\gamma)}^{E_\pi^{\max}(E_\gamma)} F(E_\gamma, E_\pi) \frac{dN(E_\pi, E_p)}{dE_\pi} \text{ (GeV}^{-1} \text{ cm}^2) \quad (D.21)
\]

\[
\frac{d\sigma(E_e, E_p)}{dE_e} = \langle \zeta\sigma_{\pi\pm}(E_p) \rangle \int_{E_{\pi\pm}^{\min}(E_e)}^{E_{\pi\pm}^{\max}(E_e)} F_{\pi\pm}(E_e, E_{\pi\pm}) \frac{dN(E_{\pi\pm}, E_p)}{dE_{\pi\pm}} \text{ (GeV}^{-1} \text{ cm}^2) \quad (D.22)
\]

where the functions \( F(E_\gamma, E_\pi) \) and \( F_{\pi\pm}(E_e, E_{\pi\pm}) \) are given by Equations D.8 and D.13 respectively. The minimum pion energy contributing to the production of a secondary with energy \( E_s \) (i.e. \( E_s = E_\gamma \) or \( E_e \)) is \( E_{\pi\pm}^{\min} = E_\gamma + m_{\pi\pm}/4E_\gamma \) for \( \gamma \)-rays, and \( E_{\pi\pm}^{\min} = m_{\pi\pm} \) for \( E_e \leq E \equiv \frac{1}{4}m_\mu \gamma_{\mu}(1 + \beta_{\mu}^2) \), \( E_{\pi\pm}^{\max} = \frac{1}{4}m_{\pi\pm} (E_e/E + E/E_e) \) if \( E_e > E \) for electrons/positrons with \( \gamma_{\mu} = (m_{\pi\pm}^2 + m_\mu^2)/2m_{\pi\pm}m_\mu \simeq 1.039 \) and \( \beta_{\mu} \simeq 0.2712 \) the muon Lorentz factor and velocity in the pion rest frame respectively. The maximum pion energy for either process can be found by equating \( s^{1/2} = \left[ \frac{2m_\chi^2 + E_{\pi\max}^*(s) - m_\pi^2}{m_\pi} \right]^{1/2} + \frac{E_{\pi\max}^*(s)}{s} \), where \( s \) is the square of the total CMS energy, \( E_{\pi\max}^*(s) \) is the maximum pion energy in the CMS, and \( m_\chi \) depends on the reaction channel (see Section D.1 for the definition of these terms and values for \( m_\chi \)). For the inclusive cross-section for each process, define the parameter

\[
\eta = \frac{p_{\pi\max}^*(s)}{m_\pi} = \frac{[(E_{\pi\max}^*(s) - m_\pi^2)^{1/2}]^2}{m_\pi} = \frac{[(s - m_\pi^2 - m_\chi^2)^2 - 4m_\pi^2m_\chi^2]^{1/2}}{2m_\pi s^{1/2}} \quad (D.23)
\]

where \( m_\pi \) is the mass of either the neutral or charged pion. The inclusive cross-section for each process is then given by the following formulae, obtained by Dermer (1986a) and Mori (1997) (for neutral pions) from fits to experimental cross-section data:

\[
\langle \sigma_{\pi^\pm = \pi^\pm X}(E_p) \rangle \text{ (mb)} = \begin{cases} 
0.032\eta^2 + 0.040\eta^6 + 0.047\eta^8 & p_p^{\text{thr}} \leq p_p < 0.96 \\
32.6(p_p - 0.8)^{3.21} & 0.96 \leq p_p < 1.27 \\
5.40(p_p - 0.8)^{0.81} & 1.27 \leq p_p < 8.0 \\
32.0\ln p_p + 48.5p_p^{-1/2} - 59.5 & 8.0 \leq p_p < 1000 \\
163(s/1876 \text{ GeV}^2)^{0.21} & 1000 \leq p_p 
\end{cases} \quad (D.24)
\]
\[
\left\langle \sigma_{pp \rightarrow \pi^+ X} (E_p) \right\rangle \text{ (mb) } = \begin{cases} 
0.95 \eta^4 + 0.099 \eta^6 + 0.204 \eta^8 & p_p^{\text{thr}} \leq p_p < 0.95 \\
0.67 \eta^{4.7} + 0.3 & 0.95 \leq p_p < 1.29 \\
22.0(p_p - 1.27)^{0.15} & 1.29 \leq p_p < 4.0 \\
27.0 \ln p_p + 57.9 p_p^{-1/2} - 40.9 & 4.0 \leq p_p 
\end{cases} \tag{D.25}
\]

\[
\left\langle \sigma_{pp \rightarrow \pi^+ d} (E_p) \right\rangle \text{ (mb) } = \begin{cases} 
0.18 \eta + 0.95 \eta^3 - 0.016 \eta^9 & E_k^{\text{thr}} \leq E_k < 0.65 \\
0.56 E_k^{-3.9} & 0.65 \leq E_k < 1.43 \\
0.34 E_k^{-2.5} & 1.43 \leq E_k 
\end{cases} \tag{D.26}
\]

\[
\left\langle \sigma_{pp \rightarrow \pi^- X} (E_p) \right\rangle \text{ (mb) } = \begin{cases} 
2.33(p_p - 1.65)^{1.2} & 1.65 \leq p_p < 2.81 \\
0.32 p_p^{2.1} & 2.81 \leq p_p < 5.52 \\
28.2 \ln p_p + 74.2 p_p^{-1/2} - 69.3 & 5.52 \leq p_p 
\end{cases} \tag{D.27}
\]

where \( p_p \) and \( E_k \) are in GeV units. Note that for the process \( pp \rightarrow \pi^+ X \), it is understood for the inclusive cross-section given by Equation D.25 that the process \( pp \rightarrow \pi^+ d \) is excluded. The cross-sections given by Equations D.24 through D.27 are shown in Figure D.1. The production cross-sections for \( \gamma \)-rays, and electrons and positrons, obtained using Equation D.21 and D.22 are shown in Figure D.2.
Figure D.1: Inclusive cross-sections as a function of incident proton momentum for the processes $pp \rightarrow \pi^0X$ (solid line), $pp \rightarrow \pi^+X$ (dashed line), $pp \rightarrow \pi^+d$ (dash-dotted line), and $pp \rightarrow \pi^-X$ (dotted line).
Figure D.2: Differential cross sections for the processes (a) $pp \rightarrow \pi^0X \rightarrow 2\gamma$, (b) $pp \rightarrow \pi^+X \rightarrow e^+$, and (c) $pp \rightarrow \pi^-X \rightarrow e^-$ as functions of the incident proton and secondary particle energy, calculated using the method described in the text. Cross-sections have been cut off for values less than $10^{-30}$ cm$^2$ GeV$^{-1}$ for display purposes only.
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